EXERCÍCIOS 1.3 | 2) ÁLGEBRA LINEAR, BERNARD KOLMAN, EDITORA GUANABARA, 3ª EDIÇÃO

EXERCÍCIO 1 (PÁGINA 27)

1.A $A+B=B+A$

$$A+B=\begin{bmatrix}1 & 2 & -2\\ 3 & 4 & 5\end{bmatrix}+\begin{bmatrix}2 & 0 & 1\\ 3 & -2 & 5\end{bmatrix}$$

$$=\begin{bmatrix}1+2 & 2+0 & -2+1\\ 3+3 & 4+(-2) & 5+5\end{bmatrix}$$

$$=\begin{bmatrix}3 & 2 & 1\\ 6 & 2 & 10\end{bmatrix}$$

$$B+A=\begin{bmatrix}2 & 0 & 1\\ 3 & -2 & 5\end{bmatrix}+\begin{bmatrix}1 & 2 & -2\\ 3 & 4 & 5\end{bmatrix}$$

$$=\begin{bmatrix}2+1 & 0+2 & 1+(-2)\\ 3+3 & -2+4 & 5+5\end{bmatrix}$$

$$=\begin{bmatrix}3 & 2 & -1\\ 6 & 2 & 10\end{bmatrix}$$

1.B $A+(B+C)=(A+B)+C$

$$B+C=\begin{bmatrix}2 & 0 & 1\\ 3 & -2 & 5\end{bmatrix}+\begin{bmatrix}-4 & -6 & 1\\ 2 & 3 & 0\end{bmatrix}$$

$$=\begin{bmatrix}2+(-4) & 0+(-6) & 1+1\\ 3+2 & -2+3 & 5+0\end{bmatrix}$$

$$=\begin{bmatrix}-2 & -6 & 2\\ 5 & 1 & 5\end{bmatrix}$$

$$A+(B+C)=\begin{bmatrix}1 & 2 & -2\\ 3 & 4 & 5\end{bmatrix}+\begin{bmatrix}-2 & -6 & 2\\ 5 & 1 & 5\end{bmatrix}$$
$$=\begin{bmatrix}1+(-2) & 2+(-6) & -2+2\\ 3+5 & 4+1 & 5+5\end{bmatrix}$$
$$=\begin{bmatrix}-1 & -4 & 0\\ 8 & 5 & 10\end{bmatrix}$$

$$(A+B)=\begin{bmatrix}1 & 2 & -2\\ 3 & 4 & 5\end{bmatrix}+\begin{bmatrix}2 & 0 & 1\\ 3 & -2 & 5\end{bmatrix}$$
$$=\begin{bmatrix}1+2 & 2+0 & -2+1\\ 3+3 & 4+(-2) & 5+5\end{bmatrix}$$
$$=\begin{bmatrix}3 & 2 & -1\\ 6 & 2 & 10\end{bmatrix}$$

$$(A+B)+C=\begin{bmatrix}3 & 2 & -1\\ 6 & 2 & 10\end{bmatrix}+\begin{bmatrix}-4 & -6 & 1\\ 2 & 3 & 0\end{bmatrix}$$
$$=\begin{bmatrix}3+(-4) & 2+(-6) & -1+1\\ 6+2 & 2+3 & 10+0\end{bmatrix}$$
$$=\begin{bmatrix}-1 & -4 & 0\\ 8 & 5 & 10\end{bmatrix}$$

1.C $A+0=A$

$$A+0=\begin{bmatrix}1 & 2 & -2\\ 3 & 4 & 5\end{bmatrix}+\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$
$$=\begin{bmatrix}1+0 & 2+0 & -2+0\\ 3+0 & 4+0 & 5+0\end{bmatrix}$$
$$=\begin{bmatrix}1 & 2 & -2\\ 3 & 4 & 5\end{bmatrix}$$

1.D $A + D = 0 \Rightarrow D = -A$

$$A + D = \begin{bmatrix} 1 & 2 & -2 \\ 3 & 4 & 5 \end{bmatrix} + \begin{bmatrix} -1 & -2 & 2 \\ -3 & -4 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 1+(-1) & 2+(-2) & -2+2 \\ 3+(-3) & 4+(-4) & 5+(-5) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

EXERCÍCIO 2 (PÁGINA 27)

$A(BC) = (AB)C$

$$BC = \begin{bmatrix} -1 & 3 & 2 \\ 1 & -3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 3 & -1 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1(1)+3(3)+2(1) & -1(0)+3(-1)+2(2) \\ 1(1)+(-3)(3)+4(1) & 1(0)+(-3)(-1)+4(2) \end{bmatrix}$$

$$= \begin{bmatrix} -1+9+2 & 0-3+4 \\ 1-9+4 & 0+3+8 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 1 \\ -4 & 11 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \times \begin{bmatrix} 10 & 1 \\ -4 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 1(10)+3(-4) & 1(1)+3(11) \\ 2(10)+(-1)(-4) & 2(1)+(-1)(11) \end{bmatrix}$$

$$= \begin{bmatrix} 10-12 & 1+33 \\ 20+4 & 2-11 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 34 \\ 24 & -9 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \times \begin{bmatrix} -1 & 3 & 2 \\ 1 & -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1(-1)+3(1) & 1(3)+3(-3) & 1(2)+3(4) \\ 2(-1)+(-1)(1) & 2(3)+(-1)(-3) & 2(2)+(-1)(4) \end{bmatrix}$$

$$= \begin{bmatrix} -1+3 & 3-9 & 2+12 \\ -2-1 & 6+3 & 4-4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -6 & 14 \\ -3 & 9 & 0 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} 2 & -6 & 14 \\ -3 & 9 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 3 & -1 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2(1)+(-6)(3)+14(1) & 2(0)+(-6)(-1)+14(2) \\ -3(1)+9(3)+0(1) & -3(0)+9(-1)+0(2) \end{bmatrix}$$

$$= \begin{bmatrix} 2-18+14 & 0+6+28 \\ -3+27+0 & 0-9+0 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 34 \\ 24 & -9 \end{bmatrix}$$

EXERCÍCIO 3 (PÁGINA 27)

$$A(B+C) = AB + AC$$

$$B+C = \begin{bmatrix} 2 & -3 & 2 \\ 3 & -1 & -2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 2 \\ 1 & 3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+0 & -3+1 & 2+2 \\ 3+1 & -1+3 & -2+(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 & 4 \\ 4 & 2 & -4 \end{bmatrix}$$

EXERCÍCIO 4 (PÁGINA 27)

4.A $r(s)A = (rs)A$

$$(s)A = (-2)\begin{bmatrix} 4 & 2 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} -2(4) & -2(2) \\ -2(1) & -2(-3) \end{bmatrix} = \begin{bmatrix} -8 & -4 \\ -2 & 6 \end{bmatrix}$$

$$r(s)A = (6)\begin{bmatrix} -8 & -4 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} 6(-8) & 6(-4) \\ 6(-2) & 6(6) \end{bmatrix} = \begin{bmatrix} -48 & -24 \\ -12 & 36 \end{bmatrix}$$

$$(rs) = 6(-2) = -12$$

$$(rs)A = (-12)\begin{bmatrix} 4 & 2 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} -12(4) & -12(2) \\ -12(1) & -12(-3) \end{bmatrix} = \begin{bmatrix} -48 & -24 \\ -12 & 36 \end{bmatrix}$$

