EXERCÍCIOS 1.2 ÁLGEBRA LINEAR, BERNARD KOLMAN, EDITORA GUANABARA, 3ª EDIÇÃO

EXERCÍCIO 1 (PÁGINA 17)

1.A $a_{12} = -3$
$a_{22} = -5$
$a_{23} = 4$

1.B $b_{11} = 4$
$b_{31} = 5$

1.C $c_{13} = 2$
$c_{31} = 6$
$c_{33} = -1$

EXERCÍCIO 2 (PÁGINA 17)

2.A

$$C+E = \begin{bmatrix} 3+2 & -1-4 & 3+5 \\ 4+0 & 1+1 & 5+4 \\ 2+3 & 1+2 & 3+1 \end{bmatrix} = \begin{bmatrix} 5 & -5 & 8 \\ 4 & 2 & 9 \\ 5 & 3 & 4 \end{bmatrix}$$

2.B

$$AB = \begin{bmatrix} 1(1)+2(2)+3(3) & 1(0)+2(1)+3(2) \\ 2(1)+1(2)+4(3) & 2(0)+1(1)+4(2) \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+9 & 0+2+6 \\ 2+2+12 & 0+1+8 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 8 \\ 16 & 9 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1(1)+0(2) & 1(2)+0(1) & 1(3)+0(4) \\ 2(1)+1(2) & 2(2)+1(1) & 2(3)+1(4) \\ 3(1)+2(2) & 3(2)+2(1) & 3(3)+2(4) \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 2+0 & 3+0 \\ 2+2 & 4+1 & 6+4 \\ 3+4 & 6+2 & 9+8 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 10 \\ 7 & 8 & 17 \end{bmatrix}$$

2.C

$$2D = \begin{bmatrix} 2(3) & 2(-2) \\ 2(2) & 2(4) \end{bmatrix} = \begin{bmatrix} 6 & -4 \\ 4 & 8 \end{bmatrix}$$

$$3F = \begin{bmatrix} 3(-4) & 3(15) \\ 3(2) & 3(3) \end{bmatrix} = \begin{bmatrix} -12 & 45 \\ 6 & 9 \end{bmatrix}$$

$$2D - 3F = \begin{bmatrix} 6-(-12) & -4-45 \\ 4-6 & 8-9 \end{bmatrix} = \begin{bmatrix} 18 & -49 \\ -2 & -1 \end{bmatrix}$$

2.D

$$CB = \begin{bmatrix} 3(1)+(-1)(2)+3(3) & 3(0)+(-1)(1)+3(2) \\ 4(1)+1(2)+5(3) & 4(0)+1(1)+5(2) \\ 2(1)+1(2)+3(3) & 2(0)+1(1)+3(2) \end{bmatrix}$$

$$= \begin{bmatrix} 3-2+9 & 0-1+6 \\ 4+2+15 & 0+1+10 \\ 2+2+9 & 0+1+6 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 5 \\ 21 & 11 \\ 13 & 7 \end{bmatrix}$$

CB + D NÃO ESTÁ DEFINIDO POIS O PRODUTO CB É DE FORMA DIFERENTE DE D.

2.E

$$AB = \begin{bmatrix} 1(1)+2(2)+3(3) & 1(0)+2(1)+3(2) \\ 2(1)+1(2)+4(3) & 2(0)+1(1)+4(2) \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+9 & 0+2+6 \\ 2+2+12 & 0+1+8 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 8 \\ 16 & 9 \end{bmatrix}$$

$$DF = \begin{bmatrix} 3(-4)+(-2)(2) & 3(5)+(-2)(3) \\ 2(-4)+4(2) & 2(5)+4(3) \end{bmatrix}$$

$$DF = \begin{bmatrix} -12-4 & 15-6 \\ -8+8 & 10+12 \end{bmatrix}$$

$$= \begin{bmatrix} -16 & 9 \\ 0 & 22 \end{bmatrix}$$

$$AB + DF = \begin{bmatrix} 14+(-16) & 8+9 \\ 16+0 & 9+22 \end{bmatrix} = \begin{bmatrix} -2 & 17 \\ 16 & 31 \end{bmatrix}$$

2.F $2A = \begin{bmatrix} 2(1) & 2(2) & 2(3) \\ 2(2) & 2(1) & 2(4) \end{bmatrix} = \begin{bmatrix} 2 & 4 & 6 \\ 4 & 2 & 8 \end{bmatrix}$

$$(3)(2A) = \begin{bmatrix} 3(2) & 3(4) & 3(6) \\ 3(4) & 3(2) & 3(8) \end{bmatrix} = \begin{bmatrix} 6 & 12 & 18 \\ 12 & 6 & 24 \end{bmatrix}$$

$$6A = \begin{bmatrix} 6(1) & 6(2) & 6(3) \\ 6(2) & 6(1) & 6(4) \end{bmatrix} = \begin{bmatrix} 6 & 12 & 18 \\ 12 & 6 & 24 \end{bmatrix}$$

EXERCÍCIO 3 (PÁGINA 17)

3.A

$$BD = \begin{bmatrix} 1(3)+0(2) & 1(-2)+0(4) \\ 2(3)+1(2) & 2(-2)+1(4) \\ 3(3)+2(2) & 3(-2)+2(4) \end{bmatrix}$$

$$= \begin{bmatrix} 3+0 & -2+0 \\ 6+2 & -4+4 \\ 9+4 & -6+8 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 8 & 0 \\ 13 & 2 \end{bmatrix}$$

$$A(BD) = \begin{bmatrix} 1(3)+2(8)+3(13) & 1(-2)+2(0)+3(2) \\ 2(3)+1(8)+4(13) & 2(-2)+1(0)+4(2) \end{bmatrix}$$

$$= \begin{bmatrix} 3+16+39 & -2+0+6 \\ 6+8+52 & -4+0+8 \end{bmatrix} = \begin{bmatrix} 58 & 4 \\ 66 & 4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1(1)+2(2)+3(3) & 1(0)+2(1)+3(2) \\ 2(1)+1(2)+4(3) & 2(0)+1(1)+4(2) \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+9 & 0+2+6 \\ 2+2+12 & 0+1+8 \end{bmatrix} = \begin{bmatrix} 14 & 8 \\ 16 & 9 \end{bmatrix}$$

$$(AB)D = \begin{bmatrix} 14(3)+8(2) & 14(-2)+8(4) \\ 16(3)+9(2) & 16(-2)+9(4) \end{bmatrix}$$

$$= \begin{bmatrix} 42+16 & -28+32 \\ 48+18 & -32+36 \end{bmatrix} = \begin{bmatrix} 58 & 4 \\ 66 & 4 \end{bmatrix}$$

