EXERCÍCIOS 1.4 ⇒ ÁLGEBRA LINEAR, BERNARD KOLMAN, EDITORA GUANABARA, 3ª EDIÇÃO

EXERCÍCIO 1 (PÁGINA 40)

A, E, G

EXERCÍCIO 2 (PÁGINA 41)

2.A

$$A_1 = \begin{bmatrix} 1 & 0 & 3 \\ 5 & -1 & 5 \\ 4 & 2 & 2 \\ -3 & 1 & 4 \end{bmatrix}$$

2.B

$$A_2 = \begin{bmatrix} 1 & 0 & 3 \\ -3 & 1 & 4 \\ 4 & 2 & 2 \\ 5 & -1 & 5 \end{bmatrix} x(3) = \begin{bmatrix} 1 & 0 & 3 \\ -9 & 3 & 12 \\ 4 & 2 & 2 \\ 5 & -1 & 5 \end{bmatrix}$$

2.C

$$A_3 = \begin{bmatrix} 1 & 0 & 3 \\ -3 & 1 & 4 \\ 4 & 2 & 2 \\ 5 & -1 & 5 \end{bmatrix} \begin{matrix} x(-3) \\ + \end{matrix} = \begin{bmatrix} 1 & 0 & 3 \\ -3 & 1 & 4 \\ 4 & 2 & 2 \\ 2 & -1 & -4 \end{bmatrix}$$

EXERCÍCIO 3 (PÁGINA 41)

3.A

$$A_1 = \begin{bmatrix} 2 & 0 & 4 & 2 \\ -1 & 3 & 1 & 1 \\ 3 & -2 & 5 & 6 \end{bmatrix}$$

3.B

$$A_2 = \begin{bmatrix} 2 & 0 & 4 & 2 \\ 3 & -2 & 5 & 6 \\ -1 & 3 & 1 & 1 \end{bmatrix} \times(-4) = \begin{bmatrix} 2 & 0 & 4 & 2 \\ -12 & 8 & -20 & -24 \\ -1 & 3 & 1 & 1 \end{bmatrix}$$

3.C

$$A_3 = \begin{bmatrix} 2 & 0 & 4 & 2 \\ 3 & -2 & 5 & 6 \\ -1 & 3 & 1 & 1 \end{bmatrix} \begin{matrix} + \\ \times(2) \end{matrix} = \begin{bmatrix} 0 & 6 & 6 & 4 \\ 3 & -2 & 5 & 6 \\ -1 & 3 & 1 & 1 \end{bmatrix}$$

EXERCÍCIO 4 (PÁGINA 41)

$$A_1 = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 1 & 2 & -1 \\ 5 & 2 & -3 & 4 \end{bmatrix} \begin{matrix} \times(2) \\ + \end{matrix} = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 1 & 2 & -1 \\ 5 & 4 & 1 & 2 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 1 & 2 & -1 \\ 5 & 2 & -3 & 4 \end{bmatrix} \begin{matrix} + \\ \times(2) \end{matrix} = \begin{bmatrix} 12 & 3 & -3 & 12 \\ 0 & 1 & 2 & -1 \\ 5 & 2 & -3 & 4 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 1 & 2 & -1 \\ 5 & 2 & -3 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 2 & -3 & 4 \\ 0 & 1 & 2 & -1 \\ 2 & -1 & 3 & 4 \end{bmatrix} \begin{matrix} \times(-1) \\ + \end{matrix} = \begin{bmatrix} 5 & 2 & -3 & 4 \\ -5 & -1 & 5 & -5 \\ 2 & -1 & 3 & 4 \end{bmatrix}$$

EXERCÍCIO 5 (PÁGINA 41)

$$A_1 = \begin{bmatrix} 4 & 3 & 7 & 5 \\ -1 & 2 & -1 & 3 \\ 2 & 0 & 1 & 4 \end{bmatrix} \begin{matrix} + \\ \times(-2) \end{matrix} = \begin{bmatrix} 0 & 3 & 5 & -3 \\ -1 & 2 & -1 & 3 \\ 2 & 0 & 1 & 4 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 4 & 3 & 7 & 5 \\ -1 & 2 & -1 & 3 \\ 2 & 0 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 3 & 7 & 5 \\ 2 & 0 & 1 & 4 \\ -1 & 2 & -1 & 3 \end{bmatrix} \begin{matrix} + \\ \times(2) \end{matrix} = \begin{bmatrix} 4 & 3 & 7 & 5 \\ 0 & 4 & -1 & 10 \\ -1 & 2 & -1 & 3 \end{bmatrix}$$

$$A_3=\begin{bmatrix} 4 & 3 & 7 & 5 \\ -1 & 2 & -1 & 3 \\ 2 & 0 & 1 & 4 \end{bmatrix} \begin{matrix} \times(-2) \\ + \\ \end{matrix} = \begin{bmatrix} 4 & 3 & 7 & 5 \\ -1 & 2 & -1 & 3 \\ -6 & -6 & -13 & -6 \end{bmatrix}$$

EXERCÍCIO 6 (PÁGINA 41)

$$C=\begin{bmatrix} 0 & 0 & -1 & 2 & 3 \\ 0 & 2 & 3 & 4 & 5 \\ 0 & 1 & 3 & -1 & 2 \\ 0 & 3 & 2 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 2 & 3 \\ 0 & 1 & 3 & -1 & 2 \\ 0 & 2 & 3 & 4 & 5 \\ 0 & 3 & 2 & 4 & 1 \end{bmatrix} \begin{matrix} \\ \times(-2) \\ + \\ \end{matrix}$$

$$C=\begin{bmatrix} 0 & 0 & -1 & 2 & 3 \\ 0 & 1 & 3 & -1 & 2 \\ 0 & 0 & -3 & 6 & 1 \\ 0 & 3 & 2 & 4 & 1 \end{bmatrix} \begin{matrix} \\ \times(-3) \\ \\ + \end{matrix} = \begin{bmatrix} 0 & 0 & -1 & 2 & 3 \\ 0 & 1 & 3 & -1 & 2 \\ 0 & 0 & -3 & 6 & 1 \\ 0 & 0 & -7 & 7 & -5 \end{bmatrix} \begin{matrix} + \\ \\ \times\left(-\frac{1}{3}\right) \\ \end{matrix}$$

$$C=\begin{bmatrix} 0 & 0 & 0 & 0 & \frac{8}{3} \\ 0 & 1 & 3 & -1 & 2 \\ 0 & 0 & -3 & 6 & 1 \\ 0 & 0 & -7 & 7 & -5 \end{bmatrix} \begin{matrix} \\ \\ \times\left(-\frac{1}{3}\right) \\ \end{matrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{8}{3} \\ 0 & 1 & 3 & -1 & 2 \\ 0 & 0 & 1 & -2 & -\frac{1}{3} \\ 0 & 0 & -7 & 7 & -5 \end{bmatrix} \begin{matrix} \\ + \\ \times(-3) \\ \end{matrix}$$

$$C=\begin{bmatrix} 0 & 0 & 0 & 0 & \frac{8}{3} \\ 0 & 1 & 0 & 5 & 1 \\ 0 & 0 & 1 & -2 & -\frac{1}{3} \\ 0 & 0 & -7 & 7 & -5 \end{bmatrix} \begin{matrix} \\ \\ \times(7) \\ + \end{matrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{8}{3} \\ 0 & 1 & 0 & 5 & 1 \\ 0 & 0 & 1 & -2 & -\frac{1}{3} \\ 0 & 0 & 0 & -7 & -\frac{22}{3} \end{bmatrix} \begin{matrix} \\ \\ \\ \times\left(-\frac{1}{7}\right) \end{matrix}$$

$$C=\begin{bmatrix} 0 & 0 & 0 & 0 & \frac{8}{3} \\ 0 & 1 & 0 & 5 & 1 \\ 0 & 0 & 1 & -2 & -\frac{1}{3} \\ 0 & 0 & 0 & 1 & \frac{22}{21} \end{bmatrix} \begin{matrix} \\ \\ + \\ \times(2) \end{matrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{8}{3} \\ 0 & 1 & 0 & 5 & 1 \\ 0 & 0 & 1 & 0 & \frac{55}{63} \\ 0 & 0 & 0 & 1 & \frac{22}{21} \end{bmatrix} \begin{matrix} \\ + \\ \\ \times(-5) \end{matrix}$$

$$C=\begin{bmatrix} 0 & 0 & 0 & 0 & \frac{8}{3} \\ 0 & 1 & 0 & 0 & -\frac{89}{21} \\ 0 & 0 & 1 & 0 & \frac{55}{63} \\ 0 & 0 & 0 & 1 & \frac{22}{21} \end{bmatrix}$$

