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EXERCÍCIOS 1.5 → ÁLGEBRA LINEAR, BERNARD KOLMAN, EDITORA GUANABARA, 3ª EDIÇÃO

EXERCÍCIO 1 (PÁGINA 53)

$A = \begin{bmatrix} 2 & 1 \\ -2 & 3 \end{bmatrix}$ $\quad A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$AA^{-1} = I_2$

$\begin{bmatrix} 2 & 1 \\ -2 & 3 \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 2a+c & 2b+d \\ -2a+3c & -2b+3d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{cases} \cancel{2a}+c = 1 \\ -\cancel{2a}+3c = 0 \end{cases}$
$4c = 1$

$\begin{cases} \cancel{2b}+d = 0 \\ -\cancel{2b}+3d = 1 \end{cases}$
$4d = 1$

$4c = 1$

$c = \frac{1}{4}$

$4d = 1$

$d = \frac{1}{4}$

$2a+c = 1$

$2a + \frac{1}{4} = 1$

$2a = 1 - \frac{1}{4}$

$2a = \frac{4-1}{4}$

$2a = \frac{3}{4}$

$a = \frac{3}{2(4)}$

$= \frac{3}{8}$

$2b+d = 0$

$2b + \frac{1}{4} = 0$

$2b = -\frac{1}{4}$

$b = -\frac{1}{2(4)}$

$= -\frac{1}{8}$

tilibra

$$A^{-1} = \begin{bmatrix} 3/8 & -1/8 \\ 1/4 & 1/4 \end{bmatrix}$$

EXERCÍCIO 2 (PÁGINA 53)

$$A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A A^{-1} = I_2$$

$$\begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2a+c & 2b+d \\ -4a-2c & -4b-2d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{cases} 2a+c = 1 \times (2) \\ -4a-2c = 0 \end{cases} \sim \begin{cases} \cancel{4a}+\cancel{2c} = 2 \\ \cancel{-4a}-\cancel{2c} = 0 \end{cases}$$
$$0 = 2$$

$$\begin{cases} 2b+d = 0 \times (2) \\ -4b-2d = 1 \end{cases} \sim \begin{cases} \cancel{4b}+\cancel{2d} = 0 \\ \cancel{-4b}-\cancel{2d} = 1 \end{cases}$$
$$0 = 1$$

ESTES SISTEMAS LINEARES NÃO TÊM SOLUÇÕES, DE MANEIRA QUE 'A' NÃO TEM INVERSA. ASSIM, 'A' É UMA MATRIZ SINGULAR.

EXERCÍCIO 3 (PÁGINA 53)

$$A = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$AA^{-1} = I_2$

$$\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a+c & b+d \\ 3a+4c & 3b+4d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{cases} a+c=1 \times(-3) \\ 3a+4c=0 \end{cases} \sim \begin{cases} \cancel{-3a}-3c=-3 \\ \cancel{3a}+4c=0 \end{cases}$$
$$c=-3$$

$$\begin{cases} b+d=0 \times(-3) \\ 3b+4d=1 \end{cases} \sim \begin{cases} \cancel{-3b}-3d=0 \\ \cancel{3b}+4d=1 \end{cases}$$
$$d=1$$

ESTES SISTEMAS LINEARES TÊM SOLUÇÕES, DE MANEIRA QUE 'A' TEM INVERSA. ASSIM, 'A' É UMA MATRIZ NÃO-SINGULAR.

$a+c=1$
$a+(-3)=1$
$a-3=1$
$a=1+3$
$=4$

$b+d=0$
$b+1=0$
$b=-1$

$$A^{-1} = \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}$$

EXERCÍCIO 4 (PÁGINA 53)

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 2 & 3 \\ 2 & 2 & 1 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$A\,A^{-1} = I_3$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 3 & 2 & 3 \\ 2 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a+2d-g & b+2e-h & c+2f-i \\ 3a+2d+3g & 3b+2e+3h & 3c+2f+3i \\ 2a+2d+g & 2b+2e+h & 2c+2f+i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{cases} a+2d-g=1 \\ 3a+2d+3g=0 \\ 2a+2d+g=0 \end{cases} \sim \begin{cases} a+2d-g=1 \\ 2a+2d+g=0 \\ 3a+2d+3g=0 \end{cases}$$

$$\begin{array}{r} 2a+2d+g=0 \\ -a-2d+g=-1 \\ \hline a+0+2g=-1 \end{array} \qquad \begin{array}{r} a+0+2g=-1 \\ -a-2d+g=-1 \\ \hline 0-2d+3g=-2 \end{array} \qquad -2d+3g=-2$$