3-B

$$C+E = \begin{bmatrix} 3+2 & -1+(-4) & 3+5 \\ 4+0 & 1+1 & 5+4 \\ 2+3 & 1+2 & 3+1 \end{bmatrix} = \begin{bmatrix} 5 & -5 & 8 \\ 4 & 2 & 9 \\ 5 & 3 & 4 \end{bmatrix}$$

$$A(C+E) = \begin{bmatrix} 1(5)+2(4)+3(5) & 1(-5)+2(2)+3(3) & 1(8)+2(9)+3(4) \\ 2(5)+1(4)+4(5) & 2(-5)+1(2)+4(3) & 2(8)+1(9)+4(4) \end{bmatrix}$$

$$= \begin{bmatrix} 5+8+15 & -5+4+9 & 8+18+12 \\ 10+4+20 & -10+2+12 & 16+9+16 \end{bmatrix}$$

$$= \begin{bmatrix} 28 & 8 & 38 \\ 34 & 4 & 41 \end{bmatrix}$$

$$AC = \begin{bmatrix} 1(3)+2(4)+3(2) & 1(-1)+2(1)+3(1) & 1(3)+2(5)+3(3) \\ 2(3)+1(4)+4(2) & 2(-1)+1(1)+4(1) & 2(3)+1(5)+4(3) \end{bmatrix}$$

$$= \begin{bmatrix} 3+8+6 & -1+2+3 & 3+10+9 \\ 6+4+8 & -2+1+4 & 6+5+12 \end{bmatrix} = \begin{bmatrix} 17 & 4 & 22 \\ 18 & 3 & 23 \end{bmatrix}$$

$$AE = \begin{bmatrix} 1(2)+2(0)+3(3) & 1(-4)+2(1)+3(2) & 1(5)+2(4)+3(1) \\ 2(2)+1(0)+4(3) & 2(-4)+1(1)+4(2) & 2(5)+1(4)+4(1) \end{bmatrix}$$

$$= \begin{bmatrix} 2+0+9 & -4+2+6 & 5+8+3 \\ 4+0+12 & -8+1+8 & 10+4+4 \end{bmatrix} = \begin{bmatrix} 11 & 4 & 16 \\ 16 & 1 & 18 \end{bmatrix}$$

$$AC + AE = \begin{bmatrix} 17+11 & 4+4 & 22+16 \\ 18+16 & 3+1 & 23+18 \end{bmatrix} = \begin{bmatrix} 28 & 8 & 38 \\ 34 & 4 & 41 \end{bmatrix}$$

3.C $$3A = \begin{bmatrix} 3(1) & 3(2) & 3(3) \\ 3(2) & 3(1) & 3(4) \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 6 & 3 & 12 \end{bmatrix}$$

$$2A = \begin{bmatrix} 2(1) & 2(2) & 2(3) \\ 2(2) & 2(1) & 2(4) \end{bmatrix} = \begin{bmatrix} 2 & 4 & 6 \\ 4 & 2 & 8 \end{bmatrix}$$

$$3A + 2A = \begin{bmatrix} 3+2 & 6+4 & 9+6 \\ 6+4 & 3+2 & 12+8 \end{bmatrix} = \begin{bmatrix} 5 & 10 & 15 \\ 10 & 5 & 20 \end{bmatrix}$$

$$5A = \begin{bmatrix} 5(1) & 5(2) & 5(3) \\ 5(2) & 5(1) & 5(4) \end{bmatrix} = \begin{bmatrix} 5 & 10 & 15 \\ 10 & 5 & 20 \end{bmatrix}$$

3.D $$DF = \begin{bmatrix} 3(-4)+(-2)(2) & 3(5)+(-2)(3) \\ 2(-4)+4(2) & 2(5)+4(3) \end{bmatrix}$$

$$= \begin{bmatrix} -12-4 & 15-6 \\ -8+8 & 10+12 \end{bmatrix} = \begin{bmatrix} -16 & 9 \\ 0 & 22 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1(1)+2(2)+3(3) & 1(0)+2(1)+3(2) \\ 2(1)+1(2)+4(3) & 2(0)+1(1)+4(2) \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+9 & 0+2+6 \\ 2+2+12 & 0+1+8 \end{bmatrix} = \begin{bmatrix} 14 & 8 \\ 16 & 9 \end{bmatrix}$$

$$DF + AB = \begin{bmatrix} -16+14 & 9+8 \\ 0+16 & 22+9 \end{bmatrix} = \begin{bmatrix} -2 & 17 \\ 16 & 31 \end{bmatrix}$$

3.E EF + 2A NÃO ESTÁ DEFINIDO POIS O PRODUTO EF NÃO É POSSÍVEL DEVIDO AO NÚMERO DE LINHAS DE 'F' SER DIFE-

RENTE DO NÚMERO DE COLUNAS DE 'E'.

3.F

$$3C=\begin{bmatrix}3(3) & 3(-1) & 3(3)\\ 3(4) & 3(1) & 3(5)\\ 3(2) & 3(1) & 3(3)\end{bmatrix}=\begin{bmatrix}9 & -3 & 9\\ 12 & 3 & 15\\ 6 & 3 & 9\end{bmatrix}$$

$$(-4)(3C)=\begin{bmatrix}-4(9) & -4(-3) & -4(9)\\ -4(12) & -4(3) & -4(15)\\ -4(6) & -4(3) & -4(9)\end{bmatrix}=\begin{bmatrix}-36 & 12 & -36\\ -48 & -12 & -60\\ -24 & -12 & -36\end{bmatrix}$$

$$(-12)C=\begin{bmatrix}-12(3) & -12(-1) & -12(3)\\ -12(4) & -12(1) & -12(5)\\ -12(2) & -12(1) & -12(3)\end{bmatrix}=\begin{bmatrix}-36 & 12 & -36\\ -48 & -12 & -60\\ -24 & -12 & -36\end{bmatrix}$$

EXERCÍCIO 4 (PÁGINA 17)

4.A

$$2B=\begin{bmatrix}2(1) & 2(0)\\ 2(2) & 2(1)\\ 2(3) & 2(2)\end{bmatrix}=\begin{bmatrix}2 & 0\\ 4 & 2\\ 6 & 4\end{bmatrix}$$

$$3F=\begin{bmatrix}3(-4) & 3(5)\\ 3(2) & 3(3)\end{bmatrix}=\begin{bmatrix}-12 & 15\\ 6 & 9\end{bmatrix}$$

$2B-3F$ NÃO ESTÁ DEFINIDO POIS AS MATRIZES TÊM FORMAS DIFERENTES.

4.B

$$EB=\begin{bmatrix}2(1)+(-4)(2)+5(3) & 2(0)+(-4)(1)+5(2)\\ 0(1)+1(2)+4(3) & 0(0)+1(1)+4(2)\\ 3(1)+2(2)+1(3) & 3(0)+2(1)+1(2)\end{bmatrix}$$

$$=\begin{bmatrix}2-8+15 & 0-4+10\\ 0+2+12 & 0+1+8\\ 3+4+3 & 0+2+2\end{bmatrix}=\begin{bmatrix}9 & 6\\ 14 & 9\\ 10 & 4\end{bmatrix}$$

$$FA = \begin{bmatrix} -4(1)+5(2) & -4(2)+5(1) & -4(3)+5(4) \\ 2(1)+3(2) & 2(2)+3(1) & 2(3)+3(4) \end{bmatrix}$$

$$= \begin{bmatrix} -4+10 & -8+5 & -12+20 \\ 2+6 & 4+3 & 6+12 \end{bmatrix} = \begin{bmatrix} 6 & -3 & 8 \\ 8 & 7 & 18 \end{bmatrix}$$

EB + FA NÃO ESTÁ DEFINIDO POIS AS MATRIZES TÊM FORMAS DIFERENTES.

4.C A(B+D) NÃO É POSSÍVEL POIS O PRODUTO (B+D) NÃO ESTÁ DEFINIDO DEVIDO A DIFERENÇA EXISTENTE ENTRE A FORMA DESSAS MATRIZES.