EXERCÍCIO 7 (PÁGINA 42)

$$C=\begin{bmatrix}1&-2&0&2\\2&-3&-1&5\\1&3&2&5\\1&1&0&2\end{bmatrix}\begin{matrix}\times(-2)\\+\\ \\ \\\end{matrix}=\begin{bmatrix}1&-2&0&2\\0&1&-1&1\\1&3&2&5\\1&1&0&2\end{bmatrix}\begin{matrix}\times(-1)\\ \\+\\ \\\end{matrix}$$

$$C=\begin{bmatrix}1&-2&0&2\\0&1&-1&1\\0&5&2&3\\1&1&0&2\end{bmatrix}\begin{matrix}\times(-1)\\ \\+\\ \\\end{matrix}=\begin{bmatrix}1&-2&0&2\\0&1&-1&1\\0&5&2&3\\0&3&0&0\end{bmatrix}\begin{matrix}+\\ \times(2)\\ \\ \\\end{matrix}=$$

$$C=\begin{bmatrix}1&0&-2&4\\0&1&-1&1\\0&5&2&3\\0&3&0&0\end{bmatrix}\begin{matrix}\\ \times(-5)\\+\\ \\\end{matrix}=\begin{bmatrix}1&0&-2&4\\0&1&-1&1\\0&0&7&-2\\0&3&0&0\end{bmatrix}\begin{matrix}\\ \times(-3)\\ \\+\\\end{matrix}$$

$$C=\begin{bmatrix}1&0&-2&4\\0&1&-1&1\\0&0&7&-2\\0&0&3&-3\end{bmatrix}\begin{matrix}\\ \\ \times(\frac{1}{7})\\ \\\end{matrix}=\begin{bmatrix}1&0&-2&4\\0&1&-1&1\\0&0&1&-2/7\\0&0&-3&-3\end{bmatrix}\begin{matrix}\\ +\\ \times(2)\\ \\\end{matrix}$$

$$C=\begin{bmatrix}1&0&0&25/7\\0&1&-1&1\\0&0&1&-2/7\\0&0&3&-3\end{bmatrix}\begin{matrix}\\ +\\ \times(1)\\ \\\end{matrix}=\begin{bmatrix}1&0&0&25/7\\0&1&0&5/7\\0&0&1&-2/7\\0&0&3&-3\end{bmatrix}\begin{matrix}\\ \\ \times(-3)\\+\\\end{matrix}$$

$$C=\begin{bmatrix}1&0&0&25/7\\0&1&0&5/7\\0&0&1&-2/7\\0&0&0&-\frac{15}{7}\end{bmatrix}\begin{matrix}\\ \\ \\ \times(-\frac{7}{15})\end{matrix}=\begin{bmatrix}1&0&0&25/7\\0&1&0&5/7\\0&0&1&-2/7\\0&0&0&1\end{bmatrix}\begin{matrix}\\ \\+\\ \times(\frac{2}{7})\end{matrix}$$

$$C=\begin{bmatrix}1&0&0&25/7\\0&1&0&5/7\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{matrix}\\ +\\ \\ \times(-\frac{5}{7})\end{matrix}=\begin{bmatrix}1&0&0&25/7\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{matrix}+\\ \\ \\ \times(-\frac{25}{7})\end{matrix}$$

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$$C=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

EXERCÍCIO 8 (PÁGINA 81)

8.A
$$\begin{cases}x+y+2z=-1\\x-2y+z=-5\\3x+y+z=3\end{cases}$$

$$\left[\begin{array}{ccc|c}1&1&2&-1\\1&-2&1&-5\\3&1&1&3\end{array}\right]\times(-1)\,+\;=\left[\begin{array}{ccc|c}1&1&2&-1\\0&-3&-1&-4\\3&1&1&3\end{array}\right]\times(-3)\,+\;=$$

$$=\left[\begin{array}{ccc|c}1&1&2&-1\\0&-3&-1&-4\\0&-2&-5&6\end{array}\right]\times\left(-\tfrac{1}{3}\right)=\left[\begin{array}{ccc|c}1&1&2&-1\\0&1&1/3&4/3\\0&-2&-5&6\end{array}\right]\;+\;\times(-1)=$$

$$=\left[\begin{array}{ccc|c}1&0&5/3&-7/3\\0&1&1/3&4/3\\0&-2&-5&6\end{array}\right]\times(2)\,+\;=\left[\begin{array}{ccc|c}1&0&5/3&-7/3\\0&1&1/3&4/3\\0&0&-13/3&26/3\end{array}\right]\times\left(-\tfrac{3}{13}\right)=$$

$$=\left[\begin{array}{ccc|c}1&0&5/3&-7/3\\0&1&1/3&4/3\\0&0&1&-2\end{array}\right]\;+\;\times\left(-\tfrac{1}{3}\right)=\left[\begin{array}{ccc|c}1&0&5/3&-7/3\\0&1&0&2\\0&0&1&-2\end{array}\right]\;+\;\times\left(-\tfrac{5}{3}\right)=$$

$$=\left[\begin{array}{ccc|c}1&0&0&1\\0&1&0&2\\0&0&1&-2\end{array}\right]$$

$x=1$, $y=2$ e $z=-2$

$x=1$, $y=2$ e $z=-2$ é a única solução do sistema linear dado.

8.B
$$\begin{cases} x+y+3z+2w=7 \\ 2x-y+4w=8 \\ 3y+6z=8 \end{cases}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 3 & 2 & 7 \\ 2 & -1 & 0 & 4 & 8 \\ 0 & 3 & 6 & 0 & 8 \end{array}\right] \begin{array}{l} \times(-2) \\ \hookleftarrow + \end{array} = \left[\begin{array}{cccc|c} 1 & 1 & 3 & 2 & 7 \\ 0 & -3 & -6 & 0 & -6 \\ 0 & 3 & 6 & 0 & 8 \end{array}\right] \left(\times -\tfrac{1}{3}\right) =$$

$$= \left[\begin{array}{cccc|c} 1 & 1 & 3 & 2 & 7 \\ 0 & 1 & 2 & 0 & 2 \\ 0 & 3 & 6 & 0 & 8 \end{array}\right] \begin{array}{l} \hookleftarrow + \\ \times(-1) \end{array} = \left[\begin{array}{cccc|c} 1 & 0 & 1 & 2 & 5 \\ 0 & 1 & 2 & 0 & 2 \\ 0 & 3 & 6 & 0 & 8 \end{array}\right] \begin{array}{l} \times(-3) \\ \hookleftarrow + \end{array} =$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 1 & 2 & 5 \\ 0 & 1 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 & 2 \end{array}\right]$$

$$\begin{cases} x+0y+z+2w=5 \\ 0x+y+2z+0w=2 \\ 0x+0y+0z+0w=2 \end{cases}$$

O SISTEMA LINEAR NÃO POSSUI SOLUÇÃO, POIS $0x+0y+0z+0w=2$ NÃO É POSSÍVEL.