$$\begin{array}{r} 3a+2d+3g=0 \\ -a-2d+g=-1 \\ \hline 2a+0+4g=-1 \end{array} \qquad \begin{array}{r} 2a+0+4g=-1 \\ -a-2d+g=-1 \\ \hline a-2d+5g=-2 \end{array} \qquad \begin{array}{r} a-2d+5g=-2 \\ -a-2d+g=-1 \\ \hline 0-4d+6g=-3 \end{array}$$

$$-4d+6g=-3$$

$$\begin{cases} -2d+3g=-2 \quad \times(-2) \\ -4d+6g=-3 \end{cases} \sim \begin{cases} \cancel{4d}-\cancel{6g}=4 \\ -\cancel{4d}+\cancel{6g}=-3 \end{cases}$$

$$0=1$$

$$\begin{cases} b+2e-h=0 \\ 3b+2e+3h=1 \\ 2b+2e+h=0 \end{cases} \sim \begin{cases} b+2e-h=0 \\ 2b+2e+h=0 \\ 3b+2e+3h=1 \end{cases}$$

$$\begin{array}{r} 2b+2e+h=0 \\ -b-2e+h=0 \\ \hline b+0+2h=0 \end{array} \qquad \begin{array}{r} \cancel{b}+0+2h=0 \\ -\cancel{b}-2e+h=0 \\ \hline 0-2e+3h=0 \end{array} \qquad -2e+3h=0$$

$$\begin{array}{r} 3b+2e+3h=1 \\ -b-2e+h=0 \\ \hline 2b+0+4h=1 \end{array} \qquad \begin{array}{r} 2b+0+4h=1 \\ -b-2e+h=0 \\ \hline b-2e+5h=1 \end{array} \qquad \begin{array}{r} \cancel{b}-2e+5h=1 \\ -\cancel{b}-2e+h=0 \\ \hline 0-4e+6h=1 \end{array}$$

$$-4e+6h=1$$

$$\begin{cases} -2e+3h=0 & \times(-2) \\ -4e+6h=1 \end{cases} \sim \begin{array}{r} \cancel{4e}-\cancel{6h}=0 \\ -\cancel{4e}+\cancel{6h}=1 \\ \hline 0=1 \end{array}$$

$$\begin{cases} x+2f-i=0 \\ 3x+2f+3i=0 \\ 2x+2f+i=1 \end{cases} \sim \begin{cases} x+2f-i=0 \\ 2x+2f+i=1 \\ 3x+2f+3i=0 \end{cases}$$

$$\begin{array}{r} 2x+2f+i=1 \\ -x-2f+i=0 \\ \hline x+0+2i=1 \end{array} \qquad \begin{array}{r} \cancel{x}+0+2i=1 \\ -\cancel{x}-2f+i=0 \\ \hline 0-2f+3i=1 \end{array} \qquad -2f+3i=1$$

$$\begin{array}{r} 3x+2f+3i=0 \\ -x-2f+i=0 \\ \hline 2x+0+4i=0 \end{array} \qquad \begin{array}{r} 2x+0+4i=0 \\ -x-2f+i=0 \\ \hline x-2f+5i=0 \end{array} \qquad \begin{array}{r} \cancel{x}-2f+5i=0 \\ -\cancel{x}-2f+i=0 \\ \hline 0-4f+6i=0 \end{array}$$

$$-4f+6i=0$$

$$\begin{cases} -2f + 3i = 1 & \times(-2) \\ -4f + 6i = 0 \end{cases} \sim \begin{cases} 4f - 6i = -2 \\ -4f + 6i = 0 \end{cases}$$
$$0 = -2$$

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EXERCÍCIO 5 (PÁGINA 53)

5.A

$$[A \mid I_2] = \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ -2 & 6 & 0 & 1 \end{array}\right] \begin{array}{l} \times(2) \\ \hookleftarrow + \end{array}$$

$$= \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 12 & 2 & 1 \end{array}\right] \begin{array}{l} \\ \times(1/12) \end{array}$$

$$= \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & 1/6 & 1/12 \end{array}\right] \begin{array}{l} \hookleftarrow + \\ \times(-3) \end{array}$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & 1/2 & -1/4 \\ 0 & 1 & 1/6 & 1/12 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1/2 & -1/4 \\ 1/6 & 1/12 \end{bmatrix}$$

5.B

$$[A \mid I_3] = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \times(-1) \\ \hookleftarrow + \\ \\ \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \\ 0 & -1 & -1 & -1 & 1 & 0 \end{array}\right] \begin{array}{l} \hookleftarrow + \\ \times(-2) \\ \\ \end{array}$$

$$[A \mid I_3] = \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & -2 \\ 0 & 1 & 2 & 0 & 0 & 1 \\ 0 & -1 & -1 & -1 & 1 & 0 \end{array}\right] \begin{array}{l} \\ \times(1) \\ + \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & -2 \\ 0 & 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \begin{array}{l} \\ + \\ \times(1) \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \begin{array}{l} \\ + \\ \times(-2) \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 2 & -2 & -1 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