4.D $D+F = \begin{bmatrix} 3+(-4) & -2+5 \\ 2+2 & 4+3 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 4 & 7 \end{bmatrix}$

$$(D+F)A = \begin{bmatrix} -1(1)+3(2) & -1(2)+3(1) & -1(3)+3(4) \\ 4(1)+7(2) & 4(2)+7(1) & 4(3)+7(4) \end{bmatrix}$$

$$= \begin{bmatrix} -1+6 & -2+3 & -3+12 \\ 4+14 & 8+7 & 12+28 \end{bmatrix} = \begin{bmatrix} 5 & 1 & 9 \\ 18 & 15 & 40 \end{bmatrix}$$

4.E $3F = \begin{bmatrix} 3(-4) & 3(5) \\ 3(2) & 3(3) \end{bmatrix} = \begin{bmatrix} -12 & 15 \\ 6 & 9 \end{bmatrix}$

$$4F = \begin{bmatrix} 4(-4) & 4(5) \\ 4(2) & 4(3) \end{bmatrix} = \begin{bmatrix} -16 & 20 \\ 8 & 12 \end{bmatrix}$$

$$3F + 4F = \begin{bmatrix} -12+(-16) & 15+20 \\ 6+8 & 9+12 \end{bmatrix} = \begin{bmatrix} -28 & 35 \\ 14 & 21 \end{bmatrix}$$

$$7.F = \begin{bmatrix} 7(-4) & 7(5) \\ 7(2) & 7(3) \end{bmatrix} = \begin{bmatrix} -28 & 35 \\ 14 & 21 \end{bmatrix}$$

4.F AC + DE NÃO É POSSÍVEL POIS O PRODUTO 'DE' NÃO ESTÁ DEFINIDO POIS AS MATRIZES QUE O COMPÕE SÃO DE FORMA DIFERENTE.

EXERCÍCIO 5 (PÁGINA 17)

5.A

$$A^T = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 4 \end{bmatrix} \qquad (A^T)^T = A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$$

5.B

$$C + E = \begin{bmatrix} 3+2 & -1+(-4) & 3+5 \\ 4+0 & 1+1 & 5+4 \\ 2+3 & 1+2 & 3+1 \end{bmatrix} = \begin{bmatrix} 5 & -5 & 8 \\ 4 & 2 & 9 \\ 5 & 3 & 4 \end{bmatrix}$$

$$(C+E)^T = \begin{bmatrix} 5 & 4 & 5 \\ -5 & 2 & 3 \\ 8 & 9 & 4 \end{bmatrix}$$

$$C^T = \begin{bmatrix} 3 & 4 & 2 \\ -1 & 1 & 1 \\ 3 & 5 & 3 \end{bmatrix} \qquad E^T = \begin{bmatrix} 2 & 0 & 3 \\ -4 & 1 & 2 \\ 5 & 4 & 1 \end{bmatrix}$$

$$C^T + E^T = \begin{bmatrix} 3+2 & 4+0 & 2+3 \\ -1+(-4) & 1+1 & 1+2 \\ 3+5 & 5+4 & 3+1 \end{bmatrix} = \begin{bmatrix} 5 & 4 & 5 \\ -5 & 2 & 3 \\ 8 & 9 & 4 \end{bmatrix}$$

5.C

$$AB = \begin{bmatrix} 1(1)+2(2)+3(3) & 1(0)+2(1)+3(2) \\ 2(1)+1(2)+4(3) & 2(0)+1(1)+4(2) \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+9 & 0+2+6 \\ 2+2+12 & 0+1+8 \end{bmatrix} = \begin{bmatrix} 14 & 8 \\ 16 & 9 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 14 & 16 \\ 8 & 9 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}_{2\times 3} \qquad A^T = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 4 \end{bmatrix}_{3\times 2}$$

$$B^T A^T = \begin{bmatrix} 1(1)+2(2)+3(3) & 1(2)+2(1)+3(4) \\ 0(1)+1(2)+2(3) & 0(2)+1(1)+2(4) \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+9 & 2+2+12 \\ 0+2+6 & 0+1+8 \end{bmatrix} = \begin{bmatrix} 14 & 16 \\ 8 & 9 \end{bmatrix}$$

5.E $(2BC+F)^T$ NÃO É POSSÍVEL POIS O PRODUTO 'BC' NÃO ESTÁ DEFINIDO DEVIDO ESSAS MATRIZES APRESENTAREM FORMAS DIFERENTES.

5.F $B^T = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}_{2\times 3}$

$$B^T C = \begin{bmatrix} 1(3)+2(4)+3(2) & 1(-1)+2(1)+3(1) & 1(3)+2(5)+3(3) \\ 0(3)+1(4)+2(2) & 0(-1)+1(1)+2(1) & 0(3)+1(5)+2(3) \end{bmatrix}$$

$$= \begin{bmatrix} 3+8+6 & -1+2+3 & 3+10+9 \\ 0+4+4 & 0+1+2 & 0+5+6 \end{bmatrix} = \begin{bmatrix} 17 & 4 & 22 \\ 8 & 3 & 11 \end{bmatrix}$$

$$B^T C + A = \begin{bmatrix} 17+1 & 4+2 & 22+3 \\ 8+2 & 3+1 & 4+11 \end{bmatrix} = \begin{bmatrix} 18 & 6 & 25 \\ 10 & 4 & 15 \end{bmatrix}$$

EXERCÍCIO 6 (PÁGINA 18)

6.A

$$3C = \begin{bmatrix} 3(3) & 3(-1) & 3(3) \\ 3(4) & 3(1) & 3(5) \\ 3(2) & 3(1) & 3(3) \end{bmatrix} = \begin{bmatrix} 9 & -3 & 9 \\ 12 & 3 & 15 \\ 6 & 3 & 9 \end{bmatrix}$$

$$2E = \begin{bmatrix} 2(2) & 2(-4) & 2(5) \\ 2(0) & 2(1) & 2(4) \\ 2(3) & 2(2) & 2(1) \end{bmatrix} = \begin{bmatrix} 4 & -8 & 10 \\ 0 & 2 & 8 \\ 6 & 4 & 2 \end{bmatrix}$$

$$3C-2E = \begin{bmatrix} 9-4 & -3-(-8) & 9-10 \\ 0-12 & 3-2 & 15-8 \\ 6-6 & 3-4 & 9-2 \end{bmatrix} = \begin{bmatrix} 5 & 5 & -1 \\ -12 & 1 & 7 \\ 0 & -1 & 7 \end{bmatrix}$$

$$(3C-2E)^T = \begin{bmatrix} 5 & -12 & 0 \\ 5 & 1 & -1 \\ -1 & 7 & 7 \end{bmatrix}_{3\times 3}$$

$$(3C-2E)^T B = \begin{bmatrix} 5(1)+(-12)(2)+0(3) & 5(0)+(-12)(1)+0(2) \\ 5(1)+1(2)+(-1)(3) & 5(0)+1(1)+(-1)(2) \\ (-1)(1)+7(2)+7(3) & (-1)(0)+7(1)+7(2) \end{bmatrix}$$

$$= \begin{bmatrix} 5-24+0 & 0-12+0 \\ 5+2-3 & 0+1-2 \\ -1+14+21 & 0+7+14 \end{bmatrix} = \begin{bmatrix} -19 & -12 \\ 4 & -1 \\ 34 & 21 \end{bmatrix}$$

6.B $D+F = \begin{bmatrix} 3+(-4) & -2+5 \\ 2+2 & 4+3 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 4 & 7 \end{bmatrix}_{2\times 2}$

$$A^T = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 4 \end{bmatrix}_{3\times 2}$$

$A^T(D+F)$ NÃO ESTÁ DEFINIDA POIS AS MATRIZES POSSUEM FORMAS DIFERENTES.