4.B $(r+s)A = rA + sA$

$$(r+s) = 6 + (-2) = 6 - 2 = 4$$

$$(r+s)A = (4)\begin{bmatrix} 4 & 2 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 4(4) & 4(2) \\ 4(1) & 4(-3) \end{bmatrix} = \begin{bmatrix} 16 & 8 \\ 4 & -12 \end{bmatrix}$$

$$rA = 6\begin{bmatrix} 4 & 2 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 6(4) & 6(2) \\ 6(1) & 6(-3) \end{bmatrix} = \begin{bmatrix} 24 & 12 \\ 6 & -18 \end{bmatrix}$$

$$sA = -2\begin{bmatrix} 4 & 2 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} -2(4) & -2(2) \\ -2(1) & -2(-3) \end{bmatrix} = \begin{bmatrix} -8 & -4 \\ -2 & 6 \end{bmatrix}$$

$$rA + sA = \begin{bmatrix} 24 & 12 \\ 6 & -18 \end{bmatrix} + \begin{bmatrix} -8 & -4 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} 24+(-8) & 12+(-4) \\ 6+(-2) & -18+6 \end{bmatrix} = \begin{bmatrix} 16 & 8 \\ 4 & -12 \end{bmatrix}$$

4.C $r(A+B) = rA + rB$

$$(A+B) = \begin{bmatrix} 4 & 2 \\ 1 & -3 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 4+0 & 2+2 \\ 1+(-4) & -3+3 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ -3 & 0 \end{bmatrix}$$

$$r(A+B) = (6)\begin{bmatrix} 4 & 4 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 6(4) & 6(4) \\ 6(-3) & 6(0) \end{bmatrix} = \begin{bmatrix} 24 & 24 \\ -18 & 0 \end{bmatrix}$$

$$rA = (6)\begin{bmatrix} 4 & 2 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 6(4) & 6(2) \\ 6(1) & 6(-3) \end{bmatrix} = \begin{bmatrix} 24 & 12 \\ 6 & -18 \end{bmatrix}$$

$$rB = (6)\begin{bmatrix} 0 & 2 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 6(0) & 6(2) \\ 6(-4) & 6(3) \end{bmatrix} = \begin{bmatrix} 0 & 12 \\ -24 & 18 \end{bmatrix}$$

$$rA + rB = \begin{bmatrix} 24 & 12 \\ 6 & -18 \end{bmatrix} + \begin{bmatrix} 0 & 12 \\ -24 & 18 \end{bmatrix} = \begin{bmatrix} 24+0 & 12+12 \\ 6+(-24) & -18+18 \end{bmatrix} = \begin{bmatrix} 24 & 24 \\ -18 & 0 \end{bmatrix}$$

EXERCÍCIO 5 (PÁGINA 27)

$A(rB) = r(AB) = (rA)B$

$$(rB) = (-3)\begin{bmatrix} -1 & 3 & 2 \\ 1 & -3 & 4 \end{bmatrix} = \begin{bmatrix} -3(-1) & -3(3) & -3(2) \\ -3(1) & -3(-3) & -3(4) \end{bmatrix} = \begin{bmatrix} 3 & -9 & -6 \\ -3 & 9 & -12 \end{bmatrix}$$

$$A(rB) = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \times \begin{bmatrix} 3 & -9 & -6 \\ -3 & 9 & -12 \end{bmatrix}$$

$$= \begin{bmatrix} 1(3)+3(-3) & 1(-9)+3(9) & 1(-6)+3(-12) \\ 2(3)+(-1)(-3) & 2(-9)+(-1)(9) & 2(-6)+(-1)(-12) \end{bmatrix}$$

$$= \begin{bmatrix} 3-9 & -9+27 & -6-36 \\ 6+3 & -18-9 & -12+12 \end{bmatrix} = \begin{bmatrix} -6 & 18 & -42 \\ 9 & -27 & 0 \end{bmatrix}$$

$$(AB) = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \times \begin{bmatrix} -1 & 3 & 2 \\ 1 & -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1(-1)+3(1) & 1(3)+3(-3) & 1(2)+3(4) \\ 2(-1)+(-1)(1) & 2(3)+(-1)(-3) & 2(2)+(-1)(4) \end{bmatrix}$$

$$= \begin{bmatrix} -1+3 & 3-9 & 2+12 \\ -2-1 & 6+3 & 4-4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -6 & 14 \\ -3 & 9 & 0 \end{bmatrix}$$

$$r(AB) = (-3)\begin{bmatrix} 2 & -6 & 14 \\ -3 & 9 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -3(2) & -3(-6) & -3(14) \\ -3(-3) & -3(9) & -3(0) \end{bmatrix} = \begin{bmatrix} -6 & 18 & -42 \\ 9 & -27 & 0 \end{bmatrix}$$

$$(rA) = -3\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -3(1) & -3(3) \\ -3(2) & -3(-1) \end{bmatrix} = \begin{bmatrix} -3 & -9 \\ -6 & 3 \end{bmatrix}$$

$$(rA)B = \begin{bmatrix} -3 & -9 \\ -6 & 3 \end{bmatrix} \times \begin{bmatrix} -1 & 3 & 2 \\ 1 & -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -3(-1)+(-9)(1) & -3(3)+(-9)(-3) & -3(2)+(-9)(4) \\ -6(-1)+3(1) & -6(3)+3(-3) & -6(2)+3(4) \end{bmatrix}$$

$$= \begin{bmatrix} 3-9 & -9+27 & -6-36 \\ 6+3 & -18-9 & -12+12 \end{bmatrix} = \begin{bmatrix} -6 & 18 & -42 \\ 9 & -27 & 0 \end{bmatrix}$$

EXERCÍCIO 6 (PÁGINA 27)

6.a $(A+B)^T = A^T + B^T$

$$(A+B) = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & -3 \end{bmatrix} + \begin{bmatrix} 4 & 2 & -1 \\ -2 & 1 & 5 \end{bmatrix}$$

$$(A+B)=\begin{bmatrix}1+4 & 3+2 & 2+(-1)\\ 2+(-2) & 1+1 & -3+5\end{bmatrix}=\begin{bmatrix}5 & 5 & 1\\ 0 & 2 & 2\end{bmatrix}$$

$$(A+B)^T=\begin{bmatrix}5 & 0\\ 5 & 2\\ 1 & 2\end{bmatrix}$$

$$A^T=\begin{bmatrix}1 & 2\\ 3 & 1\\ 2 & -3\end{bmatrix} \qquad B^T=\begin{bmatrix}4 & -2\\ 2 & 1\\ -1 & 5\end{bmatrix}$$

$$A^T+B^T=\begin{bmatrix}1 & 2\\ 3 & 1\\ 2 & -3\end{bmatrix}+\begin{bmatrix}4 & -2\\ 2 & 1\\ -1 & 5\end{bmatrix}=\begin{bmatrix}1+4 & 2+(-2)\\ 3+2 & 1+1\\ 2+(-1) & -3+5\end{bmatrix}=\begin{bmatrix}5 & 0\\ 5 & 2\\ 1 & 2\end{bmatrix}$$

EXERCÍCIO 7 (PÁGINA 27)

$$(AB)^T=B^TA^T$$

$$(AB)=\begin{bmatrix}1 & 3 & 2\\ 2 & 1 & -3\end{bmatrix}\times\begin{bmatrix}3 & -1\\ 2 & 4\\ 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}1(3)+3(2)+2(1) & 1(-1)+3(4)+2(2)\\ 2(3)+1(2)+(-3)(1) & 2(-1)+1(4)+(-3)(2)\end{bmatrix}$$

$$=\begin{bmatrix}3+6+2 & -1+12+4\\ 6+2-3 & -2+4-6\end{bmatrix}$$

$$=\begin{bmatrix}11 & 15\\ 5 & -4\end{bmatrix}$$

$$(AB)^T=\begin{bmatrix}11 & 5\\ 15 & -4\end{bmatrix}$$

$$B^T = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 4 & 2 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 2 & -3 \end{bmatrix}$$

$$B^T . A^T = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 4 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 3(1)+2(3)+1(2) & 3(2)+2(1)+1(-3) \\ (-1)(1)+4(3)+2(2) & (-1)(2)+4(1)+2(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 3+6+2 & 6+2-3 \\ -1+12+4 & -2+4-6 \end{bmatrix} = \begin{bmatrix} 11 & 5 \\ 15 & -4 \end{bmatrix}$$