5.C

$$[A \mid I_4] = \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 2 & -1 & 2 & 0 & 1 & 0 & 0 \\ 1 & -1 & 2 & 1 & 0 & 0 & 1 & 0 \\ 1 & 3 & 3 & 2 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \times(-1) \\ + \\ \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & -1 & 1 & 0 & 0 \\ 1 & -1 & 2 & 1 & 0 & 0 & 1 & 0 \\ 1 & 3 & 3 & 2 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \times(-1) \\ \\ + \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & -1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 & -1 & 0 & 1 & 0 \\ 1 & 3 & 3 & 2 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \times(-1) \\ \\ + \\ \\ \end{array}$$

$$[A \mid I_4] = \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & -1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 2 & 2 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \leftarrow + \\ \times(-1) \\ \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 3 & 0 & 2 & -1 & 0 & 0 \\ 0 & 1 & -2 & 1 & -1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 2 & 2 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \times(2) \\ \leftarrow + \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 3 & 0 & 2 & -1 & 0 & 0 \\ 0 & 1 & -2 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -3 & 2 & -3 & 2 & 1 & 0 \\ 0 & 2 & 2 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \times(-2) \\ \\ \leftarrow + \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 3 & 0 & 2 & -1 & 0 & 0 \\ 0 & 1 & -2 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -3 & 2 & -3 & 2 & 1 & 0 \\ 0 & 0 & 6 & -1 & 1 & -2 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \\ \times\left(-\frac{1}{3}\right) \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 3 & 0 & 2 & -1 & 0 & 0 \\ 0 & 1 & -2 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2/3 & 1 & -2/3 & -1/3 & 0 \\ 0 & 0 & 6 & -1 & 1 & -2 & 0 & 1 \end{array}\right] \begin{array}{l} \leftarrow + \\ \\ \times(-3) \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 2 & -1 & 1 & 1 & 0 \\ 0 & 1 & -2 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2/3 & 1 & -2/3 & -1/3 & 0 \\ 0 & 0 & 6 & -1 & 1 & -2 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \leftarrow + \\ \times(2) \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 2 & -1 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1/3 & 1 & -1/3 & -2/3 & 0 \\ 0 & 0 & 1 & -2/3 & 1 & -2/3 & -1/3 & 0 \\ 0 & 0 & 6 & -1 & 1 & -2 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \\ \times(-6) \\ \leftarrow + \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 2 & -1 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1/3 & 1 & -1/3 & -2/3 & 0 \\ 0 & 0 & 1 & -2/3 & 1 & -2/3 & -1/3 & 0 \\ 0 & 0 & 0 & 3 & -5 & 2 & 2 & 1 \end{array}\right] \begin{array}{l} \\ \\ \\ \times\left(\frac{1}{3}\right) \end{array}$$

$$[A \mid I_4] = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 2 & -1 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1/3 & 1 & -1/3 & -2/3 & 0 \\ 0 & 0 & 1 & -2/3 & 1 & -2/3 & -1/3 & 0 \\ 0 & 0 & 0 & 1 & -5/3 & 2/3 & 2/3 & 1/3 \end{array}\right] \begin{matrix} \\ + \\ \\ \times(-2) \end{matrix}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 7/3 & -1/3 & -1/3 & -2/3 \\ 0 & 1 & 0 & -1/3 & 1 & -1/3 & -2/3 & 0 \\ 0 & 0 & 1 & -2/3 & 1 & -2/3 & -1/3 & 0 \\ 0 & 0 & 0 & 1 & -5/3 & 2/3 & 2/3 & 1/3 \end{array}\right] \begin{matrix} \\ \\ + \\ \times(1/3) \end{matrix}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 7/3 & -1/3 & -1/3 & -2/3 \\ 0 & 1 & 0 & 0 & 4/9 & -1/9 & -4/9 & 1/9 \\ 0 & 0 & 1 & -2/3 & 1 & -2/3 & -1/3 & 0 \\ 0 & 0 & 0 & 1 & -5/3 & 2/3 & 2/3 & 1/3 \end{array}\right] \begin{matrix} \\ \\ + \\ \times(2/3) \end{matrix}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 7/3 & -1/3 & -1/3 & -2/3 \\ 0 & 1 & 0 & 0 & 4/9 & -1/9 & -4/9 & 1/9 \\ 0 & 0 & 1 & 0 & -1/9 & -2/9 & 1/9 & 2/9 \\ 0 & 0 & 0 & 1 & -5/3 & 2/3 & 2/3 & 1/3 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 7/3 & -1/3 & -1/3 & -2/3 \\ 4/9 & -1/9 & -4/9 & 1/9 \\ -1/9 & -2/9 & 1/9 & 2/9 \\ -5/3 & 2/3 & 2/3 & 1/3 \end{bmatrix}$$

EXERCÍCIO 6 (PÁGINA 34)

6.A $[A \mid I_2] = \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2 & 6 & 0 & 1 \end{array}\right] \begin{matrix} \times(-2) \\ + \end{matrix}$

$= \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 0 & -2 & -1 \end{array}\right]$

'A' É UMA MATRIZ SINGULAR, OU SEJA, NÃO TEM INVERSA.