6.C

$$2E=\begin{bmatrix}2(2) & 2(-4) & 2(5)\\ 2(0) & 2(1) & 2(4)\\ 2(3) & 2(2) & 2(1)\end{bmatrix}=\begin{bmatrix}4 & -8 & 10\\ 0 & 2 & 8\\ 6 & 4 & 2\end{bmatrix}_{3\times 3}$$

$$A^T=\begin{bmatrix}1 & 2\\ 2 & 1\\ 3 & 4\end{bmatrix}_{3\times 2}$$

$$(2E)A^T=\begin{bmatrix}4(1)+(-8)(2)+10(3) & 4(2)+(-8)(1)+10(4)\\ 0(1)+2(2)+8(3) & 0(2)+2(1)+8(4)\\ 6(1)+4(2)+2(3) & 6(2)+4(1)+2(4)\end{bmatrix}$$

$$=\begin{bmatrix}4-16+30 & 8-8+40\\ 0+4+24 & 0+2+32\\ 6+8+6 & 12+4+8\end{bmatrix}=\begin{bmatrix}18 & 40\\ 28 & 34\\ 20 & 24\end{bmatrix}$$

6.D $(BC)^T$ NÃO É POSSÍVEL POIS O PRODUTO "BC" NÃO ESTÁ DEFINIDO DEVIDO A DIFERENÇA DE FORMA DAS DUAS MATRIZES

$$C^T=\begin{bmatrix}3 & 4 & 2\\ -1 & 1 & 1\\ 3 & 5 & 3\end{bmatrix}_{3\times 3} \qquad B^T=\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 2\end{bmatrix}_{2\times 3}$$

C^TB^T NÃO ESTÁ DEFINIDO POIS AS MATRIZES DESTE PRODUTO TÊM FORMAS DIFERENTES.

6.E $$B^T=\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 2\end{bmatrix}$$

$$B^T+A=\begin{bmatrix}1+1 & 2+2 & 3+3\\ 0+2 & 1+1 & 2+4\end{bmatrix}=\begin{bmatrix}2 & 4 & 6\\ 2 & 2 & 6\end{bmatrix}_{2\times 3}$$

$$(B^T+A)C=\begin{bmatrix}2(3)+4(4)+6(2) & 2(-1)+4(1)+6(1) & 2(3)+4(5)+6(3)\\ 2(3)+2(4)+6(2) & 2(-1)+2(1)+6(1) & 2(3)+2(5)+6(3)\end{bmatrix}$$

$$=\begin{bmatrix}6+16+12 & -2+4+6 & 6+20+18\\ 6+8+12 & -2+2+6 & 6+10+18\end{bmatrix}$$

$$=\begin{bmatrix}34 & 8 & 32\\ 26 & 6 & 22\end{bmatrix}$$

6-F $(D^T+E)F$ NÃO É POSSÍVEL POIS O SOMATÓRIO (D^T+E) NÃO ESTÁ DEFINIDO DEVIDO A DIFERENÇA ENTRE AS FORMAS DESSAS MATRIZES.

EXERCÍCIO 7 (PÁGINA 18)

$$DI=\begin{bmatrix}3(1)+(-2)(0) & 3(0)+(-2)(1)\\ 2(1)+4(0) & 2(0)+4(1)\end{bmatrix}$$

$$=\begin{bmatrix}3+0 & 0-2\\ 2+0 & 0+4\end{bmatrix}=\begin{bmatrix}3 & -2\\ 2 & 4\end{bmatrix}$$

$$ID=\begin{bmatrix}1(3)+0(2) & 1(-2)+0(4)\\ 0(3)+1(2) & 0(-2)+1(4)\end{bmatrix}$$

$$=\begin{bmatrix}3+0 & -2+0\\ 0+2 & 0+4\end{bmatrix}=\begin{bmatrix}3 & -2\\ 2 & 4\end{bmatrix}$$

$$DI=ID=D$$

EXERCÍCIO 8 (PÁGINA 18)

$$\begin{bmatrix}a+b & c+d\\ c-d & a-b\end{bmatrix}=\begin{bmatrix}4 & 6\\ 10 & 2\end{bmatrix}$$

$$\begin{cases} a + b = 4 \\ a - b = 2 \end{cases}$$
$2a + 0 = 6$

$2a = 6$
$a = \frac{6}{2}$
$= 3$

$a + b = 4$
$3 + b = 4$
$b = 4 - 3$
$= 1$

$$\begin{cases} c + d = 6 \\ c - d = 10 \end{cases}$$
$2c + 0 = 16$

$2c = 16$
$c = \frac{16}{2}$
$= 8$

$c + d = 6$
$8 + d = 6$
$d = 6 - 8$
$= -2$

EXERCÍCIO 9 (PÁGINA 18)

$$\begin{bmatrix} a + 2b & 2a - b \\ 2c + d & c - 2d \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 4 & -3 \end{bmatrix}$$

$$\begin{cases} a + 2b = 4 \\ 2a - b = -2 \ (2) \end{cases} \sim \begin{cases} a + 2b = 4 \\ 2a - 2b = -4 \end{cases}$$
$3a + 0 = 0$

$3a = 0$
$a = 0$

$a + 2b = 4$
$0 + 2b = 4$
$2b = 4$
$b = \frac{4}{2}$
$= 2$

$$\begin{cases} 2c + d = 4 \ (2) \\ c - 2d = -3 \end{cases} \sim \begin{cases} 4c + 2d = 8 \\ c - 2d = -3 \end{cases}$$
$5c = 5$

$5c = 5$
$c = \frac{5}{5}$
$= 1$

$c - 2d = -3$
$1 - 2d = -3$
$-2d = -3 - 1$
$-2d = -4$
$d = \frac{-4}{-2}$
$= 2$

EXERCÍCIO 10 (PÁGINA 18)

$$AB = \begin{bmatrix} 1(2)+2(-3) & 1(-1)+2(4) \\ 3(2)+2(-3) & 3(-1)+2(4) \end{bmatrix}$$

$$= \begin{bmatrix} 2-6 & -1+8 \\ 6-6 & -3+8 \end{bmatrix} = \begin{bmatrix} -4 & 7 \\ 0 & 5 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2(1)+(-1)(3) & 2(2)+(-1)(2) \\ (-3)(1)+4(3) & -3(2)+4(2) \end{bmatrix}$$

$$= \begin{bmatrix} 2-3 & 4-2 \\ -3+12 & -6+8 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 9 & 2 \end{bmatrix}$$

EXERCÍCIO 11 (PÁGINA 18)

$A0 = 0$

EXERCÍCIO 12 (PÁGINA 18)

12.A
$$\begin{bmatrix} 2 & 0 & 0 & 1 \\ 3 & 2 & 3 & 0 \\ 2 & 3 & -4 & 0 \\ 1 & 0 & 3 & 0 \end{bmatrix}$$

12.B
$$\begin{bmatrix} 2 & 0 & 0 & 1 \\ 3 & 2 & 3 & 0 \\ 2 & 3 & -4 & 0 \\ 1 & 0 & 3 & 0 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \\ 3 \\ 5 \end{bmatrix}$$