EXERCÍCIO 8 (PÁGINA 27)

$$AB = \begin{bmatrix} -2 & 3 \\ 2 & -3 \end{bmatrix} \times \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -2(3)+3(2) & -2(6)+3(4) \\ 2(3)+(-3)(2) & 2(6)+(-3)(4) \end{bmatrix}$$

$$= \begin{bmatrix} -6+6 & -12+12 \\ 6-6 & 12-12 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

EXERCÍCIO 9 (PÁGINA 28)

$$AB = \begin{bmatrix} -2 & 3 \\ 2 & -3 \end{bmatrix} \times \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -2(-1)+3(2) & -2(3)+3(0) \\ 2(-1)+(-3)(2) & 2(3)+(-3)(0) \end{bmatrix}$$

$$= \begin{bmatrix} 2+6 & -6+0 \\ -2-6 & 6+0 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -6 \\ -8 & 6 \end{bmatrix}$$

$$AC = \begin{bmatrix} -2 & 3 \\ 2 & -3 \end{bmatrix} \times \begin{bmatrix} -4 & -3 \\ 0 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} (-2)(-4)+3(0) & -2(-3)+3(-4) \\ 2(-4)+(-3)(0) & 2(-3)+(-3)(-4) \end{bmatrix}$$

$$= \begin{bmatrix} 8+0 & 6-12 \\ -8+0 & -6+12 \end{bmatrix} = \begin{bmatrix} 8 & -6 \\ -8 & 6 \end{bmatrix}$$

EXERCÍCIO 10 (PÁGINA 28)

$$A^2 = A.A$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0(0)+1(1) & 0(1)+1(0) \\ 1(0)+0(1) & 1(1)+0(0) \end{bmatrix} = \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & 1+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

EXERCÍCIO 11 (PÁGINA 28)

$$A^2 = A.A$$

$$= \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix} \times \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4(4)+2(1) & 4(2)+2(3) \\ 1(4)+3(1) & 1(2)+3(3) \end{bmatrix} = \begin{bmatrix} 16+2 & 8+6 \\ 4+3 & 2+9 \end{bmatrix} = \begin{bmatrix} 18 & 14 \\ 7 & 11 \end{bmatrix}$$

$$A^3 = A^2.A$$

$$= \begin{bmatrix} 18 & 14 \\ 7 & 11 \end{bmatrix} \times \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 18(4)+14(1) & 18(2)+14(3) \\ 7(4)+11(1) & 7(2)+11(3) \end{bmatrix}$$

$$= \begin{bmatrix} 72+14 & 36+42 \\ 28+11 & 14+33 \end{bmatrix} = \begin{bmatrix} 86 & 78 \\ 39 & 47 \end{bmatrix}$$

11.A $A^2+3A=\begin{bmatrix}18 & 14\\ 7 & 11\end{bmatrix}+(3)\begin{bmatrix}4 & 2\\ 1 & 3\end{bmatrix}$

$=\begin{bmatrix}18 & 14\\ 7 & 11\end{bmatrix}+\begin{bmatrix}3(4) & 3(2)\\ 3(1) & 3(3)\end{bmatrix}$

$=\begin{bmatrix}18 & 14\\ 7 & 11\end{bmatrix}+\begin{bmatrix}12 & 6\\ 3 & 9\end{bmatrix}$

$=\begin{bmatrix}18+12 & 14+6\\ 7+3 & 11+9\end{bmatrix}=\begin{bmatrix}30 & 20\\ 10 & 20\end{bmatrix}$

11.B $2A^3+3A^2+4A+5I_2$

$(2)\begin{bmatrix}86 & 78\\ 39 & 47\end{bmatrix}+(3)\begin{bmatrix}18 & 14\\ 7 & 11\end{bmatrix}+(4)\begin{bmatrix}4 & 2\\ 1 & 3\end{bmatrix}+(5)\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\begin{bmatrix}2(86) & 2(78)\\ 2(39) & 2(47)\end{bmatrix}+\begin{bmatrix}3(18) & 3(14)\\ 3(7) & 3(11)\end{bmatrix}+\begin{bmatrix}4(4) & 4(2)\\ 4(1) & 4(3)\end{bmatrix}+\begin{bmatrix}5(1) & 5(0)\\ 5(0) & 5(1)\end{bmatrix}$

$\begin{bmatrix}172 & 156\\ 78 & 94\end{bmatrix}+\begin{bmatrix}54 & 42\\ 21 & 33\end{bmatrix}+\begin{bmatrix}16 & 8\\ 4 & 12\end{bmatrix}+\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix}$

$\begin{bmatrix}172+54+16+5 & 156+42+8+0\\ 78+21+4+0 & 94+33+12+5\end{bmatrix}=\begin{bmatrix}247 & 206\\ 103 & 144\end{bmatrix}$

EXERCÍCIO 12 (PÁGINA 28)

$A^2=A.A$

$=\begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}\times\begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}$

$=\begin{bmatrix}1(1)+(-1)(2) & 1(-1)+(-1)(3)\\ 2(1)+3(2) & 2(-1)+3(3)\end{bmatrix}$

$=\begin{bmatrix}1-2 & -1-3\\ 2+6 & -2+9\end{bmatrix}$

$=\begin{bmatrix}-1 & -4\\ 8 & 7\end{bmatrix}$

$$A^3 = A^2 \cdot A$$

$$= \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1(1)+(-4)(2) & -1(-1)+(-4)(3) \\ 8(1)+7(2) & 8(-1)+7(3) \end{bmatrix}$$

$$= \begin{bmatrix} -1-8 & 1-12 \\ 8+14 & -8+21 \end{bmatrix} = \begin{bmatrix} -9 & -11 \\ 22 & 13 \end{bmatrix}$$

12.A $A^2 - 2A = \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} - (2)\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$$= \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} - \begin{bmatrix} 2(1) & 2(-1) \\ 2(2) & 2(3) \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} - \begin{bmatrix} 2 & -2 \\ 4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -1-2 & -4-(-2) \\ 8-4 & 7-6 \end{bmatrix} = \begin{bmatrix} -3 & -2 \\ 4 & 1 \end{bmatrix}$$

12.B $3A^3 - 2A^2 + 5A - 4I_2$

$$(3)\begin{bmatrix} -9 & -11 \\ 22 & 13 \end{bmatrix} - (2)\begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} + (5)\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} - (4)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3(-9) & 3(-11) \\ 3(22) & 3(13) \end{bmatrix} - \begin{bmatrix} 2(-1) & 2(-4) \\ 2(8) & 2(7) \end{bmatrix} + \begin{bmatrix} 5(1) & 5(-1) \\ 5(2) & 5(3) \end{bmatrix} - \begin{bmatrix} 4(1) & 4(0) \\ 4(0) & 4(1) \end{bmatrix}$$

$$\begin{bmatrix} -27 & -33 \\ 66 & 39 \end{bmatrix} - \begin{bmatrix} -2 & -8 \\ 16 & 14 \end{bmatrix} + \begin{bmatrix} 5 & -5 \\ 10 & 15 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} -27-(-2)+5-4 & -33-(-8)+(-5)-0 \\ 66-16+10-0 & 39-14+15-4 \end{bmatrix}$$

$$\begin{bmatrix} -24 & -30 \\ 60 & 36 \end{bmatrix}$$

EXERCÍCIO 13 (PÁGINA 28)