8.C
$$\begin{cases} x+2y-4z=3 \\ x-2y+3z=-1 \\ 2x+3y-z=5 \\ 4x+3y-2z=7 \\ 3x+2y-6z=7 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -4 & 3 \\ 1 & -2 & 3 & -1 \\ 2 & 3 & -1 & 5 \\ 4 & 3 & -2 & 7 \\ 3 & 2 & -6 & 7 \end{array}\right] \begin{array}{l} \times(-1) \\ \hookleftarrow + \end{array} = \left[\begin{array}{ccc|c} 1 & 2 & -4 & 3 \\ 0 & -4 & 7 & -4 \\ 2 & 3 & -1 & 5 \\ 4 & 3 & -2 & 7 \\ 3 & 2 & -6 & 7 \end{array}\right] \begin{array}{l} \times(-2) \\ \\ \hookleftarrow + \end{array} =$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -4 & 3 \\ 0 & -4 & 7 & -4 \\ 0 & -1 & 7 & -1 \\ 4 & 3 & -2 & 7 \\ 5 & 2 & -6 & 7 \end{array}\right] \times(-4) + = \left[\begin{array}{ccc|c} 1 & 2 & -4 & 3 \\ 0 & -4 & 7 & -4 \\ 0 & -1 & 7 & -1 \\ 0 & -5 & 14 & -5 \\ 5 & 2 & -6 & 7 \end{array}\right] \times(-5) + =$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -4 & 3 \\ 0 & -4 & 7 & -4 \\ 0 & -1 & 7 & -1 \\ 0 & -5 & 14 & -5 \\ 0 & -8 & 14 & -8 \end{array}\right] \left(\times -\tfrac{1}{4}\right) = \left[\begin{array}{ccc|c} 1 & 2 & -4 & 3 \\ 0 & 1 & -7/4 & 1 \\ 0 & -1 & 7 & -1 \\ 0 & -5 & 14 & -5 \\ 0 & -8 & 14 & -8 \end{array}\right] \times(-2) + =$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -1/2 & 1 \\ 0 & 1 & -7/4 & -1 \\ 0 & -1 & 7 & -1 \\ 0 & -5 & 14 & -5 \\ 0 & -8 & 14 & -8 \end{array}\right] \times(1) + = \left[\begin{array}{ccc|c} 1 & 0 & -1/2 & 1 \\ 0 & 1 & -7/4 & -1 \\ 0 & 0 & 21/4 & -2 \\ 0 & -5 & 14 & -5 \\ 0 & -8 & 14 & -8 \end{array}\right] \times(5) + =$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -1/2 & 1 \\ 0 & 1 & -7/4 & -1 \\ 0 & 0 & 21/4 & -2 \\ 0 & 0 & 21/4 & -10 \\ 0 & -8 & 14 & -8 \end{array}\right] \times(8) + = \left[\begin{array}{ccc|c} 1 & 0 & -1/2 & 1 \\ 0 & 1 & -7/4 & -1 \\ 0 & 0 & 21/4 & -2 \\ 0 & 0 & 21/4 & -10 \\ 0 & 0 & 0 & -16 \end{array}\right] \times\left(\tfrac{4}{21}\right) =$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -1/2 & 1 \\ 0 & 1 & -7/4 & -1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 21/4 & -10 \\ 0 & 0 & 0 & -16 \end{array}\right] + \times\left(\tfrac{1}{2}\right) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -7/4 & -1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 21/4 & -10 \\ 0 & 0 & 0 & -16 \end{array}\right] + \times\left(\tfrac{7}{4}\right) =$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -9/2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 21/4 & -10 \\ 0 & 0 & 0 & -16 \end{array}\right] \times\left(-\tfrac{21}{4}\right) + = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -9/2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & -16 \end{array}\right]$$

$$\begin{cases} x + 0y + 0z = 0 \\ 0x + y + 0z = -9/2 \\ 0x + 0y + z = -2 \\ 0x + 0y + 0z = 1/2 \\ 0x + 0y + 0z = -16 \end{cases}$$

O SISTEMA LINEAR NÃO TEM SOLUÇÃO. POIS $0x+0y+0z = 1/2$ E $0x+0y+0z=-16$ NÃO SÃO POSSÍVEL.

EXERCÍCIO 9 (PÁGINA 81)

9.A
$$\begin{cases} x + y + 2z + 3w = 13 \\ x - 2y + z + w = 8 \\ 3x + y + z - w = 1 \end{cases}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 1 & -2 & 1 & 1 & 8 \\ 3 & 1 & 1 & -1 & 1 \end{array}\right] \times(-1) + = \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & -3 & -1 & -2 & -5 \\ 3 & 1 & 1 & -1 & 1 \end{array}\right] \times(-3) + =$$

$$= \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & -3 & -1 & -2 & -5 \\ 0 & -2 & -5 & -10 & -38 \end{array}\right] \times\left(-\tfrac{1}{3}\right) = \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & 1 & 1/3 & 2/3 & 5/3 \\ 0 & -2 & -5 & -10 & -38 \end{array}\right] + \times(-1) =$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 5/3 & 7/3 & 35/3 \\ 0 & 1 & 1/3 & 2/3 & 5/3 \\ 0 & -2 & -5 & -10 & -38 \end{array}\right] \times(2) + = \left[\begin{array}{cccc|c} 1 & 0 & 5/3 & 7/3 & \frac{35}{3} \\ 0 & 1 & 1/3 & 2/3 & 5/3 \\ 0 & 0 & -\frac{13}{3} & -\frac{26}{3} & \frac{104}{3} \end{array}\right] \times\left(-\tfrac{3}{13}\right) =$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 5/3 & 7/3 & 35/3 \\ 0 & 1 & 1/3 & 2/3 & 5/3 \\ 0 & 0 & 1 & 2 & 8 \end{array}\right] + \times\left(-\tfrac{1}{3}\right) = \left[\begin{array}{cccc|c} 1 & 0 & 5/3 & 7/3 & 35/3 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 2 & 8 \end{array}\right] + \times\left(-\tfrac{5}{3}\right) =$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 2 & 8 \end{array}\right]$$

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$$\begin{cases} x + 0y + 0z - w = -2 \\ 0x + y + 0z + 0w = -1 \\ 0x + 0y + z + 2w = 8 \end{cases}$$

$$\begin{cases} x - w = -2 \\ y = -1 \\ z + 2w = 8 \end{cases}$$

$w = r$

r = QUALQUER NÚMERO REAL

$z + 2w = 8$ $\quad$ $x - w = -2$

$z + 2r = 8$ $\quad$ $x - r = -2$

$z = 8 - 2r$ $\quad$ $x = r - 2$

$x = r - 2$

$y = -1$

$z = 8 - 2r$

$w = r$

O SISTEMA POSSUI INFINITAS SOLUÇÕES.

9.B
$$\begin{cases} x + y + z = 1 \\ x + y - 2z = 3 \\ 2x + y + z = 2 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 1 & -2 & 3 \\ 2 & 1 & 1 & 2 \end{array}\right] \begin{array}{l} x(-1) \\ \leftarrow + \\ \end{array} = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & -3 & 2 \\ 2 & 1 & 1 & 2 \end{array}\right] \begin{array}{l} x(-2) \\ + \\ \leftarrow \end{array} =$$

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$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & -3 & 2 \\ 0 & -1 & -1 & 0 \end{array}\right] \times(-1/3) = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -2/3 \\ 0 & -1 & -1 & 0 \end{array}\right] \times(-1) =$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 0 & 5/3 \\ 0 & 0 & 1 & -2/3 \\ 0 & -1 & -1 & 0 \end{array}\right] \times(1) = \left[\begin{array}{ccc|c} 1 & 1 & 0 & 5/3 \\ 0 & 0 & 1 & -2/3 \\ 0 & -1 & 0 & -2/3 \end{array}\right]$$

$$\begin{cases} X + Y + 0Z = 5/3 \\ 0X + 0Y + Z = -2/3 \\ 0X - Y + 0Z = -2/3 \end{cases} \sim \begin{cases} X + Y = 5/3 \\ Z = -2/3 \\ -Y = -2/3 \end{cases} \sim \begin{cases} X + Y = 5/3 \\ Z = -2/3 \\ Y = 2/3 \end{cases}$$

$$X + Y = \frac{5}{3} \qquad X + \frac{2}{3} = \frac{5}{3} \rightarrow X = \frac{5}{3} - \frac{2}{3} = \frac{3}{3} \rightarrow X = 1$$

$$X = 1,\ Y = \frac{2}{3} \text{ E } Z = \frac{-2}{3}$$

$X = 1$, $Y = \frac{2}{3}$ E $Z = -\frac{2}{3}$ É A ÚNICA SOLUÇÃO DO SISTEMA LINEAR.