12.C
$$\left[\begin{array}{cccc|c} 2 & 0 & 0 & 1 & 7 \\ 3 & 2 & 3 & 0 & -2 \\ 2 & 3 & -4 & 0 & 3 \\ 1 & 0 & 3 & 0 & 5 \end{array}\right]$$

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EXERCÍCIO 13 (PÁGINA 18)

$$\begin{cases} -2x - y + 4w = 5 \\ -3x + 2y + 7z + 8w = 3 \\ x + 2w = 4 \\ 3x + z + 3w = 6 \end{cases}$$

EXERCÍCIO 14 (PÁGINA 18)

$$\begin{cases} 2x - 4w = 3 \\ y + 2w = 5 \\ x + 3y + 4w = -1 \end{cases}$$

EXERCÍCIO 15 (PÁGINA 18)

15.A

$$\begin{bmatrix} 3 & -1 & 2 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \\ 4 & 0 & -1 \end{bmatrix}$$

15.B

$$\begin{bmatrix} 3 & -1 & 2 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \\ 4 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 7 \\ 4 \end{bmatrix}$$

15.C

$$\left[\begin{array}{ccc|c} 3 & -1 & 2 & 4 \\ 2 & 1 & 0 & 2 \\ 0 & 1 & 3 & 7 \\ 4 & 0 & -1 & 4 \end{array}\right]$$

EXERCÍCIO 16 (PÁGINA 18)

16.A

$$AB_1 = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 4 & -2 & 3 \\ 2 & 1 & 5 \end{bmatrix} \times \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1(1)+(-1)(3)+2(4) \\ 3(1)+2(3)+4(4) \\ 4(1)+(-2)(3)+3(4) \\ 2(1)+1(3)+5(4) \end{bmatrix}$$

$$= \begin{bmatrix} 1-3+8 \\ 3+6+16 \\ 4-6+12 \\ 2+3+20 \end{bmatrix} = \begin{bmatrix} 6 \\ 25 \\ 10 \\ 25 \end{bmatrix}$$

16.B

$$AB_3 = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 4 & -2 & 3 \\ 2 & 1 & 5 \end{bmatrix} \times \begin{bmatrix} -1 \\ -3 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1(-1)+(-1)(-3)+2(5) \\ 3(-1)+2(-3)+4(5) \\ 4(-1)+(-2)(-3)+3(5) \\ 2(-1)+1(-3)+5(5) \end{bmatrix}$$

$$= \begin{bmatrix} -1+3+10 \\ -3-6+20 \\ -4+6+15 \\ -2-3+25 \end{bmatrix} = \begin{bmatrix} 12 \\ 11 \\ 17 \\ 20 \end{bmatrix}$$

EXERCÍCIO 17 (PÁGINA 18)

17.A

$$AB_2 = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 4 & -2 & 3 \\ 2 & 1 & 5 \end{bmatrix} \times \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1(0)+(-1)(3)+2(2) \\ 3(0)+2(3)+4(2) \\ 4(0)+(-2)(3)+3(2) \\ 2(0)+1(3)+5(2) \end{bmatrix}$$

$$= \begin{bmatrix} 0-3+4 \\ 0+6+8 \\ 0-6+6 \\ 0+3+10 \end{bmatrix} = \begin{bmatrix} 1 \\ 14 \\ 0 \\ 13 \end{bmatrix}$$

17.B

$$AB_4 = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 4 & -2 & 3 \\ 2 & 1 & 5 \end{bmatrix} \times \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1(2)+(-1)(4)+2(1) \\ 3(2)+2(4)+4(1) \\ 4(2)+(-2)(4)+3(1) \\ 2(2)+1(4)+5(1) \end{bmatrix}$$

$$= \begin{bmatrix} 2-4+2 \\ 6+8+4 \\ 8-8+3 \\ 4+4+5 \end{bmatrix} = \begin{bmatrix} 0 \\ 18 \\ 3 \\ 13 \end{bmatrix}$$

EXERCÍCIO 18 (PÁGINA 18)

$$AB = \begin{bmatrix} 2(9)+2(10) & 2(10)+2(12) \\ 3(9)+4(10) & 3(10)+4(12) \end{bmatrix} = \begin{bmatrix} 18+20 & 20+24 \\ 27+40 & 30+48 \end{bmatrix} = \begin{bmatrix} 38 & 44 \\ 67 & 78 \end{bmatrix}$$

ESSES COEFICIENTES REPRESENTAM O CUSTO POR HORA DE CADA UM DOS PRODUTOS EM CADA FÁBRICA.

EXERCÍCIO 19 (PÁGINA 18)

$$AB = \begin{bmatrix} 300(8)+100(7)+150(15) & 300(12)+100(9)+150(10) \\ 200(8)+250(7)+400(15) & 200(12)+250(9)+400(10) \end{bmatrix}$$

$$= \begin{bmatrix} 2.400+700+2.250 & 3.600+900+1.500 \\ 1.600+1.750+6.000 & 2.400+2.250+4.000 \end{bmatrix}$$

$$= \begin{bmatrix} 5.350 & 6.000 \\ 9.350 & 8.650 \end{bmatrix}$$

ESSES COEFICIENTES REPRESENTAM O CUSTO DIÁRIO (EM DÓLAR) QUE CADA FÁBRICA TEM PARA ELIMINAR OS POLUENTES GERADOS PELOS PRODUTOS 'P' E 'Q'.

EXERCÍCIO 20 (PÁGINA 19)

20.A

$$AB_1 = \begin{bmatrix} 80 & 120 \\ 100 & 200 \end{bmatrix} \times \begin{bmatrix} 20 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} 80(20)+120(10) \\ 100(20)+200(10) \end{bmatrix}$$

2.800 GRAMAS

$$= \begin{bmatrix} 1.600+1.200 \\ 2.000+2.000 \end{bmatrix} = \begin{bmatrix} 2.800 \\ 4.000 \end{bmatrix}$$

20.B

$$AB_2 = \begin{bmatrix} 80 & 120 \\ 100 & 200 \end{bmatrix} \times \begin{bmatrix} 20 \\ 20 \end{bmatrix}$$

$$= \begin{bmatrix} 80(20)+120(20) \\ 100(20)+200(20) \end{bmatrix}$$

6.000 GRAMAS

$$= \begin{bmatrix} 1.600+2.400 \\ 2.000+4.000 \end{bmatrix} = \begin{bmatrix} 4.000 \\ 6.000 \end{bmatrix}$$

EXERCÍCIO 21 (PÁGINA 19)

21.A
$$AB_1 = \begin{bmatrix} 200 & 150 & 120 \\ 50 & 40 & 25 \end{bmatrix} \times \begin{bmatrix} 220 \\ 300 \\ 120 \end{bmatrix}$$

$$= \begin{bmatrix} 200(220) + 150(300) + 120(120) \\ 50(220) + 40(300) + 25(120) \end{bmatrix}$$

$$= \begin{bmatrix} 44.000 + 45.000 + 14.400 \\ 11.000 + 12.000 + 3.000 \end{bmatrix}$$