13.A $X_1 = A X_0$

$$= \begin{bmatrix} 1/3 & 2/3 \\ 2/3 & 3/5 \end{bmatrix} \times \begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix}$$

$$= \begin{bmatrix} 1/3(2/3) + 2/5(1/3) \\ 2/3(2/3) + 3/5(1/3) \end{bmatrix} = \begin{bmatrix} 2/9 + \frac{2}{15} \\ 4/9 + 1/5 \end{bmatrix} = \begin{bmatrix} \frac{10+6}{45} \\ \frac{20+9}{45} \end{bmatrix} = \begin{bmatrix} 16/45 \\ 29/45 \end{bmatrix}$$

13.B $X_0 = \begin{bmatrix} a \\ b \end{bmatrix}$ $\quad a + b = 1$

$a = 1 - b$

$A X_0 = X_0$

$$\begin{bmatrix} 1/3 & 2/5 \\ 2/3 & 3/5 \end{bmatrix} \times \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\begin{bmatrix} 1/3\, a + 2/5\, b \\ 2/3\, a + 3/5\, b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\frac{1}{3}a + \frac{2}{5}b = a$$

$$\frac{1}{3}a - a + \frac{2}{5}b = 0$$

$$\frac{a - 3a}{3} + \frac{2}{5}b = 0 \rightarrow -\frac{2}{3}a + \frac{2}{5}b = 0$$

$$\frac{2}{3}a + \frac{3}{5}b = b$$

$$\frac{2}{3}a + \frac{3}{5}b - b = 0$$

$$\frac{2}{3}a + \frac{3b - 5b}{5} = 0 \rightarrow \frac{2}{3}a - \frac{2}{5}b = 0$$

$-\frac{2}{3}a + \frac{2}{5}b = 0$

$-\frac{2}{3}(1-b) + \frac{2}{5}b = 0$

$-\frac{2}{3} + \frac{2}{3}b + \frac{2}{5}b = 0$

$\frac{2}{3}b + \frac{2}{5}b = \frac{2}{3}$

$\frac{10b + 6b = 10}{\cancel{15}}$

$16b = 10$

$b = \frac{\cancel{10}^{5}}{\cancel{16}_{8}} = \frac{5}{8}$

$a = 1 - b = 1 - \frac{5}{8} = \frac{8-5}{8}$

$a = \frac{3}{8}$

CADEIA DE MARKOV

LOGO, A DISTRIBUIÇÃO ESTÁVEL É: $X_0 = \begin{bmatrix} 3/8 \\ 5/8 \end{bmatrix}$

EXERCÍCIO 14 (PÁGINA 28)

$$A = \begin{array}{c} \\ \\ \end{array}\begin{array}{cc} M & N \end{array}$$

$$A = \begin{bmatrix} 1/3 & 1/2 \\ 2/3 & 1/2 \end{bmatrix} \begin{array}{c} M \\ N \end{array}$$

14.A $X_1 = A X_0$

$= \begin{bmatrix} 1/3 & 1/2 \\ 2/3 & 1/2 \end{bmatrix} \times \begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix}$

$= \begin{bmatrix} 1/3(1/3) + 1/2(2/3) \\ 2/3(1/3) + 1/2(2/3) \end{bmatrix} = \begin{bmatrix} 1/9 + 1/3 \\ 2/9 + 1/3 \end{bmatrix} = \begin{bmatrix} \frac{1+3}{9} \\ \frac{2+3}{9} \end{bmatrix} = \begin{bmatrix} 4/9 \\ 5/9 \end{bmatrix}$

14.B $X_0 = \begin{bmatrix} a \\ b \end{bmatrix}$ $a + b = 1$

$a = 1 - b$

$A X_0 = X_0$

$$\begin{bmatrix} 1/3 & 1/2 \\ 2/3 & 1/2 \end{bmatrix} \times \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\begin{bmatrix} 1/3\, a + 1/2\, b \\ 2/3\, a + 1/2\, b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\frac{1}{3} a + \frac{1}{2} b = a$$
$$\frac{1}{3} a - a + \frac{1}{2} b = 0$$
$$\frac{a - 3a}{3} + \frac{1}{2} b = 0 \rightarrow -\frac{2}{3} a + \frac{1}{2} b = 0$$

$$\frac{2}{3} a + \frac{1}{2} b = b$$
$$\frac{2}{3} a + \frac{1}{2} b - b = 0$$
$$\frac{2}{3} a + \frac{b - 2b}{2} = 0 \rightarrow \frac{2}{3} a - \frac{1}{2} b = 0$$

CADEIA DE MARKOV

$$\frac{2}{3} a - \frac{1}{2} b = 0$$
$$\frac{2}{3} (1 - b) - \frac{1}{2} b = 0$$
$$\frac{2}{3} - \frac{2}{3} b - \frac{1}{2} b = 0 \rightarrow -\frac{2}{3} b - \frac{1}{2} b = -\frac{2}{3}$$
$$\frac{-4b - 3b = -4}{6}$$
$$-7b = -4 \rightarrow b = \frac{-4}{-7} = \frac{4}{7}$$

$$a = 1 - b = 1 - \frac{4}{7} \rightarrow a = \frac{7 - 4}{7} = \frac{3}{7}$$

LOGO, A DISTRIBUIÇÃO ESTÁVEL É: $X_0 = \begin{bmatrix} 3/7 \\ 4/7 \end{bmatrix}$

EXERCÍCIO T.1 (PÁGINA 28)

T.1-A $A + (B + C) = (A + B) + C$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mm} \end{bmatrix}_{m \times m} \qquad B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mm} \end{bmatrix}_{m \times m}$$

$$C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1m} \\ c_{21} & c_{22} & \cdots & c_{2m} \\ \vdots & \vdots & & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mm} \end{bmatrix}_{m \times m}$$

$$B + C = \begin{bmatrix} b_{11}+c_{11} & b_{12}+c_{12} & \cdots & b_{1m}+c_{1m} \\ b_{21}+c_{21} & b_{22}+c_{22} & \cdots & b_{2m}+c_{2m} \\ \vdots & \vdots & & \vdots \\ b_{m1}+c_{m1} & b_{m2}+c_{m2} & \cdots & b_{mm}+c_{mm} \end{bmatrix}_{m \times m}$$

$$A + (B + C) = \begin{bmatrix} a_{11}+b_{11}+c_{11} & a_{12}+b_{12}+c_{12} & \cdots & a_{1m}+b_{1m}+c_{1m} \\ a_{21}+b_{21}+c_{21} & a_{22}+b_{22}+c_{22} & \cdots & a_{2m}+b_{2m}+c_{2m} \\ \vdots & \vdots & & \vdots \\ a_{m1}+b_{m1}+c_{m1} & a_{m2}+b_{m2}+c_{m2} & \cdots & a_{mm}+b_{mm}+c_{mm} \end{bmatrix}_{m \times m}$$

$$A + B = \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \cdots & a_{1m}+b_{1m} \\ a_{21}+b_{21} & a_{22}+b_{22} & \cdots & b_{2m}+b_{2m} \\ \vdots & \vdots & & \vdots \\ a_{m1}+b_{m1} & a_{m2}+b_{m2} & \cdots & a_{mm}+b_{mm} \end{bmatrix}_{m \times m}$$

$$(A+B)+C=\begin{bmatrix} a_{11}+b_{11}+c_{11} & a_{12}+b_{12}+c_{12} & \cdots & a_{1n}+b_{1n}+c_{1n} \\ a_{21}+b_{21}+c_{21} & a_{22}+b_{22}+c_{22} & \cdots & a_{2n}+b_{2n}+c_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1}+b_{m1}+c_{m1} & a_{m2}+b_{m2}+c_{m2} & \cdots & a_{mn}+b_{mn}+c_{mn} \end{bmatrix}_{m\times n}$$