9.C
$$\begin{cases} 2x + Y + Z - 2w = 1 \\ 3x - 2Y + Z - 6w = -2 \\ x + Y - Z - w = -1 \\ 6x + Z - 9w = -2 \\ 5x - Y + 2Z - 8w = 3 \end{cases}$$

$$\left[\begin{array}{cccc|c} 2 & 1 & 1 & -2 & 1 \\ 3 & -2 & 1 & -6 & -2 \\ 1 & 1 & -1 & -1 & -1 \\ 6 & 0 & 1 & -9 & -2 \\ 5 & -1 & 2 & -8 & 3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 2 & 1 & 1 & -2 & 1 \\ 3 & -2 & 1 & -6 & -2 \\ 5 & -1 & 2 & -8 & 3 \\ 6 & 0 & 1 & -9 & -2 \end{array}\right] \times(-2)$$

$$= \left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 0 & -1 & 3 & 0 & 3 \\ 3 & -2 & 1 & -6 & -2 \\ 5 & -1 & 2 & -8 & 3 \\ 6 & 0 & 1 & -9 & -2 \end{array}\right] \begin{array}{l} \times(-3) \\ + \end{array} = \left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 0 & -1 & 3 & 0 & 3 \\ 0 & -5 & 4 & -3 & 1 \\ 5 & -1 & 2 & -8 & 3 \\ 6 & 0 & -1 & -9 & -2 \end{array}\right] \begin{array}{l} \times(-5) \\ + \end{array} =$$

$$= \left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 0 & -1 & 3 & 0 & 3 \\ 0 & -5 & 4 & -3 & 1 \\ 0 & -6 & 7 & -3 & 8 \\ 6 & 0 & 1 & -9 & -2 \end{array}\right] \begin{array}{l} \times(-6) \\ + \end{array} = \left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 0 & -1 & 3 & 0 & 3 \\ 0 & -5 & 4 & -3 & 1 \\ 0 & -6 & 7 & -3 & 8 \\ 0 & -6 & 7 & -3 & 4 \end{array}\right] \times(-1) =$$

$$= \left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 0 & 1 & -3 & 0 & -3 \\ 0 & -5 & 4 & -3 & 1 \\ 0 & -6 & 7 & -3 & 8 \\ 0 & -6 & 7 & -3 & 4 \end{array}\right] \begin{array}{l} + \\ \times(-1) \end{array} = \left[\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 2 \\ 0 & 1 & -3 & 0 & -3 \\ 0 & -5 & 4 & -3 & 1 \\ 0 & -6 & 7 & -3 & 8 \\ 0 & -6 & 7 & -3 & 4 \end{array}\right] \begin{array}{l} \times(5) \\ + \end{array} =$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 2 \\ 0 & 1 & -3 & 0 & -3 \\ 0 & 0 & -11 & -3 & -14 \\ 0 & -6 & 7 & -3 & 8 \\ 0 & -6 & 7 & -3 & 4 \end{array}\right] \begin{array}{l} \times(6) \\ + \end{array} = \left[\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 2 \\ 0 & 1 & -3 & 0 & -3 \\ 0 & 0 & -11 & -3 & -14 \\ 0 & 0 & -11 & -3 & 10 \\ 0 & -6 & 7 & -3 & 4 \end{array}\right] \begin{array}{l} \times(6) \\ + \end{array} =$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 2 \\ 0 & 1 & -3 & 0 & -3 \\ 0 & 0 & -11 & -3 & -14 \\ 0 & 0 & -11 & -3 & 10 \\ 0 & 0 & -11 & -3 & -14 \end{array}\right] \times\left(-\frac{1}{11}\right) = \left[\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 2 \\ 0 & 1 & -3 & 0 & -3 \\ 0 & 0 & 1 & 3/11 & 14/11 \\ 0 & 0 & -11 & -3 & 10 \\ 0 & 0 & -11 & -3 & -14 \end{array}\right] \begin{array}{l} + \\ \times(-2) \end{array} =$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{17}{11} & -\frac{6}{11} \\ 0 & 1 & -3 & 0 & -3 \\ 0 & 0 & 1 & 3/11 & 14/11 \\ 0 & 0 & -11 & -3 & 10 \\ 0 & 0 & -11 & -3 & -14 \end{array}\right] \begin{array}{l} + \\ \times(3) \end{array} = \left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{17}{11} & -\frac{6}{11} \\ 0 & 1 & 0 & 9/11 & 9/11 \\ 0 & 0 & 1 & 3/11 & 14/11 \\ 0 & 0 & -11 & -3 & 10 \\ 0 & 0 & -11 & -3 & -14 \end{array}\right] \begin{array}{l} \times(11) \\ + \end{array} =$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & -17/11 & -6/11 \\ 0 & 1 & 0 & 9/11 & 9/11 \\ 0 & 0 & 1 & 3/11 & 15/11 \\ 0 & 0 & 0 & 0 & 24 \\ 0 & 0 & -11 & -3 & -15 \end{array}\right] \times(11) = \left[\begin{array}{cccc|c} 1 & 0 & 0 & -17/11 & -6/11 \\ 0 & 1 & 0 & 9/11 & 9/11 \\ 0 & 0 & 1 & 3/11 & 15/11 \\ 0 & 0 & 0 & 0 & 24 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

O SISTEMA LINEAR NÃO TEM SOLUÇÃO POIS $0x+0y+0z+0w=24$ NÃO É POSSÍVEL.

EXERCÍCIO 10 (PÁGINA 42)

10.A
$$\begin{cases} 2x-y+z=3 \\ x-3y+z=4 \\ -5x \quad -2z=-5 \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & -1 & 1 & 3 \\ 1 & -3 & 1 & 4 \\ -5 & 0 & -2 & -5 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 2 & -1 & 1 & 3 \\ -5 & 0 & -2 & -5 \end{array}\right] \times(-2) =$$

$$= \left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & 5 & -1 & -5 \\ -5 & 0 & -2 & -5 \end{array}\right] \times(5) = \left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & 5 & -1 & -5 \\ 0 & -15 & 3 & 15 \end{array}\right] \times(1/5) =$$

$$= \left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & 1 & -1/5 & -1 \\ 0 & -15 & 3 & 15 \end{array}\right] \times(3) = \left[\begin{array}{ccc|c} 1 & 0 & 2/5 & 1 \\ 0 & 1 & -1/5 & -1 \\ 0 & -15 & 3 & 15 \end{array}\right] \times(15) =$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 2/5 & 1 \\ 0 & 1 & -1/5 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} x+0y+2/5\,z=1 \\ 0x+y-1/5\,z=-1 \\ 0x+0y+0z=0 \end{cases} \sim \begin{cases} x+2/5\,z=1 \\ y-1/5\,z=-1 \end{cases}$$

$z = r$

r = QUALQUER NÚMERO REAL

$x + \frac{2}{5}z = 1$ → $x = 1 - \frac{2}{5}r$

$x + \frac{2}{5}r = 1$

$y - \frac{1}{5}z = -1$ → $y = \frac{1}{5}r - 1$

$y - \frac{1}{5}r = -1$

$x = 1 - \frac{2}{5}r$, $y = \frac{1}{5}r - 1$ E $z = r$

O SISTEMA POSSUI INFINITAS SOLUÇÕES.

10.B
$$\begin{cases} x + y + z + w = 6 \\ 2x + y - z = 3 \\ 3x + y + 2w = 6 \end{cases}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 2 & 1 & -1 & 0 & 3 \\ 3 & 1 & 0 & 2 & 6 \end{array}\right] \begin{array}{l} \times(-2) \\ \leftarrow + \\ \\ \end{array} = \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & -1 & -3 & -2 & -9 \\ 3 & 1 & 0 & 2 & 6 \end{array}\right] \begin{array}{l} \times(-3) \\ + \\ \leftarrow \end{array} =$$

$$= \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & -1 & -3 & -2 & -9 \\ 0 & -2 & -3 & -1 & -12 \end{array}\right] \begin{array}{l} \\ \times(-1) \\ \\ \end{array} = \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & 1 & 3 & 2 & 9 \\ 0 & -2 & -3 & -1 & -12 \end{array}\right] \begin{array}{l} \leftarrow + \\ \times(-1) \\ \\ \end{array} =$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & -2 & -1 & -12 \\ 0 & 1 & 3 & 2 & 9 \\ 0 & -2 & -3 & -1 & -12 \end{array}\right] \begin{array}{l} \\ \times(2) \\ \leftarrow + \end{array} = \left[\begin{array}{cccc|c} 1 & 0 & -2 & -1 & -12 \\ 0 & 1 & 3 & 2 & 9 \\ 0 & 0 & 3 & 3 & 6 \end{array}\right] \begin{array}{l} \\ \\ \times(1/3) \end{array} =$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & -2 & -1 & -12 \\ 0 & 1 & 3 & 2 & 9 \\ 0 & 0 & 1 & 1 & 2 \end{array}\right] \begin{matrix} \\ +\\ \times(2) \end{matrix} = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & -8 \\ 0 & 1 & 3 & 2 & 9 \\ 0 & 0 & 1 & 1 & 2 \end{array}\right] \begin{matrix} \\ +\\ \times(-3) \end{matrix} =$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & -8 \\ 0 & 1 & 0 & -1 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{array}\right]$$

$$\begin{cases} x + 0y + 0z + w = -8 \\ 0x + y + 0z - w = 3 \\ 0x + 0y + z + w = 2 \end{cases} \sim \begin{cases} x + w = -8 \\ y - w = 3 \\ z + w = 2 \end{cases}$$

$w = r$

r = QUALQUER NÚMERO REAL

$x + w = -8$	$y - w = 3$	$z + w = 2$
$x + r = -8$	$y - r = 3$	$z + r = 2$
$x = -8 - r$	$y = 3 + r$	$z = 2 - r$

$x = -8 - r$, $y = 3 + r$, $z = 2 - r$ E $w = r$

O SISTEMA LINEAR POSSUI INFINITAS SOLUÇÕES.