US$ 103.400,00

$$= \begin{bmatrix} 103.400 \\ 26.000 \end{bmatrix}$$

21.B
$$AB_3 = \begin{bmatrix} 200 & 150 & 120 \\ 50 & 40 & 25 \end{bmatrix} \times \begin{bmatrix} 100 \\ 120 \\ 250 \end{bmatrix}$$

$$= \begin{bmatrix} 200(100) + 150(120) + 120(250) \\ 50(100) + 40(120) + 25(250) \end{bmatrix}$$

$$= \begin{bmatrix} 20.000 + 18.000 + 30.000 \\ 5.000 + 4.800 + 6.250 \end{bmatrix}$$

$$= \begin{bmatrix} 68.000 \\ 16.050 \end{bmatrix}$$

US$ 16.050,00

EXERCÍCIO T.1 (PÁGINA 19)

T.1.A

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1p} \\ a_{21} & a_{22} & \cdots & a_{2p} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mp} \end{bmatrix}_{m \times p} \quad B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ \vdots & \vdots & & \vdots \\ b_{p1} & b_{p2} & \cdots & b_{pm} \end{bmatrix}_{p \times m}$$

SUPONDO QUE A PRIMEIRA LINHA DA MATRIZ 'A' SEJA UMA

LINHA DE ZEROS, TEMOS:

$$AB = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & a_{2p} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mp} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ \vdots & \vdots & & \vdots \\ b_{p1} & b_{p2} & \cdots & b_{pm} \end{bmatrix}$$

$$= \begin{bmatrix} 0(b_{11})+0(b_{21})+\cdots+0(b_{p1}) & \cdots & 0(b_{1m})+0(b_{2m})+\cdots+0(b_{pm}) \\ a_{21}(b_{11})+a_{22}(b_{21})+\cdots+a_{2p}(b_{p1}) & \cdots & a_{21}(b_{1m})+a_{22}(b_{2m})+\cdots+a_{2p}(b_{pm}) \\ \vdots & & \vdots \\ a_{m1}(b_{11})+a_{m2}(b_{21})+\cdots+a_{mp}(b_{p1}) & \cdots & a_{m1}(b_{1m})+a_{m2}(b_{2m})+\cdots+a_{mp}(b_{pm}) \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+\cdots+0 & \cdots & 0+0+\cdots+0 \\ & & \\ & & \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & \cdots & 0 \\ & & & \\ & & & \end{bmatrix}$$

T.1.B SUPONDO QUE A PRIMEIRA COLUNA DA MATRIZ 'B' SEJA UMA COLUNA DE ZEROS, TEMOS

$$AB = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1p} \\ a_{21} & a_{22} & \cdots & a_{2p} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mp} \end{bmatrix} \begin{bmatrix} 0 & b_{12} & \cdots & b_{1m} \\ 0 & b_{22} & \cdots & b_{2m} \\ \vdots & \vdots & & \vdots \\ 0 & b_{p2} & \cdots & b_{pm} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}(0)+a_{12}(0)+\cdots+a_{1p}(0) & \cdots & a_{11}(b_{1m})+a_{12}(b_{2m})+\cdots+a_{1p}(b_{pm}) \\ a_{21}(0)+a_{22}(0)+\cdots+a_{2p}(0) & \cdots & a_{21}(b_{1m})+a_{22}(b_{2m})+\cdots+a_{2p}(b_{pm}) \\ \vdots & & \vdots \\ a_{m1}(0)+a_{m2}(0)+\cdots+a_{mp}(0) & \cdots & a_{m1}(b_{1m})+a_{m2}(b_{2m})+\cdots+a_{mp}(b_{pm}) \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+\cdots+0 \\ 0+0+\cdots+0 \\ \vdots \\ 0+0+\cdots+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

EXERCÍCIO T.2 (PÁGINA 19)

EM QUE, $a_{ij} \neq 0$ SE $i=j$ E $a_{ij}=0$ SE $i \neq j$.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1p} \\ a_{21} & a_{22} & \cdots & a_{2p} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mp} \end{bmatrix}_{m \times p}$$

EM QUE, $a_{ij} \neq 0$ SE $i=j$ E $a_{ij}=0$ SE $i \neq j$.

$$B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ \vdots & \vdots & & \vdots \\ b_{p1} & b_{p2} & \cdots & b_{pm} \end{bmatrix}_{p \times m}$$

T.2.A

$$A+B = \begin{bmatrix} a_{11}+b_{11} & 0+0 & \cdots & 0+0 \\ 0+0 & a_{22}+b_{22} & \cdots & 0+0 \\ \vdots & \vdots & & \vdots \\ 0+0 & 0+0 & \cdots & a_{mp}+b_{pm} \end{bmatrix}$$ => EM QUE, $m \times p = p \times m$

$$= \begin{bmatrix} a_{11}+b_{11} & 0 & \cdots & 0 \\ 0 & a_{22}+b_{22} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & a_{mp}+b_{pm} \end{bmatrix}$$

T.2.B

$$AB = \begin{bmatrix} a_{11}(b_{11})+0(0)+\cdots+0(0) & \cdots & a_{11}(0)+0(0)+\cdots+0(b_{pm}) \\ 0(b_{11})+a_{22}(0)+\cdots+0(0) & \cdots & 0(0)+a_{22}(0)+\cdots+0(b_{pm}) \\ \vdots & & \vdots \\ 0(b_{11})+0(0)+\cdots+a_{mp}(0) & \cdots & 0(0)+0(0)+\cdots+a_{mp}(b_{pm}) \end{bmatrix}$$

$$AB = \begin{bmatrix} a_{11}b_{11} + 0 + \dots + 0 & \dots & 0 + 0 + \dots + 0 \\ 0 + 0 + \dots + 0 & \dots & 0 + 0 + \dots + 0 \\ \vdots \quad \vdots \quad \vdots & & \vdots \quad \vdots \quad \vdots \\ 0 + 0 + \quad + 0 & \dots & 0 + 0 + \dots + a_{mp}b_{pm} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}b_{11} & 0 & \dots & 0 \\ 0 & a_{22}b_{22} & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & a_{mp}b_{pm} \end{bmatrix}$$

EXERCÍCIO T.3 (PÁGINA 19)

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1p} \\ a_{21} & a_{22} & \dots & a_{2p} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mp} \end{bmatrix}_{m \times p}$$

SEJA A MATRIZ 'A' DADA. EM QUE $a_{ij} = k_A$ SE $i = j$ E $a_{ij} = 0$ SE $i \neq j$ ($k_A \Rightarrow$ CONSTANTE)

$$B = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \vdots & \vdots & & \vdots \\ b_{p1} & b_{p2} & \dots & b_{pm} \end{bmatrix}_{p \times m}$$

SEJA A MATRIZ 'B' DADA. EM QUE $b_{ij} = k_B$ SE $i = j$ E $b_{ij} = 0$ SE $i \neq j$ ($k_B \Rightarrow$ CONSTANTE)

T.3.A CONSIDERANDO QUE $m \times p = p \times m$ TEMOS:

$$A + B = \begin{bmatrix} a_{11} + b_{11} & 0 + 0 & \dots & 0 + 0 \\ 0 + 0 & a_{22} + b_{22} & \dots & 0 + 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 + 0 & 0 + 0 & & a_{mp} + b_{pm} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} + b_{11} & 0 & \dots & 0 \\ 0 & a_{22} + b_{22} & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & a_{mp} + b_{pm} \end{bmatrix}$$