T.1.B

$$A=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m\times n}$$

$$D=-A=\begin{bmatrix} -a_{11} & -a_{12} & \cdots & -a_{1n} \\ -a_{21} & -a_{22} & \cdots & -a_{2n} \\ \vdots & \vdots & & \vdots \\ -a_{m1} & -a_{m2} & \cdots & -a_{mn} \end{bmatrix}_{m\times n}$$

$$A+D=A+(-A)=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}+\begin{bmatrix} -a_{11} & -a_{12} & \cdots & -a_{1n} \\ -a_{21} & -a_{22} & \cdots & -a_{2n} \\ \vdots & \vdots & & \vdots \\ -a_{m1} & -a_{m2} & \cdots & -a_{mn} \end{bmatrix}$$

$$=\begin{bmatrix} a_{11}+(-a_{11}) & a_{12}+(-a_{12}) & \cdots & a_{1n}+(-a_{1n}) \\ a_{21}+(-a_{21}) & a_{22}+(-a_{22}) & \cdots & a_{2n}+(-a_{2n}) \\ \vdots & \vdots & & \vdots \\ a_{m1}+(-a_{m1}) & a_{m2}+(-a_{m2}) & \cdots & a_{mn}+(-a_{mn}) \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

EXERCÍCIO T.2 (PÁGINA 28)

$$A=\begin{bmatrix}-1 & 0 & 3\\ 4 & 2 & 5\end{bmatrix}_{2\times 3} \qquad B=\begin{bmatrix}2 & 3 & 5 & 6\\ -1 & 0 & -2 & 4\\ 1 & 5 & -3 & 0\end{bmatrix}_{3\times 4}$$

$$C=\begin{bmatrix}1 & -1 & 1\\ 3 & 0 & 3\\ -1 & 2 & -2\\ 4 & 0 & 5\end{bmatrix}_{4\times 3}$$

$$BC=\begin{bmatrix}2 & 3 & 5 & 6\\ -1 & 0 & -2 & 4\\ 1 & 5 & -3 & 0\end{bmatrix}\times\begin{bmatrix}1 & -1 & 1\\ 3 & 0 & 3\\ -1 & 2 & -2\\ 4 & 0 & 5\end{bmatrix}$$

$$=\begin{bmatrix}2(1)+3(3)+5(-1)+6(4) & 2(-1)+3(0)+5(2)+6(0) & 2(1)+3(3)+5(-2)+6(5)\\ (-1)(1)+0(3)+(-2)(-1)+4(4) & (-1)(-1)+0(0)+(-2)(2)+4(0) & (-1)(1)+0(3)+(-2)(-2)+4(5)\\ 1(1)+5(3)+(-3)(-1)+0(4) & 1(-1)+5(0)+(-3)(2)+0(0) & 1(1)+5(3)+(-3)(-2)+0(5)\end{bmatrix}$$

$$=\begin{bmatrix}2+9-5+24 & -2+0+10+0 & 2+9-10+30\\ -1+0+2+16 & 1+0-4+0 & -1+0+4+20\\ 1+15+3+0 & -1+0-6+0 & 1+15+6+0\end{bmatrix}=\begin{bmatrix}30 & 8 & 31\\ 17 & -3 & 23\\ 19 & -7 & 22\end{bmatrix}$$

$$A(BC)=\begin{bmatrix}-1 & 0 & 3\\ 4 & 2 & 5\end{bmatrix}\times\begin{bmatrix}30 & 8 & 31\\ 17 & -3 & 23\\ 19 & -7 & 22\end{bmatrix}$$

$$=\begin{bmatrix}(-1)(30)+0(17)+3(19) & (-1)(8)+0(-3)+3(-7) & (-1)(31)+0(23)+3(22)\\ 4(30)+2(17)+5(19) & 4(8)+2(-3)+5(-7) & 4(31)+2(23)+5(22)\end{bmatrix}$$

$$=\begin{bmatrix}-30+0+57 & -8+0-21 & -31+0+66\\ 120+34+95 & 32-6-35 & 124+46+110\end{bmatrix}$$

$$=\begin{bmatrix}27 & -29 & 35\\ 249 & -9 & 280\end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & 0 & 3 \\ 4 & 2 & 5 \end{bmatrix} \times \begin{bmatrix} 2 & 3 & 5 & 6 \\ -1 & 0 & -2 & 4 \\ 1 & 5 & -3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (-1)(2)+0(-1)+3(1) & (-1)(3)+0(0)+3(5) & (-1)(5)+0(-2)+3(-3) & (-1)(6)+0(4)+3(0) \\ 4(2)+2(-1)+5(1) & 4(3)+2(0)+5(5) & 4(5)+2(-2)+5(-3) & 4(6)+2(4)+5(0) \end{bmatrix}$$

$$= \begin{bmatrix} -2+0+3 & -3+0+15 & -5+0-9 & -6+0+0 \\ 8-2+5 & 12+0+25 & 20-4-15 & 24+8+0 \end{bmatrix} = \begin{bmatrix} 1 & 12 & -14 & -6 \\ 11 & 37 & 1 & 32 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} 1 & 12 & -14 & -6 \\ 11 & 37 & 1 & 32 \end{bmatrix} \times \begin{bmatrix} 1 & -1 & 1 \\ 3 & 0 & 3 \\ -1 & 2 & -2 \\ 4 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1(1)+12(3)+(-14)(-1)+(-6)(4) & 1(-1)+12(0)+(-14)(2)+(-6)(0) & 1(1)+12(3)+(-14)(-2)+(-6)(5) \\ 11(1)+37(3)+1(-1)+32(4) & 11(-1)+37(0)+1(2)+32(0) & 11(1)+37(3)+1(-2)+32(5) \end{bmatrix}$$

$$= \begin{bmatrix} 1+36+14-24 & -1+0-28+0 & 1+36+28-30 \\ 11+111-1+128 & -11+0+2+0 & 11+111-2+160 \end{bmatrix}$$

$$= \begin{bmatrix} 27 & -29 & 35 \\ 249 & -9 & 280 \end{bmatrix}$$

EXERCÍCIO T.3 (PÁGINA 28)

T.3.A $A(B+C) = AB + AC$

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1p} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mp} \end{bmatrix}_{m \times p} \qquad B = \begin{bmatrix} b_{11} & \cdots & b_{1m} \\ \vdots & & \vdots \\ b_{p1} & \cdots & b_{pm} \end{bmatrix}_{p \times m}$$

$$C = \begin{bmatrix} c_{11} & \cdots & c_{1m} \\ \vdots & & \vdots \\ c_{p1} & \cdots & c_{pm} \end{bmatrix}_{p \times m}$$

$$B+C=\begin{bmatrix} b_{11} & \cdots & b_{1m} \\ \vdots & & \vdots \\ b_{p1} & \cdots & b_{pm} \end{bmatrix}+\begin{bmatrix} c_{11} & \cdots & c_{1m} \\ \vdots & & \vdots \\ c_{p1} & \cdots & c_{pm} \end{bmatrix}$$

$$=\begin{bmatrix} b_{11}+c_{11} & \cdots & b_{1m}+c_{1m} \\ \vdots & & \vdots \\ b_{p1}+c_{p1} & \cdots & b_{pm}+c_{pm} \end{bmatrix}_{p\times m}$$

$$A(B+C)=\begin{bmatrix} a_{11} & \cdots & a_{1p} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mp} \end{bmatrix}\times\begin{bmatrix} b_{11}+c_{11} & \cdots & b_{1m}+c_{1m} \\ \vdots & & \vdots \\ b_{p1}+c_{p1} & \cdots & b_{pm}+c_{pm} \end{bmatrix}$$