10.C

$$\begin{cases} 2x - y + z = 3 \\ 3x + y - 2z = -2 \\ x - y + z = 7 \\ x + 3y + 7z = 13 \\ x - 7y - 5z = 12 \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & -1 & 1 & 3 \\ 3 & 1 & -2 & -2 \\ 1 & -1 & 1 & 7 \\ 1 & 5 & 7 & 13 \\ 1 & -7 & -5 & 12 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & 7 \\ 1 & 5 & 7 & 13 \\ 1 & -7 & -5 & 12 \\ 2 & -1 & 1 & 3 \\ 3 & 1 & -2 & -2 \end{array}\right] \begin{array}{l} \times(-1) \\ \leftarrow + \\ \\ \\ \\ \end{array} =$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 1 & 7 \\ 0 & 6 & 6 & 6 \\ 1 & -7 & -5 & 12 \\ 2 & -1 & 1 & 3 \\ 3 & 1 & -2 & -2 \end{array}\right] \begin{array}{l} \times(-1) \\ + \\ \leftarrow \\ \\ \\ \end{array} = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 7 \\ 0 & 6 & 6 & 6 \\ 0 & -6 & -6 & 5 \\ 2 & -1 & 1 & 3 \\ 3 & 1 & -2 & -2 \end{array}\right] \begin{array}{l} \times(-2) \\ \\ + \\ \leftarrow \\ \\ \end{array} =$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 1 & 7 \\ 0 & 6 & 6 & 6 \\ 0 & -6 & -6 & 5 \\ 0 & 1 & -1 & -11 \\ 3 & 1 & -2 & -2 \end{array}\right] \begin{array}{l} \times(-3) \\ \\ + \\ \\ \leftarrow \end{array} = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 7 \\ 0 & 6 & 6 & 6 \\ 0 & -6 & -6 & 5 \\ 0 & 1 & -1 & -11 \\ 0 & 4 & -5 & -23 \end{array}\right] \begin{array}{l} \\ \times(1/6) \\ \\ \\ \\ \end{array} =$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 1 & 7 \\ 0 & 1 & 1 & 1 \\ 0 & -6 & -6 & 5 \\ 0 & 1 & -1 & -11 \\ 0 & 4 & 5 & -23 \end{array}\right] \begin{array}{l} \leftarrow + \\ \times(1) \\ \\ \\ \\ \end{array} = \left[\begin{array}{ccc|c} 1 & 0 & 2 & 8 \\ 0 & 1 & 1 & 1 \\ 0 & -6 & -6 & 5 \\ 0 & 1 & -1 & -11 \\ 0 & 4 & 5 & -23 \end{array}\right] \begin{array}{l} \\ \times(6) \\ \leftarrow + \\ \\ \\ \end{array} =$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 2 & 8 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 11 \\ 0 & 1 & -1 & -11 \\ 0 & 4 & 5 & -23 \end{array}\right] \begin{array}{l} \\ \times(-1) \\ + \\ \leftarrow \\ \\ \end{array} = \left[\begin{array}{ccc|c} 1 & 0 & 2 & 8 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 11 \\ 0 & 0 & -2 & -12 \\ 0 & 4 & 5 & -23 \end{array}\right] \begin{array}{l} \\ \times(-4) \\ \\ + \\ \leftarrow \end{array} =$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 2 & 8 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 11 \\ 0 & 0 & -2 & -12 \\ 0 & 0 & 1 & -27 \end{array}\right] \begin{array}{l} \\ \\ \\ \times(-1/2) \\ \\ \end{array} = \left[\begin{array}{ccc|c} 1 & 0 & 2 & 8 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 11 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 1 & -27 \end{array}\right] \begin{array}{l} \leftarrow \\ + \\ \\ \times(-2) \\ \\ \end{array}$$

$$
= \left[\begin{array}{ccc|c} 1 & 0 & 0 & -8 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 11 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 1 & -27 \end{array}\right] \begin{array}{l} \\ + \\ \\ \times(-1) \\ \\ \end{array} = \left[\begin{array}{ccc|c} 1 & 0 & 0 & -8 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & 11 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 1 & -27 \end{array}\right] \begin{array}{l} \\ \\ \\ \times(-1) \\ + \end{array} =
$$

$$
= \left[\begin{array}{ccc|c} 1 & 0 & 0 & -8 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & 11 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & -33 \end{array}\right]
$$

$$
\begin{cases} x + 0y + 0z = -8 \\ 0x + y + 0z = -5 \\ 0x + 0y + 0z = 11 \\ 0x + 0y + z = 6 \\ 0x + 0y + 0z = -33 \end{cases}
$$

O SISTEMA LINEAR NÃO POSSUI SOLUÇÃO POIS $0x+0y+0z=11$ E $0x+0y+0z=-33$ NÃO É POSSÍVEL

EXERCÍCIO 11 (PÁGINA 92)

$$
\begin{cases} x + y - z = 2 \\ x + 2y + z = 3 \\ x + y + (a^2-5)z = a \end{cases}
$$

$$
\left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 1 & 2 & 1 & 3 \\ 1 & 1 & a^2-5 & a \end{array}\right]
$$

11.A

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 1 & 2 & 1 & 3 \\ 1 & 1 & a^2-5 & a \end{array}\right] \times(-1) \; + = \left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & 1 & 2 & 1 \\ 1 & 1 & a^2-5 & a \end{array}\right] \times(-1) \; + =$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & a^2-4 & a-2 \end{array}\right] \times(-1) = \left[\begin{array}{ccc|c} 1 & 0 & -3 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & a^2-4 & a-2 \end{array}\right]$$

$a^2-4=0$

$a^2=4$

$a=\pm\sqrt{4}$

$=\pm 2$

$\cancel{a-2=2-2}$

$\cancel{=0}$

$a-2=-2-2$

$=-4$

$a=-2$

11.B

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & a^2-4 & a-2 \end{array}\right]$$

$a^2-4=0$

$a^2=4$

$a=\pm\sqrt{4}$

$=\pm 2$

$a \neq \pm 2$

11.C

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & a^2-4 & a-2 \end{array}\right]$$

$a^2-4=0$

$a^2=4$

$a=\pm\sqrt{4}$

$=\pm 2$

$a=2$

EXERCÍCIO 12 (PÁGINA 42)

$$\begin{cases} x + y \qquad z = 2 \\ 2x + 3y \qquad 2z = 5 \\ 2x + 3y + (a^2-1)z = a+1 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 3 & 2 & 5 \\ 2 & 3 & a^2-1 & a+1 \end{array}\right]$$

12.A

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 3 & 2 & 5 \\ 2 & 3 & a^2-1 & a+1 \end{array}\right] \times(-2) \; + = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 2 & 3 & a^2-1 & a+1 \end{array}\right] \times(-2) \; + =$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & a^2-3 & a-3 \end{array}\right] + \; \times(-1) = \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 1 & a^2-3 & a-3 \end{array}\right] \times(-1) \; + =$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & a^2-2 & a-2 \end{array}\right]$$

$a^2 - 2 = 0$

$a^2 = 2 \rightarrow a = \pm\sqrt{2}$

$a = \pm\sqrt{2}$

12.B

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & a^2-2 & a-2 \end{array}\right]$$

$a^2 - 2 = 0$

$a^2 = 2 \rightarrow a = \pm\sqrt{2}$

$a \neq \pm\sqrt{2}$

12.C
$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & a^2-2 & a-2 \end{array}\right]$$

$a^2-2=0$
$a^2=2 \Rightarrow a=\pm\sqrt{2}$

ESSE SISTEMA NÃO POSSUI INFINITAS SOLUÇÕES.