T.3.B

$$AB = \begin{bmatrix} a_{11}(b_{11}) + 0(0) + \dots + 0(0) & \dots & a_{11}(0) + 0(0) + \dots + 0(b_{pm}) \\ 0(b_{11}) + a_{22}(0) + \dots + 0(0) & \dots & 0(0) + a_{22}(0) + \dots + 0(b_{pm}) \\ \vdots & & \vdots \\ 0(b_{11}) + 0(0) + \dots + a_{mp}(0) & \dots & 0(0) + 0(0) + \dots + a_{mp}(b_{pm}) \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}b_{11} + 0 + \dots + 0 & \dots & 0 + 0 + \dots + 0 \\ 0 + 0 + \dots + 0 & \dots & 0 + 0 + \dots + 0 \\ \vdots & & \vdots \\ 0 + 0 + \dots + 0 & \dots & 0 + 0 + \dots + a_{mp}b_{pm} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}b_{11} & 0 & \dots & 0 \\ 0 & a_{22}b_{22} & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & & a_{mp}b_{pm} \end{bmatrix}$$

EXERCÍCIO T.5 (PÁGINA 19)

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1p} \\ a_{21} & a_{22} & \dots & a_{2p} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mp} \end{bmatrix}_{m \times p}$$

SEJA A MATRIZ 'A' DADA UMA MATRIZ TRIANGULAR SUPERIOR. EM QUE $a_{ij} = 0$ SE $i > j$.

$$B = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \vdots & \vdots & & \vdots \\ b_{p1} & b_{p2} & \dots & b_{pm} \end{bmatrix}_{p \times m}$$

SEJA A MATRIZ 'B' DADA UMA MATRIZ TRIANGULAR INFERIOR. EM QUE $a_{ij} = 0$ SE $i < j$.

T.5.A SUPONDO QUE $m \times p = p \times m$, QUE $m > p$ OU $m = p$ E QUE $p > m$ OU $m = p$. TEMOS QUE $A+B$ SERÁ IGUAL A:

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$$A+B=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1p} \\ 0 & a_{22} & \cdots & a_{2p} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & a_{mp} \end{bmatrix}+\begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ 0 & b_{22} & \cdots & b_{2m} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & b_{pm} \end{bmatrix}$$

$$=\begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \cdots & a_{1p}+b_{1m} \\ 0+0 & a_{22}+b_{22} & \cdots & a_{2p}+b_{2m} \\ \vdots & \vdots & & \vdots \\ 0+0 & 0+0 & \cdots & a_{mp}+b_{pm} \end{bmatrix}$$

$$=\begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \cdots & a_{1p}+b_{1m} \\ 0 & a_{22}+b_{22} & \cdots & a_{2p}+b_{2m} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & a_{mp}+b_{pm} \end{bmatrix}$$

$$AB=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1p} \\ a_{21} & a_{22} & \cdots & a_{2p} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mp} \end{bmatrix}\times\begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ \vdots & \vdots & & \vdots \\ b_{p1} & b_{p2} & \cdots & b_{pm} \end{bmatrix}$$

$$=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1p} \\ 0 & a_{22} & \cdots & a_{2p} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & a_{mp} \end{bmatrix}\times\begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ 0 & b_{22} & \cdots & b_{2m} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & b_{pm} \end{bmatrix}$$

$$=\begin{bmatrix} a_{11}(b_{11})+a_{12}(0)+\cdots+a_{1p}(0) & \cdots & a_{11}(b_{1m})+a_{12}(b_{2m})+\cdots+a_{1p}(b_{pm}) \\ 0(b_{11})+a_{22}(0)+\cdots+a_{2p}(0) & \cdots & 0(b_{1m})+a_{22}(b_{2m})+\cdots+a_{2p}(b_{pm}) \\ \vdots & & \vdots \\ 0(b_{11})+0(0)+\cdots+a_{mp}(0) & \cdots & 0(b_{1m})+0(b_{2m})+\cdots+a_{mp}(b_{pm}) \end{bmatrix}$$

$$=\begin{bmatrix} a_{11}b_{11}+0+\cdots+0 & a_{11}b_{12}+a_{12}b_{22}+\cdots+0 & a_{11}b_{1m}+a_{12}b_{2m}+\cdots+a_{1p}b_{pm} \\ 0+0+\cdots+0 & 0+a_{22}b_{22}+\cdots+0 & 0+a_{22}b_{2m}+\cdots+a_{2p}b_{pm} \\ \vdots & \vdots & \vdots \\ 0+0+\cdots+0 & 0+0+\cdots+0 & 0+0+\cdots+a_{mp}b_{pm} \end{bmatrix}$$

$$AB = \begin{bmatrix} a_{11} b_{11} & a_{11} b_{12} + a_{12} b_{22} & \cdots & a_{11} b_{1m} + a_{12} b_{2m} + \cdots + a_{1p} b_{pm} \\ 0 & a_{22} b_{22} & \cdots & a_{22} b_{2m} + \cdots + a_{2p} b_{pm} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & a_{mp} b_{pm} \end{bmatrix}$$

T.E.B

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1p} \\ a_{21} & a_{22} & \cdots & a_{2p} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mp} \end{bmatrix}_{m \times p}$$

SEJA A MATRIZ 'A' DADA UMA MATRIZ TRIANGULAR INFERIOR. EM QUE $a_{ij} = 0$ SE $i < j$.

$$B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ \vdots & \vdots & & \vdots \\ b_{p1} & b_{p2} & \cdots & b_{pm} \end{bmatrix}_{p \times m}$$

SEJA A MATRIZ 'B' DADA UMA MATRIZ TRIANGULAR INFERIOR. EM QUE $a_{ij} = 0$ SE $i < j$.

SUPONDO QUE $m \times p = p \times m$, QUE $m > p$ OU $m = p$ E QUE $p > m$ OU $p = m$. TEMOS QUE $A+B$ SERÁ IGUAL A:

$$A+B = \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mp} \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & \cdots & 0 \\ b_{21} & b_{22} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ b_{p1} & b_{p2} & \cdots & b_{pm} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} + b_{11} & 0+0 & \cdots & 0+0 \\ a_{21} + b_{21} & a_{22} + b_{22} & \cdots & 0+0 \\ \vdots & \vdots & & \vdots \\ a_{m1} + b_{p1} & a_{m2} + b_{p2} & \cdots & a_{mp} + b_{pm} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} + b_{11} & 0 & & 0 \\ a_{21} + b_{21} & a_{22} + b_{22} & & 0 \\ \vdots & \vdots & & \vdots \\ a_{m1} + b_{p1} & a_{m2} + b_{p2} & \cdots & a_{mp} + b_{pm} \end{bmatrix}$$

$$AB = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1p} \\ a_{21} & a_{22} & \cdots & a_{2p} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mp} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ \vdots & \vdots & & \vdots \\ b_{p1} & b_{p2} & \cdots & b_{pm} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mp} \end{bmatrix} \times \begin{bmatrix} b_{11} & 0 & \cdots & 0 \\ b_{21} & b_{22} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ b_{p1} & b_{p2} & \cdots & b_{pm} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}(b_{11}) + 0(b_{21}) + \cdots + 0(b_{p1}) & \cdots & a_{11}(0) + 0(0) + \cdots + 0(b_{pm}) \\ a_{21}(b_{11}) + a_{22}(b_{21}) + \cdots + 0(b_{p1}) & \cdots & a_{21}(0) + a_{22}(0) + \cdots + 0(b_{pm}) \\ \vdots & & \vdots \\ a_{m1}(b_{11}) + a_{m2}(b_{21}) + \cdots + a_{mp}(b_{p1}) & \cdots & a_{m1}(0) + a_{m2}(0) + \cdots + a_{mp}(b_{pm}) \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}b_{11} + 0 + \cdots + 0 & 0 + 0 + \cdots + 0 & 0 + 0 + \cdots + 0 \\ a_{21}b_{11} + a_{22}b_{21} + \cdots + 0 & 0 + a_{22}b_{22} + \cdots + 0 & 0 + 0 + \cdots + 0 \\ \vdots & \vdots & \vdots \\ a_{m1}b_{11} + a_{m2}b_{21} + \cdots + a_{mp}b_{p1} & 0 + a_{m2}b_{22} + \cdots + a_{mp}b_{p2} & 0 + 0 + \cdots + a_{mp}b_{pm} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}b_{11} & 0 & 0 \\ a_{21}b_{11} + a_{22}b_{21} & a_{22}b_{22} & 0 \\ \vdots & \vdots & \vdots \\ a_{m1}b_{11} + a_{m2}b_{21} + \cdots + a_{mp}b_{p1} & a_{m2}b_{22} + \cdots + a_{mp}b_{p2} & a_{mp}b_{pm} \end{bmatrix}$$