$$=\begin{bmatrix} a_{11}(b_{11}+c_{11})+\cdots+a_{1p}(b_{p1}+c_{p1}) & a_{11}(b_{1m}+c_{1m})+\cdots+a_{1p}(b_{pm}+c_{pm}) \\ \vdots & \vdots \\ a_{m1}(b_{11}+c_{11})+\cdots+a_{mp}(b_{p1}+c_{p1}) & a_{m1}(b_{1m}+c_{1m})+\cdots+a_{mp}(b_{pm}+c_{pm}) \end{bmatrix}$$

$$AB=\begin{bmatrix} a_{11} & \cdots & a_{1p} \\ \vdots & & \\ a_{m1} & \cdots & a_{mp} \end{bmatrix}\times\begin{bmatrix} b_{11} & \cdots & b_{1m} \\ \vdots & & \vdots \\ b_{p1} & \cdots & b_{pm} \end{bmatrix}$$

$$=\begin{bmatrix} a_{11}b_{11}+\cdots+a_{1p}b_{p1} & a_{11}b_{1m}+\cdots+a_{1p}b_{pm} \\ \vdots & \vdots \\ a_{m1}b_{11}+\cdots+a_{mp}b_{p1} & a_{m1}b_{1m}+\cdots+a_{mp}b_{pm} \end{bmatrix}$$

$$AC=\begin{bmatrix} a_{11} & \cdots & a_{1p} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mp} \end{bmatrix}\times\begin{bmatrix} c_{11} & \cdots & c_{1m} \\ \vdots & & \\ c_{p1} & \cdots & c_{pm} \end{bmatrix}$$

$$=\begin{bmatrix} a_{11}c_{11}+\cdots+a_{1p}c_{p1} & a_{11}c_{1m}+\cdots+a_{1p}c_{pm} \\ \vdots & \vdots \\ a_{m1}c_{11}+\cdots+a_{mp}c_{p1} & a_{m1}c_{1m}+\cdots+a_{mp}c_{pm} \end{bmatrix}$$

$$AB+AC=\begin{bmatrix} a_{11}(b_{11}+c_{11})+\cdots+a_{1p}(b_{p1}+c_{p1}) & a_{11}(b_{1m}+c_{1m})+\cdots+a_{1p}(b_{pm}+c_{pm}) \\ \vdots & \vdots \\ a_{m1}(b_{11}+c_{11})+\cdots+a_{mp}(b_{p1}+c_{p1}) & a_{m1}(b_{1m}+c_{1m})+\cdots+a_{mp}(b_{pm}+c_{pm}) \end{bmatrix}$$

T.3.B $(A+B)C = AC + BC$

$$A = \begin{bmatrix} a_{11} & \dots & a_{1p} \\ \vdots & & \vdots \\ a_{m1} & \dots & a_{mp} \end{bmatrix}_{m \times p} \qquad B = \begin{bmatrix} b_{11} & \dots & b_{1p} \\ \vdots & & \vdots \\ b_{m1} & \dots & b_{mp} \end{bmatrix}_{m \times p}$$

$$C = \begin{bmatrix} c_{11} & \dots & c_{1m} \\ \vdots & & \vdots \\ c_{p1} & \dots & c_{pm} \end{bmatrix}_{p \times m}$$

$$A+B = \begin{bmatrix} a_{11} & \dots & a_{1p} \\ \vdots & & \vdots \\ a_{m1} & \dots & a_{mp} \end{bmatrix} + \begin{bmatrix} b_{11} & \dots & b_{1p} \\ \vdots & & \vdots \\ b_{m1} & \dots & b_{mp} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}+b_{11} & \dots & a_{1p}+b_{1p} \\ \vdots & & \vdots \\ a_{m1}+b_{m1} & \dots & a_{mp}+b_{mp} \end{bmatrix}_{m \times p}$$

$$(A+B)C = \begin{bmatrix} a_{11}+b_{11} & \dots & a_{1p}+b_{1p} \\ \vdots & & \vdots \\ a_{m1}+b_{m1} & \dots & a_{mp}+b_{mp} \end{bmatrix} \times \begin{bmatrix} c_{11} & \dots & c_{1m} \\ \vdots & & \vdots \\ c_{p1} & \dots & c_{pm} \end{bmatrix}$$

$$= \begin{bmatrix} (a_{11}+b_{11})c_{11}+\dots+(a_{1p}+b_{1p})c_{p1} & \dots & (a_{11}+b_{11})c_{1m}+\dots+(a_{1p}+b_{1p})c_{pm} \\ \vdots & & \vdots \\ (a_{m1}+b_{m1})c_{11}+\dots+(a_{mp}+b_{mp})c_{p1} & \dots & (a_{m1}+b_{m1})c_{1m}+\dots+(a_{mp}+b_{mp})c_{pm} \end{bmatrix}$$

$$AC = \begin{bmatrix} a_{11} & \dots & a_{1p} \\ \vdots & & \vdots \\ a_{m1} & \dots & a_{mp} \end{bmatrix} \times \begin{bmatrix} c_{11} & \dots & c_{1m} \\ \vdots & & \vdots \\ c_{p1} & \dots & c_{pm} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}c_{11}+\dots+a_{1p}c_{p1} & a_{11}c_{1m}+\dots+a_{1p}c_{pm} \\ \vdots & \vdots \\ a_{m1}c_{11}+\dots+a_{mp}c_{p1} & a_{m1}c_{1m}+\dots+a_{mp}c_{pm} \end{bmatrix}$$

$$BC = \begin{bmatrix} b_{11} & \cdots & b_{1p} \\ \vdots & & \vdots \\ b_{m1} & \cdots & b_{mp} \end{bmatrix} \times \begin{bmatrix} c_{11} & \cdots & c_{1n} \\ \vdots & & \vdots \\ c_{p1} & \cdots & c_{pn} \end{bmatrix}$$

$$= \begin{bmatrix} b_{11}c_{11} + \cdots + b_{1p}c_{p1} & b_{11}c_{1n} + \cdots + b_{1p}c_{pn} \\ \vdots & \vdots \\ b_{m1}c_{11} + \cdots + b_{mp}c_{p1} & b_{m1}c_{1n} + \cdots + b_{mp}c_{pn} \end{bmatrix}$$

$$AC+BC = \begin{bmatrix} (a_{11}+b_{11})c_{11} + \cdots + (a_{1p}+b_{1p})c_{p1} \cdots (a_{11}+b_{11})c_{1n} + \cdots + (a_{1p}+b_{1p})c_{pn} \\ \vdots \quad\quad \vdots \\ (a_{m1}+b_{m1})c_{11} + \cdots + (a_{mp}+b_{mp})c_{p1} \cdots (a_{m1}+b_{m1})c_{1n} + \cdots + (a_{mp}+b_{mp})c_{pn} \end{bmatrix}$$

EXERCÍCIO T.4 (PÁGINA 28)

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mm} \end{bmatrix}_{m \times m} \qquad I_m = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}_{m \times m}$$

$$A I_m = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mm} \end{bmatrix} \times \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mm} \end{bmatrix}$$

$$I_m = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}_{m \times m}$$

$$I_m A = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix} \times \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mm} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mm} \end{bmatrix}$$

EXERCÍCIO T.5 (PÁGINA 28)

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mm} \end{bmatrix}_{m \times m}$$

T.5.A $A^p A^q = A^{p+q}$

$A^p = \underbrace{A.A\dots A}_{p \text{ FATORES}}$ E $A^q = \underbrace{A.A\dots A}_{q \text{ FATORES}}$

$A^p . A^q = \underbrace{A.A\dots A}_{p \text{ FATORES}} . \underbrace{A.A\dots A}_{q \text{ FATORES}}$