EXERCÍCIO 13 (PÁGINA 42)

$$\begin{cases} x+y+z=2 \\ x+2y+z=3 \\ x+y+(a^2-5)z=a \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 2 & 1 & 3 \\ 1 & 1 & a^2-5 & a \end{array}\right]$$

13.A
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 2 & 1 & 3 \\ 1 & 1 & a^2-5 & a \end{array}\right] x(-1) + = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & a^2-5 & a \end{array}\right] x(-1) + =$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & a^2-6 & a-2 \end{array}\right] + x(-1) = \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & a^2-6 & a-2 \end{array}\right]$$

$a^2-6=0$
$a^2=6 \Rightarrow a^2=\pm\sqrt{6}$

$a=\pm\sqrt{6}$

13.B $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & a^2-6 & a-2 \end{array}\right]$

$a^2-6=0 \rightarrow a=\pm\sqrt{6}$
$a^2=6$

$a \neq \pm\sqrt{6}$

13.C $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & a^2-6 & a-2 \end{array}\right]$

$a^2-6=0 \rightarrow a=\pm\sqrt{6}$
$a^2=6$

ESSE SISTEMA NÃO POSSUI INFINITAS SOLUÇÕES

EXERCÍCIO 14 (PÁGINA 82)

$\begin{cases} x + y = 3 \\ x + (a^2-8)y = a \end{cases}$

$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 1 & a^2-8 & a \end{array}\right]$

14.A $\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 1 & a^2-8 & a \end{array}\right] \begin{array}{l} \times(-1) \\ \hookleftarrow + \end{array} = \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & a^2-9 & a-3 \end{array}\right]$

$a^2-9=0 \rightarrow a=\pm\sqrt{9}$
$a^2=9 \qquad = \pm 3$

$a=-3$

14.B $\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & a^2-9 & a-3 \end{array}\right]$

$a^2-9=0 \rightarrow a=\pm\sqrt{9}$ $\quad a \neq \pm 3$

$a^2=9 \qquad = \pm 3$

14.C $\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & a^2-9 & a-3 \end{array}\right]$

$a^2-9=0 \rightarrow a=\pm\sqrt{9}$

$a^2=9 \qquad = \pm 3$

ESSE SISTEMA LINEAR NÃO POSSUI INFINITAS SOLUÇÕES.

EXERCÍCIO 15 (PÁGINA 42)

15.A $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 3 \\ 0 & 1 & 1 & 1 \end{array}\right] \begin{array}{l} \times(-1) \\ + \\ \\ \end{array} = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 3 \\ 0 & 1 & 1 & 1 \end{array}\right] \begin{array}{l} \\ + \\ \times(-1) \end{array} =$

$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 3 \\ 0 & 1 & 1 & 1 \end{array}\right]$

$\begin{cases} x+0y+0z=-1 \\ 0x+0y-z=3 \\ 0x+y+z=1 \end{cases}$ $\qquad \begin{array}{l} x=-1 \\ -z=3 \\ y+z=1 \end{array}$

$x=-1 \qquad -z=3 \qquad y+z=1 \qquad x=-1,\ y=4 \text{ E } z=-3$

$z=\dfrac{3}{-1} \qquad y-3=1$

$=-3 \qquad y=1+3$

$=4$

15.B
$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 2 & 0 \\ 1 & 3 & 3 & 0 \end{array}\right] \times(-1) = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -1 & -2 & 0 \\ 1 & 1 & 2 & 0 \\ 1 & 3 & 3 & 0 \end{array}\right] \times(-1) =$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 1 & 3 & 3 & 0 \end{array}\right] \times(-1) = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right] \times(-1) =$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right] \times(-2) = \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right] \times(1) =$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right] \times(-1) = \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & 0 \end{array}\right]$$

$$\begin{cases} x + 0y - z = 0 \\ 0x + y + 2z = 0 \\ 0x + 0y + z = 0 \\ 0x + 0y - 2z = 0 \end{cases} \qquad \begin{array}{l} x - z = 0 \\ y + 2z = 0 \\ z = 0 \\ -2z = 0 \end{array}$$

$$\begin{array}{l} x - z = 0 \\ x - 0 = 0 \\ x = 0 \end{array} \qquad \begin{array}{l} y + 2z = 0 \\ y + 2(0) = 0 \\ y + 0 = 0 \\ y = 0 \end{array} \qquad z = 0 \qquad x = 0,\ y = 0 \text{ e } z = 0$$

EXERCÍCIO 16 (PÁGINA 52)

16.A

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 1 & 1 & 1 & 0 \\ 3 & 7 & 9 & 0 \end{array}\right] \times(-1) + = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -1 & -2 & 0 \\ 3 & 7 & 9 & 0 \end{array}\right] \times(-3) + =$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & -3 & -6 & 0 \end{array}\right] \times(-1) = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -3 & -6 & 0 \end{array}\right] + \times(-2) =$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -3 & -6 & 0 \end{array}\right] \times(3) + = \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} x + 0y - z = 0 \\ 0x + y + 2z = 0 \\ 0x + 0y + 0z = 0 \end{cases} \qquad \begin{array}{l} x - z = 0 \\ y + 2z = 0 \end{array}$$

$x - z = 0$ $\quad$ $y + 2z = 0$ $\quad$ $x = z$, $y = -2z$ e $z = x$

$x = z$ $\quad$ $y = -2z$

16.B

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 2 & 0 & 1 & 4 \\ 1 & 0 & 2 & 5 \\ 1 & 2 & 3 & 11 \\ 2 & 1 & 4 & 12 \end{array}\right] \times(-2) + = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & -4 & -1 & -10 \\ 1 & 0 & 2 & 5 \\ 1 & 2 & 3 & 11 \\ 2 & 1 & 4 & 12 \end{array}\right] \times(-1) + =$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & -4 & -1 & -10 \\ 0 & -2 & 1 & -2 \\ 1 & 2 & 3 & 11 \\ 2 & 1 & 4 & 12 \end{array}\right] \times(-1) + = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & -4 & -1 & -10 \\ 0 & -2 & 1 & -2 \\ 0 & 0 & 2 & 4 \\ 2 & 1 & 4 & 12 \end{array}\right] \times(-2) + =$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & -4 & -1 & -10 \\ 0 & -2 & 1 & -2 \\ 0 & 0 & 2 & 4 \\ 0 & -3 & 2 & -2 \end{array}\right] \times\left(-\frac{1}{4}\right) = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & 1 & 1/4 & 10/4 \\ 0 & -2 & 1 & -2 \\ 0 & 0 & 2 & 4 \\ 0 & -3 & 2 & -2 \end{array}\right] + \times(-2) =$$

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$$=\left[\begin{array}{ccc|c} 1 & 0 & 1/2 & 2 \\ 0 & 1 & 1/4 & 10/4 \\ 0 & -2 & 1 & -2 \\ 0 & 0 & 2 & 4 \\ 0 & -3 & 2 & -2 \end{array}\right] \times(2) + = \left[\begin{array}{ccc|c} 1 & 0 & 1/2 & 2 \\ 0 & 1 & 1/4 & 10/4 \\ 0 & 0 & 3/2 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & -3 & 2 & -2 \end{array}\right] \times(3) + =$$

$$=\left[\begin{array}{ccc|c} 1 & 0 & 1/2 & 2 \\ 0 & 1 & 1/4 & 10/4 \\ 0 & 0 & 3/2 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 11/4 & 22/4 \end{array}\right]$$

$$\begin{cases} x + 0y + \frac{1}{2}z = 2 \\ 0x + y + \frac{1}{4}z = \frac{10}{4} \\ 0x + 0y + \frac{3}{2}z = 3 \\ 0x + 0y + 2z = 4 \\ 0x + 0y + \frac{11}{4}z = \frac{22}{4} \end{cases}$$

$$x + \frac{1}{2}z = 2$$
$$y + \frac{1}{4}z = \frac{10}{4}$$
$$\frac{3}{2}z = 3$$
$$2z = 4$$
$$\frac{11}{4}z = \frac{22}{4}$$

$$\frac{3}{2}z = 3 \qquad z = \frac{2}{\cancel{3}_1}(\cancel{3}) = 2$$

$$2z = 4 \qquad z = \frac{4}{2} = 2$$

$$\frac{11}{4}z = \frac{22}{4} \qquad z = \frac{\cancel{4}^1}{{}_1\cancel{11}}\left(\frac{\cancel{22}^2}{\cancel{4}_1}\right) = 2$$

$$x + \frac{1}{2}z = 2 \qquad x + \frac{1}{\cancel{2}_1}(\cancel{2}^1) = 2 \qquad x + 1 = 2 \qquad x = 2 - 1 = 1$$

$$y + \frac{1}{4}z = \frac{\cancel{10}^5}{\cancel{4}_2} \qquad y + \frac{1}{\cancel{4}_2}(\cancel{2}^1) = \frac{5}{2} \qquad y + \frac{1}{2} = \frac{5}{2}$$

$$y = \frac{5}{2} - \frac{1}{2} = \frac{\cancel{4}^2}{\cancel{2}_1} = 2$$

$x = 1,\ y = 2$ e $z = 2$

EXERCÍCIO 17 (PÁGINA 42)