EXERCÍCIO T.5 (PÁGINA 19)

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1p} \\ a_{21} & a_{22} & \cdots & a_{2p} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mp} \end{bmatrix}_{m \times p} \qquad B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ \vdots & \vdots & & \vdots \\ b_{p1} & b_{p2} & \cdots & b_{pm} \end{bmatrix}_{p \times m}$$

LOGO AB É IGUAL A:

$$AB = \begin{bmatrix} a_{11}(b_{11})+a_{12}(b_{21})+\cdots+a_{1p}(b_{p1}) & \cdots & a_{11}(b_{1m})+a_{12}(b_{2m})+\cdots+a_{1p}(b_{pm}) \\ a_{21}(b_{11})+a_{22}(b_{21})+\cdots+a_{2p}(b_{p1}) & \cdots & a_{21}(b_{1m})+a_{22}(b_{2m})+\cdots+a_{2p}(b_{pm}) \\ \vdots & & \vdots \\ a_{m1}(b_{11})+a_{m2}(b_{21})+\cdots+a_{mp}(b_{p1}) & \cdots & a_{m1}(b_{1m})+a_{m2}(b_{2m})+\cdots+a_{mp}(b_{pm}) \end{bmatrix}$$

FAZENDO $AB_j = AB_1$, EM QUE B_1 É A PRIMEIRA COLUNA DE B.

$$AB_1 = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1p} \\ a_{21} & a_{22} & \cdots & a_{2p} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mp} \end{bmatrix} \times \begin{bmatrix} b_{11} \\ b_{21} \\ \vdots \\ b_{p1} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}(b_{11})+a_{12}(b_{21})+\cdots+a_{1p}(b_{p1}) \\ a_{21}(b_{11})+a_{22}(b_{21})+\cdots+a_{2p}(b_{p1}) \\ \vdots \\ a_{m1}(b_{11})+a_{m2}(b_{21})+\cdots+a_{mp}(b_{p1}) \end{bmatrix}$$

EXERCÍCIO T.6 (PÁGINA 19)

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mm} \end{bmatrix}_{m \times m}$$

$$A^T = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & & \vdots \\ a_{1m} & a_{2m} & \cdots & a_{mm} \end{bmatrix}_{m \times m}$$

$AA^T = 0$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mm} \end{bmatrix} \times \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & & \vdots \\ a_{1m} & a_{2m} & \cdots & a_{mm} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$\begin{bmatrix} a_{11}(a_{11}) + a_{12}(a_{12}) + \cdots + a_{1m}(a_{1m}) \cdots \\ \cdots a_{21}(a_{21}) + a_{22}(a_{22}) + \cdots + a_{2m}(a_{2m}) \cdots \\ \vdots \qquad \vdots \qquad \vdots \\ \cdots a_{mm1}(a_{m1}) + a_{m2}(a_{m2}) + \cdots + a_{mm}(a_{mm}) \end{bmatrix} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

$a_{11}(a_{11}) + a_{12}(a_{12}) + \cdots + a_{1m}(a_{1m}) = 0$

$(a_{11})^2 + (a_{12})^2 + \cdots + (a_{1m})^2 = 0 \Rightarrow a_{11} = 0;\ a_{12} = 0$ E $a_{1m} = 0$

$a_{21}(a_{21}) + a_{22}(a_{22}) + \cdots + a_{2m}(a_{2m}) = 0$

$(a_{21})^2 + (a_{22})^2 + \cdots + (a_{2m})^2 = 0 \Rightarrow a_{21} = 0;\ a_{22} = 0$ E $a_{2m} = 0$

$a_{m1}(a_{m1}) + a_{m2}(a_{m2}) + \cdots + a_{mmm}(a_{mm}) = 0$

$(a_{m1})^2 + (a_{m2})^2 + \cdots + (a_{mm})^2 = 0 \Rightarrow a_{m1} = 0;\ a_{m2} = 0$ E $a_{mm} = 0$

EXERCÍCIO T. 7 (PÁGINA 19)

T.7.i $\quad \sum_{i=1}^{m} (r_i + s_i)a_i$. FAZENDO $m = 3$ TEMOS:

$(r_1 + s_1)a_1 + (r_2 + s_2)a_2 + (r_3 + s_3)a_3 = r_1 a_1 + s_1 a_1 +$
$+ r_2 a_2 + s_2 a_2 + r_3 a_3 + s_3 a_3 = r_1 a_1 + r_2 a_2 + r_3 a_3 + s_1 a_1 +$
$+ s_2 a_2 + s_3 a_3 = \sum_{i=1}^{m} r_i a_i + \sum_{i=1}^{m} s_i a_i$

T.7.ii $\sum_{i=1}^{m} c(r_i a_i)$. FAZENDO $m=3$ TEMOS:

$$c r_1 a_1 + c r_2 a_2 + c r_3 a_3 = c(r_1 a_1 + r_2 a_2 + r_3 a_3) =$$
$$= c\left(\sum_{i=1}^{m} r_i a_i\right)$$

EXERCÍCIO T.8 (PÁGINA 19)

$\sum_{i=1}^{m}\sum_{j=1}^{m} a_{ij}$. FAZENDO $m=3$ E $m=3$ TEMOS:

$$\sum_{i=1}^{3}\sum_{j=1}^{3} = \sum_{i=1}^{3}(a_{i1}+a_{i2}+a_{i3})$$
$$=(a_{11}+a_{12}+a_{13})+(a_{21}+a_{22}+a_{23})+(a_{31}+a_{33}+a_{33})$$

$\sum_{j=1}^{m}\sum_{i=1}^{m} a_{ij}$. FAZENDO $m=3$ E $m=3$ TEMOS:

$$\sum_{j=1}^{3}\sum_{i=1}^{3} = \sum_{j=1}^{3}(a_{1j}+a_{2j}+a_{3j})$$
$$=(a_{11}+a_{21}+a_{31})+(a_{12}+a_{22}+a_{32})+(a_{13}+a_{23}+a_{33})$$
$$=(a_{11}+a_{12}+a_{13})+(a_{21}+a_{22}+a_{23})+(a_{31}+a_{32}+a_{33})$$

LOGO:

$$\sum_{i=1}^{m}\sum_{j=1}^{m} a_{ij} = \sum_{j=1}^{m}\sum_{i=1}^{m} a_{ij}$$