$= A^{p+q}$

T.5.B $(A^p)^q = A^{pq}$

$A^p = \underbrace{A.A\dots A}_{p \text{ FATORES}}$ E $(A^p)^q = \underbrace{A^p.A^p\dots A^p}_{q \text{ FATORES}}$

$$(A^p)^q = \underbrace{A^p \cdot A^p \ldots A^p}_{q \text{ FATORES}}$$

$$= \underbrace{A \cdot A \ldots A}_{p \text{ FATORES}} \cdot \underbrace{AA \ldots A}_{p \text{ FATORES}} \cdots \underbrace{A \cdot A \ldots A}_{p \text{ FATORES}}$$

$$= A^{pq}$$

EXERCÍCIO T.6 (PÁGINA 28)

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1p} \\ a_{21} & a_{22} & \ldots & a_{2p} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \ldots & a_{mp} \end{bmatrix}_{m \times p} \qquad B = \begin{bmatrix} b_{11} & b_{12} & \ldots & b_{1n} \\ b_{21} & b_{22} & \ldots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{p1} & b_{p2} & \ldots & b_{pn} \end{bmatrix}_{p \times n}$$

$$(AB)^p = A^p \cdot B^p$$

$$(AB)^p = \underbrace{AB \cdot AB \ldots AB}_{p \text{ FATORES}}$$

$$= \underbrace{A \cdot A \ldots A}_{p \text{ FATORES}} \cdot \underbrace{B \cdot B \ldots B}_{p \text{ FATORES}}$$

$$= A^p \cdot B^p$$

EXERCÍCIO T.7 (PÁGINA 28)

$$(rA)^p = r^p A^p \qquad r \Rightarrow \text{ESCALAR}$$

$$(rA)^p = \underbrace{rA \cdot rA \ldots rA}_{p \text{ FATORES}}$$

$$= \underbrace{r \cdot r \ldots r}_{p \text{ FATORES}} \cdot \underbrace{A \cdot A \ldots A}_{p \text{ FATORES}}$$

$$= r^p \cdot A^p$$

EXERCÍCIO T.8 (PÁGINA 28)

T.8.A $r(sA) = (rs)A$

O (i,j)-ÉSIMO ELEMENTO DE sA É $s a_{ij}$. O (i,j)-ÉSIMO ELEMENTO DE $r(sA)$ É $r(s a_{ij})$. COMO O (i,j)-ÉSIMO ELEMENTO DE $(rs)A$ É $(rs)a_{ij}$ TEMOS O RESULTADO DESEJADO.

T.8.B $(r+s)A = rA + sA$

O (i,j)-ÉSIMO ELEMENTO DE $(r+s)A$ É $(r+s)a_{ij}$. O (i,j)-ÉSIMO ELEMENTO DE rA É $r a_{ij}$ E O (i,j)-ÉSIMO ELEMENTO DE sA É $s a_{ij}$, COMO:

$$(r+s)a_{ij} = r a_{ij} + s a_{ij} \quad (1 \leq i \leq m,\ 1 \leq j \leq n)$$

TEMOS O RESULTADO DESEJADO.

T.8.C $r(A+B) = rA + rB$ SEJAM $A = [a_{ij}]_{m\times n}$ E $B = [b_{ij}]_{m\times n}$

O (i,j)-ÉSIMO ELEMENTO DE $r(A+B)$ É $r(a_{ij} + b_{ij})$. O (i,j)-ÉSIMO ELEMENTO DE $r(A)$ É $r(a_{ij})$ E O (i,j)-ÉSIMO ELEMENTO DE $r(B)$ É $r(b_{ij})$. COMO:

$$r(a_{ij} + b_{ij}) = r(a_{ij}) + r(b_{ij}) \quad (1 \leq i \leq m, 1 \leq j \leq n)$$

TEMOS O RESULTADO DESEJADO.

T.8.D $A(rB) = r(AB) = (rA)B$ SEJAM $A = [a_{ij}]_{m\times p}$ E $B = [b_{ij}]_{p\times n}$

O (i,j)-ÉSIMO ELEMENTO DE $A(rB)$ É $a_{ij}(rb_{ij})$, O (i,j)-ÉSIMO ELEMENTO DE $r(AB)$ É $r(a_{ij}b_{ij})$ E O (i,j)-ÉSIMO ELEMENTO DE $(rA)B$ É $(ra_{ij})b_{ij}$. COMO:

$$a_{ij}(rb_{ij}) = r(a_{ij}b_{ij}) = (ra_{ij})b_{ij}$$

TEMOS O RESULTADO DESEJADO.

EXERCÍCIO T.9 (PÁGINA 28)

$(-1)A = -A$

O (i,j)-ÉSIMO ELEMENTO DE $(-1)A$ É $(-1)a_{ij}$ E O (i,j)-ÉSIMO ELEMENTO DE $-A$ É $-a_{ij}$. COMO:

$$(-1)a_{ij} = -a_{ij} \quad (1 \le i \le m, 1 \le j \le m)$$

TEMOS O RESULTADO DESEJADO.

EXERCÍCIO T.10 (PÁGINA 28)

T.10.A $(A^T)^T = A$

SEJA $A = [a_{ij}]_{m \times m}$

$A = [a_{i1} \ a_{i2} \ldots a_{im}]$

$$A^T = \begin{bmatrix} a_{1i} \\ a_{2i} \\ \vdots \\ a_{mi} \end{bmatrix} \qquad (A^T)^T = [a_{i1} \ a_{i2} \ldots a_{im}]$$

COMO $(A^T)^T = A$ TEMOS O RESULTADO DESEJADO.

T.10.B $(A+B)^T = A^T + B^T$

SEJAM $A = [a_{ij}]_{m \times n}$ E $B = [b_{ij}]_{m \times n}$

$A = [a_{i1} \ a_{i2} \cdots a_{in}]$ E $B = [b_{i1} \ b_{i2} \cdots b_{in}]$

$A + B = [a_{i1} + b_{i1} \ \ a_{i2} + b_{i2} \cdots a_{in} + b_{in}]$

$$(A+B)^T = \begin{bmatrix} a_{1i} + b_{1i} \\ a_{2i} + b_{2i} \\ \vdots \\ a_{mi} + b_{mi} \end{bmatrix}$$

$$A^T = \begin{bmatrix} a_{1i} \\ a_{2i} \\ \vdots \\ a_{mi} \end{bmatrix} \quad E \quad B^T = \begin{bmatrix} b_{1i} \\ b_{2i} \\ \vdots \\ b_{mi} \end{bmatrix}$$

$$A^T + B^T = \begin{bmatrix} a_{1i} + b_{1i} \\ a_{2i} + b_{2i} \\ \vdots \\ a_{mi} + b_{mi} \end{bmatrix}$$

COMO $(A+B)^T = A^T + B^T$ TEMOS O RESULTADO DESEJADO.

T.10.C $(rA)^T = rA^T$

SEJA $A = [a_{ij}]_{m \times n}$

$A = [a_{i1} \ a_{i2} \dots a_{im}]$

$rA = r[a_{i1} \ a_{i2} \dots a_{im}]$
$= [ra_{i1} \ ra_{i2} \dots ra_{im}]$

$$(rA)^T = \begin{bmatrix} ra_{1i} \\ ra_{2i} \\ \vdots \\ ra_{mi} \end{bmatrix}$$

$$A^T = \begin{bmatrix} a_{1i} \\ a_{2i} \\ \vdots \\ a_{mi} \end{bmatrix} \qquad rA^T = r\begin{bmatrix} a_{1i} \\ a_{2i} \\ \vdots \\ a_{mi} \end{bmatrix} = \begin{bmatrix} ra_{1i} \\ ra_{2i} \\ \vdots \\ ra_{mi} \end{bmatrix}$$

COMO $(rA)^T = rA^T$ TEMOS O RESULTADO DESEJADO.