17.A

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 8 \\ 1 & 3 & 0 & 1 & 7 \\ 1 & 0 & 2 & 1 & 3 \end{array}\right] \times(-1) \; + = \left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & -1 \\ 1 & 0 & 2 & 1 & 3 \end{array}\right] \times(-1) \; + =$$

$$= \left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & -2 & -1 & 0 & -5 \end{array}\right] + \; \times(-2) = \left[\begin{array}{cccc|c} 1 & 0 & 9 & 1 & 10 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & -2 & -1 & 0 & -5 \end{array}\right] \times(2) \; + =$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 9 & 1 & 10 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & -7 & 0 & -7 \end{array}\right] \times\left(-\frac{1}{7}\right) = \left[\begin{array}{cccc|c} 1 & 0 & 9 & 1 & 10 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 \end{array}\right]$$

$$\begin{cases} x + 0y + 9z + w = 10 \\ 0x + y - 3z + 0w = -1 \\ 0x + 0y + z + 0w = 1 \end{cases} \qquad \begin{array}{l} x + w = 10 \\ y - 3z = -1 \\ z = 1 \end{array}$$

$w = r$

$r \Rightarrow$ QUALQUER NÚMERO REAL

$$\begin{array}{lll} x + w = 10 & y - 3z = -1 & z = 1 \\ x + r = 10 & y - 3(1) = -1 & \\ x = 10 - r & y - 3 = -1 & \\ & y = -1 + 3 & \\ & \;\;\; = 2 & \end{array}$$

$x = 10 - r$, $y = 2$, $z = 1$ E $w = r$

17.B

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 2 & -1 & -3 & 3 \\ 3 & 0 & 1 & 2 \\ 3 & -3 & 0 & 7 \end{array}\right] \times(-2) \; + = \left[\begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 0 & 3 & -9 & -3 \\ 3 & 0 & 1 & 2 \\ 3 & -3 & 0 & 7 \end{array}\right] \times(-3) \; + =$$

$$= \left[\begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 0 & 3 & -9 & -3 \\ 0 & 6 & -8 & -10 \\ 3 & -3 & 0 & 7 \end{array}\right] \times(-3) + \;=\; \left[\begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 0 & 3 & -9 & -3 \\ 0 & 6 & -8 & -10 \\ 0 & 3 & -9 & -5 \end{array}\right] \times(1/3) =$$

$$= \left[\begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 0 & 1 & -3 & -1 \\ 0 & 6 & -8 & -10 \\ 0 & 3 & -9 & -5 \end{array}\right] \times(2) + \;=\; \left[\begin{array}{ccc|c} 1 & 0 & -3 & 2 \\ 0 & 1 & -3 & -1 \\ 0 & 6 & -8 & -10 \\ 0 & 3 & -9 & -5 \end{array}\right] \times(-6) + \;=$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -3 & 2 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 10 & -4 \\ 0 & 3 & -9 & -5 \end{array}\right] \times(-3) + \;=\; \left[\begin{array}{ccc|c} 1 & 0 & -3 & 2 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 10 & -4 \\ 0 & 0 & 0 & -2 \end{array}\right]$$

$$\begin{cases} x + 0y - 3z = 2 \\ 0x + y - 3z = -1 \\ 0x + 0y + 10z = -4 \\ 0x + 0y + 0z = -2 \end{cases}$$

O SISTEMA LINEAR NÃO POSSUI SOLUÇÃO POIS $0x+0y+0z=-2$ NÃO É POSSÍVEL.

EXERCÍCIO 18 (PÁGINA 42)

18.A
$$\left[\begin{array}{ccc|c} 4 & 2 & -1 & 5 \\ 3 & 3 & 6 & 1 \\ 5 & 1 & -8 & 8 \end{array}\right] \sim \left[\begin{array}{ccc|c} 3 & 3 & 6 & 1 \\ 4 & 2 & -1 & 5 \\ 5 & 1 & -8 & 8 \end{array}\right] \times(1/3) =$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1/3 \\ 4 & 2 & -1 & 5 \\ 5 & 1 & -8 & 8 \end{array}\right] \times(-4) + \;=\; \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1/3 \\ 0 & -2 & -9 & 11/3 \\ 5 & 1 & -8 & 8 \end{array}\right] \times(-5) + \;=$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1/3 \\ 0 & -2 & -9 & 11/3 \\ 0 & -4 & -18 & 22/3 \end{array}\right] \times\left(-\frac{1}{2}\right) = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1/3 \\ 0 & 1 & 9/2 & -11/6 \\ 0 & -4 & -18 & 22/3 \end{array}\right] \times(-1) =$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -5/2 & 13/6 \\ 0 & 1 & 9/2 & -11/6 \\ 0 & -4 & -18 & 22/3 \end{array}\right] \times(4) = \left[\begin{array}{ccc|c} 1 & 0 & -5/2 & 13/6 \\ 0 & 1 & 9/2 & -11/6 \\ 0 & 0 & 0 & -1/3 \end{array}\right]$$

$$\begin{cases} x + 0y - 5/2\,z = 13/6 \\ 0x + y + 9/2\,z = -11/6 \\ 0x + 0y + 0z = -1/3 \end{cases}$$

O SISTEMA LINEAR NÃO TEM SOLUÇÃO POIS $0x+0y+0z=-1/3$ NÃO É POSSÍVEL.

18-B

$$\left[\begin{array}{cccc|c} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 1 & 0 & 2 & -1 & -1 \end{array}\right] \times(-1) = \left[\begin{array}{cccc|c} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 0 & -1 & -1 & 2 & -1 \end{array}\right] \times\left(\frac{1}{2}\right) =$$

$$= \left[\begin{array}{cccc|c} 1 & 1 & 3 & -3 & 0 \\ 0 & 1 & 1/2 & -3/2 & 3/2 \\ 0 & -1 & -1 & 2 & -1 \end{array}\right] \times(-1) = \left[\begin{array}{cccc|c} 1 & 0 & 5/2 & -3/2 & -3/2 \\ 0 & 1 & 1/2 & -3/2 & 3/2 \\ 0 & -1 & -1 & 2 & -1 \end{array}\right] \times(1) =$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 5/2 & -3/2 & 3/2 \\ 0 & 1 & 1/2 & -3/2 & 3/2 \\ 0 & 0 & -1/2 & 1/2 & 1/2 \end{array}\right] \times(-2) = \left[\begin{array}{cccc|c} 1 & 0 & 5/2 & -3/2 & 3/2 \\ 0 & 1 & 1/2 & -3/2 & 3/2 \\ 0 & 0 & 1 & -1 & -1 \end{array}\right] \times\left(-\frac{5}{2}\right) =$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 4 \\ 0 & 1 & 1/2 & -3/2 & 3/2 \\ 0 & 0 & 1 & -1 & -1 \end{array}\right] \times\left(-\frac{1}{2}\right) = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 4 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & -1 & -1 \end{array}\right]$$

$$\begin{cases} x + 0y + 0z + w = 4 \\ 0x + y + 0z - w = 2 \\ 0x + 0y + z - w = -1 \end{cases} \qquad \begin{aligned} x + w &= 4 \\ y - w &= 2 \\ z - w &= -1 \end{aligned}$$

tilibra

$w = r$

$r \Rightarrow$ QUALQUER NÚMERO REAL

$x + w = 4$ $\quad$ $y - w = 2$ $\quad$ $z - w = -1$

$x + r = 4$ $\quad$ $y - r = 2$ $\quad$ $z - r = -1$

$x = 4 - r$ $\quad$ $y = 2 + r$ $\quad$ $z = -1 + r$

$x = 4 - r$, $y = 2 + r$, $z = -1 + r$ E $w = r$

O SISTEMA LINEAR POSSUI INFINITAS SOLUÇÕES.