EXERCÍCIO T.11 (PÁGINA 28)

SEJAM $A = [a_{ij}]_{m \times m}$ E $A^T = [a_{ji}]_{m \times m}$

O (i,j)-ÉSIMO ELEMENTO DE A É a_{ij} E O (j,i)-ÉSIMO ELEMENTO DE A^T É a_{ji}. SE:

$a_{ij} = a_{ji} \Rightarrow a_{ji} = a_{ij} \quad (1 \leq i \leq m, 1 \leq j \leq m)$

LOGO: $A^T = A$

TEMOS QUE A MATRIZ A É SIMÉTRICA

EXERCÍCIO T.12 (PÁGINA 28)

SEJA $A = [a_{ij}]_{m \times m}$

SE A MATRIZ A É SIMÉTRICA, SIGNIFICA QUE O (i, j)-ÉSIMO ELEMENTO DE A (a_{ij}) SERÁ IGUAL AO (j, i)-ÉSIMO ELEMENTO a_{ji}. ESSE ELEMENTO PERTENCE A A^T. LOGO, A A^T TAMBÉM É SIMÉTRICA.

EXERCÍCIO T.13 (PÁGINA 28)

T.13.A) SEJAM $A = [a_{ij}]_{m \times m}$ E $B = [b_{ij}]_{m \times m}$

$A + B = [a_{ij} + b_{ij}]_{m \times m}$

SE $a_{ij} = a_{ji}$ E $b_{ij} = b_{ji}$ SENDO ASSIM:

$$a_{ij} + b_{ij} = a_{ji} + b_{ji} \quad (1 \leq i \leq m, 1 \leq j \leq m)$$

TEMOS O RESULTADO DESEJADO.

T.13.B) SEJAM $A = [a_{ij}]_{m \times m}$ E $B = [b_{ij}]_{m \times m}$

$AB = [a_{ij} b_{ij}]_{m \times m}$ E $BA = [b_{ij} a_{ij}]_{m \times m}$

SE $a_{ij} = a_{ji}$ E $b_{ij} = b_{ji}$ $(1 \leq i \leq m, 1 \leq j \leq m)$ TEMOS:

$AB = [a_{ij} b_{ij}] = [a_{ji} b_{ji}]$ E $BA = [b_{ij} a_{ij}] = [b_{ji} a_{ji}]$

PORÉM SEJA $A = [a_{ij}]_{m \times m}$ E $B = [b_{ij}]_{m \times p}$

O PRODUTO AB ESTÁ DEFINIDO POIS O NÚMERO DE LINHAS DE B É IGUAL AO NÚMERO DE COLUNAS DE A. POR SUA VEZ, O PRODUTO BA NÃO ESTÁ DEFINIDO DEVIDO A DIFERENÇA NO NÚMERO DE COLUNAS DE B E O NÚMERO DE LINHAS DE A.

EXERCÍCIO T.15 (PÁGINA 28)

SEJA $A = [a_{ij}]_{m \times m}$ E $A^T = [-a_{ji}]_{m \times m}$

O (i,j)-ÉSIMO ELEMENTO DE A É a_{ij} E O (j,i)-ÉSIMO ELEMENTO DE A^T É $-a_{ji}$. SE:

$$a_{ij} = -a_{ji} \Rightarrow -a_{ji} = a_{ij} \quad (1 \leq i \leq m,\ 1 \leq j \leq m)$$

TEMOS O RESULTADO DESEJADO.

EXERCÍCIO T.15 (PÁGINA 28)

SEJA $A = [a_{ij}]_{m \times m}$

T.15.A $A = [a_{ij}]_{m \times m}$ E $A^T = [a_{ji}]_{m \times m}$

O (i,j)-ÉSIMO ELEMENTO DE A É a_{ij} E O (j,i)-ÉSIMO ELEMENTO DE A^T É a_{ji}. SE $a_{ij} = a_{ji}$ A E A^T SÃO SIMÉTRICAS.
↳ E VICE-VERSA

$$\begin{aligned} AA^T &= [a_{ij}\, a_{ji}] \\ &= [a_{ij}\, a_{ij}] \\ &= [a_{ij}]^2 \end{aligned}$$

LOGO AA^T É SIMÉTRICA

$A^T A = [a_{ji}\, a_{ij}]$
$= [a_{ji}\, a_{ji}]$
$= [a_{ji}]^2$

LOGO $A^T A$ É SIMÉTRICA.

T.15.B $A = [a_{ij}]_{m \times m}$ E $A^T = [a_{ji}]_{m \times m}$

O(i, j)-ÉSIMO ELEMENTO DE A É a_{ij} E O(j, i)-ÉSIMO ELEMENTO DE A^T É a_{ji}. SE $a_{ij} = a_{ji}$ A E A^T SÃO SIMÉTRICAS.
↳ E VICE-VERSA

$A + A^T = [a_{ij} + a_{ji}]$
$= [a_{ij} + a_{ij}]$
$= 2[a_{ij}]$

LOGO $A + A^T$ É SIMÉTRICA.

T.15.C $A = [a_{ij}]_{m \times m}$ E $A^T = [-a_{ji}]$

O (i, j)-ÉSIMO ELEMENTO DE A É a_{ij} E O(j, i)-ÉSIMO ELEMENTO DE A^T É a_{ji}. SE $a_{ij} = -a_{ji}$ E $-a_{ji} = a_{ij}$ A E A^T SÃO ANTI-SIMÉTRICAS

$A - A^T = [a_{ij} - (-a_{ji})]$
$= [a_{ij} + a_{ji}]$
$= [a_{ij} + a_{ij}]$
$= 2[a_{ij}]$

LOGO $A - A^T$ É SIMÉTRICA

?

tilibra

EXERCÍCIO T.16 (PÁGINA 28)

EXERCÍCIO T.17 (PÁGINA 28)

SEJA $A = [a_{ij}]_{m \times m}$, EM QUE $a_{ij} = r$ SE $i = j$ E $a_{ij} = 0$ SE $i \neq j$. ENTÃO:

$$r I_m = A$$

TEMOS O RESULTADO DESEJADO.

EXERCÍCIO T.18 (PÁGINA 28)

SEJAM $I_m = [i_{ij}]$ E $I_m^T = [i_{ji}]$

O (i, j)-ÉSIMO ELEMENTO DE I_m É i_{ij} E O (j, i)-ÉSIMO ELEMTO DE I_m^T É i_{ji}. SE:

$$i_{ij} = i_{ji} \Rightarrow i_{ji} = i_{ij} \quad (1 \leq i \leq m, 1 \leq j \leq m)$$

ENTÃO A MATRIZ I_m É UMA MATRIZ SIMÉTRICA SENDO ASSIM, TEMOS O RESULTADO DESEJADO $(I_m^T = I_m)$

EXERCÍCIO T.19 (PÁGINA 28)

SEJA $A = [a_{ij}]_{m \times n}$

$rA = r\, a_{ij}$

SE $rA = 0$, TEMOS QUE $r a_{ij} = 0$

ENTÃO:

$$r a_{ij} = 0 \qquad \text{OU} \qquad r a_{ij} = 0$$

$$r = \frac{0}{a_{ij}} = 0 \qquad\qquad a_{ij} = \frac{0}{r} = 0$$

NO SEGUNDO CASO ($a_{ij} = 0$) TEMOS:

$$A = [a_{ij}] = [0] = 0$$

EXERCÍCIO T.20 (PÁGINA 28)

EXERCÍCIO T.27 (PÁGINA 28)