EXERCÍCIO 19 (PÁGINA 42)

$$-4I_3 = -4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -4(1) & -4(0) & -4(0) \\ -4(0) & -4(1) & -4(0) \\ -4(0) & -4(0) & -4(1) \end{bmatrix} = \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

$$-A = \begin{bmatrix} -1 & 0 & -3 \\ -1 & -1 & -1 \\ 0 & -1 & 4 \end{bmatrix}$$

$$-4I_3 - A = -4I_3 + (-A) = \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix} + \begin{bmatrix} -1 & 0 & -3 \\ -1 & -1 & -1 \\ 0 & -1 & 4 \end{bmatrix} =$$

$$= \begin{bmatrix} -4+(-1) & 0+0 & 0+(-3) \\ 0+(-1) & -4+(-1) & 0+(-1) \\ 0+0 & 0+(-1) & -4+4 \end{bmatrix} = \begin{bmatrix} -5 & 0 & -3 \\ -1 & -5 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} -5 & 0 & -5 & 0 \\ -1 & -5 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} -1 & -5 & -1 & 0 \\ -5 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \times(-1) =$$

$$= \left[\begin{array}{ccc|c} 1 & 5 & 1 & 0 \\ -5 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \times(5) \hookleftarrow + = \left[\begin{array}{ccc|c} 1 & 5 & 1 & 0 \\ 0 & 25 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \times(1/25) =$$

$$= \left[\begin{array}{ccc|c} 1 & 5 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} x + 5y + z = 0 \\ 0x + y + 0z = 0 \\ 0x + 0y + 0z = 0 \end{cases} \qquad \begin{array}{l} x + 5y + z = 0 \\ y = 0 \end{array}$$

$x + 5y + z = 0$

$x + 5(0) + z = 0$

$x + 0 + z = 0$ → $x + z = 0$

$x = -z$

$z = r$

$r \Rightarrow$ QUALQUER NÚMERO REAL

$x = -z$

$= -r$

$x = -r$, $y = 0$ e $z = r$

EXERCÍCIO 20 (PÁGINA 42)

$$2I_3 = 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2(1) & 2(0) & 2(0) \\ 2(0) & 2(1) & 2(0) \\ 2(0) & 2(0) & 2(1) \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$-A = \begin{bmatrix} -1 & 0 & -5 \\ -1 & -1 & -1 \\ 0 & -1 & 4 \end{bmatrix}$$

$$2I_3 - A = 2I_3 + (-A) = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & -5 \\ -1 & -1 & -1 \\ 0 & -1 & 4 \end{bmatrix} =$$

$$= \begin{bmatrix} 2+(-1) & 0+0 & 0+(-5) \\ 0+(-1) & 2+(-1) & 0+(-1) \\ 0+0 & 0+(-1) & 2+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -5 \\ -1 & 1 & -1 \\ 0 & -1 & 6 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -5 & 0 \\ -1 & 1 & -1 & 0 \\ 0 & -1 & 6 & 0 \end{array}\right] \begin{matrix} x(1) \\ \hookleftarrow + \\ \end{matrix} = \left[\begin{array}{ccc|c} 1 & 0 & -5 & 0 \\ 0 & 1 & -6 & 0 \\ 0 & -1 & 6 & 0 \end{array}\right] \begin{matrix} \\ x(1) \\ \hookleftarrow + \end{matrix} =$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -5 & 0 \\ 0 & 1 & -6 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} x + 0y - 5z = 0 \\ 0x + y - 6z = 0 \\ 0x + 0y + 0z = 0 \end{cases} \qquad \begin{matrix} x - 5z = 0 \\ y - 6z = 0 \end{matrix}$$

$z = r$

r = QUALQUER NÚMERO REAL

$$\begin{matrix} x - 5z = 0 & \qquad & y - 6z = 0 \\ x - 5r = 0 & \qquad & y - 6r = 0 \\ x = 5r & \qquad & y = 6r \end{matrix}$$

$x = 5r$, $y = 6r$ E $z = r$

EXERCÍCIO 21 (PÁGINA 42)

CADEIRAS	MESINHAS	MESAS	
10	12	15	LIXAGEM
6	8	12	PINTURA
12	12	18	ENVERNIZAMENTO

MÁQUINA DE LIXAGEM:

16 HORAS/SEMANA = 16(60 MINUTOS) = 960 MINUTOS

MÁQUINA DE PINTURA:

11 HORAS/SEMANA = 11(60 MINUTOS) = 660 MINUTOS

MÁQUINA DE ENVERNIZAMENTO:

18 HORAS/SEMANA = 18(60 MINUTOS) = 1.080 MINUTOS

$$\begin{cases} 10x + 12y + 15z = 960 \\ 6x + 8y + 12z = 660 \quad \times(1/2) \\ 12x + 12y + 18z = 1.080 \quad \times(1/6) \end{cases} \sim \begin{cases} 10x + 12y + 15z = 960 \\ 3x + 4y + 6z = 330 \\ 2x + 2y + 3z = 180 \end{cases} \sim$$

$$\sim \begin{cases} 2x + 2y + 3z = 180 \\ 3x + 4y + 6z = 330 \\ 10x + 12y + 15z = 960 \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & 2 & 3 & 180 \\ 3 & 4 & 6 & 330 \\ 10 & 12 & 15 & 960 \end{array}\right] \times(1/2) = \left[\begin{array}{ccc|c} 1 & 1 & 3/2 & 90 \\ 3 & 4 & 6 & 330 \\ 10 & 12 & 15 & 960 \end{array}\right] \begin{array}{l} \times(-3) \\ \hookleftarrow + \\ \\ \end{array} =$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 3/2 & 90 \\ 0 & 1 & 3/2 & 60 \\ 10 & 12 & 15 & 960 \end{array}\right] \times(-10) + = \left[\begin{array}{ccc|c} 1 & 1 & 3/2 & 90 \\ 0 & 1 & 3/2 & 60 \\ 0 & 2 & 0 & 60 \end{array}\right] \times(-1) + =$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 30 \\ 0 & 1 & 3/2 & 60 \\ 0 & 2 & 0 & 60 \end{array}\right] \times(-2) + = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 30 \\ 0 & 1 & 3/2 & 60 \\ 0 & 0 & -3 & -60 \end{array}\right]$$

$$\begin{cases} x + 0y + 0z = 30 \\ 0x + y + 3/2\,z = 60 \\ 0x + 0y - 3z = -60 \end{cases} \qquad \begin{aligned} x &= 30 \\ y + 3/2\,z &= 60 \\ -3z &= -60 \end{aligned}$$

$-3z = -60$

$z = \dfrac{-60}{-3}$

$= 20$

$y + \dfrac{3}{2}z = 60$

$y + \dfrac{3}{\cancel{2}}(\cancel{20}^{10}) = 60$

$y + 3(10) = 60$

$y + 30 = 60$

$y = 60 - 30$

$= 30$

$x = 30,\ y = 30$ E $z = 20$

X => CADEIRAS — 30 CADEIRAS

Y => MESINHAS — 30 MESINHAS

Z => MESAS — 20 MESAS

EXERCÍCIO 22 (PÁGINA 42)

BROCHURA	PADRÃO	LUXO	
1	2	3	COSTURA
2	4	5	COLAGEM

MÁQUINA DE COSTURA:

6 HORAS/DIA = 6 (60 MINUTOS) = 360 MINUTOS

MÁQUINA DE COLAGEM:

11 HORAS/DIA = 11 (60 MINUTOS) = 660 MINUTOS

$$\begin{cases} x + 2y + 3z = 360 \\ 2x + 4y + 5z = 660 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 360 \\ 2 & 4 & 5 & 660 \end{array}\right] \times (-2) = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 360 \\ 0 & 0 & -1 & -60 \end{array}\right]$$

$$\begin{cases} x + 2y + 3z = 360 \\ 0x + 0y - z = -60 \end{cases} \qquad \begin{array}{l} x + 2y + 3z = 360 \\ -z = -60 \end{array}$$

$-z = -60$

$z = \dfrac{-60}{-1}$

$= 60$

$x + 2y + 3z = 360$

$x + 2y + 3(60) = 360$

$x + 2y + 180 = 360$

$x + 2y = 360 - 180 \rightarrow x + 2y = 180$

$y = r$

$r \Rightarrow$ QUALQUER NÚMERO REAL

$x + 2y = 180$

$x + 2r = 180$

$x = 180 - 2r$

$x = 180 - 2r$, $y = r$ E $z = 60$

O SISTEMA LINEAR POSSUI INFINITAS SOLUÇÕES.