6.B

$$[A \mid I_3] = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 1 & 2 & 4 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \times(-1) \\ + \\ \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \times(1/2) \\ \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 3/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} + \\ \times(-2) \\ \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 3/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} \\ + \\ \times(-3/2) \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 3/2 & 1/2 & -3/2 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 3/2 & 1/2 & -3/2 \\ -1 & 0 & 1 \end{bmatrix}$$

6.C

$$[A \mid I_4] = \left[\begin{array}{cccc|cccc} 1 & 1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & -2 & 0 & 0 & 1 & 0 \\ 0 & 3 & 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \times(-1) \\ + \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -3 & -1 & 0 & 1 & 0 \\ 0 & 3 & 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{cccc|cccc} 1 & 1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -3 & -1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 3 & 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} + \\ \times(-1) \\ \\ \end{array}$$

$$[A \mid I_4] = \begin{array}{c} \left[\begin{array}{cccc|cccc} 1 & 0 & 3 & 4 & 2 & 0 & -1 & 0 \\ 0 & 1 & -1 & -3 & -1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 3 & 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \end{array} \begin{array}{l} \\ \times(2) \\ \leftarrow + \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 3 & 4 & 2 & 0 & -1 & 0 \\ 0 & 1 & -1 & -3 & -1 & 0 & 1 & 0 \\ 0 & 0 & -2 & -6 & -2 & 1 & 2 & 0 \\ 0 & 3 & 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \times(-3) \\ + \\ \leftarrow \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 3 & 4 & 2 & 0 & -1 & 0 \\ 0 & 1 & -1 & -3 & -1 & 0 & 1 & 0 \\ 0 & 0 & -2 & -6 & -2 & 1 & 2 & 0 \\ 0 & 0 & 5 & 10 & 3 & 0 & -3 & 1 \end{array}\right] \begin{array}{l} \\ \\ \times\left(-\frac{1}{2}\right) \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 3 & 4 & 2 & 0 & -1 & 0 \\ 0 & 1 & -1 & -3 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 3 & 1 & -\frac{1}{2} & -1 & 0 \\ 0 & 0 & 5 & 10 & 3 & 0 & -3 & 1 \end{array}\right] \begin{array}{l} \leftarrow \\ + \\ \times(-3) \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -5 & -1 & \frac{3}{2} & 2 & 0 \\ 0 & 1 & -1 & -3 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 3 & 1 & -\frac{1}{2} & -1 & 0 \\ 0 & 0 & 5 & 10 & 3 & 0 & -3 & 1 \end{array}\right] \begin{array}{l} \\ \leftarrow + \\ \times(1) \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -5 & -1 & \frac{3}{2} & 2 & 0 \\ 0 & 1 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 & -\frac{1}{2} & -1 & 0 \\ 0 & 0 & 5 & 10 & -1 & \frac{3}{2} & 2 & 0 \end{array}\right] \begin{array}{l} \\ \\ \times(-5) \\ \leftarrow + \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -5 & -1 & \frac{3}{2} & 2 & 0 \\ 0 & 1 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 & -\frac{1}{2} & -1 & 0 \\ 0 & 0 & 0 & -5 & -6 & 4 & 7 & 0 \end{array}\right] \begin{array}{l} \\ \\ \\ \times\left(-\frac{1}{5}\right) \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -5 & -1 & \frac{3}{2} & 2 & 0 \\ 0 & 1 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 & -\frac{1}{2} & -1 & 0 \\ 0 & 0 & 0 & 1 & \frac{6}{5} & -\frac{4}{5} & -\frac{7}{5} & 0 \end{array}\right] \begin{array}{l} \leftarrow \\ + \\ \\ \times(5) \end{array}$$

$$[A \mid I_4] = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 5 & -\frac{5}{2} & -5 & 0 \\ 0 & 1 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 & -\frac{1}{2} & -1 & 0 \\ 0 & 0 & 0 & 1 & \frac{6}{5} & -\frac{5}{3} & -\frac{7}{5} & 0 \end{array}\right] \begin{array}{l} \\ \\ \leftarrow + \\ \times(-3) \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 5 & -\frac{5}{2} & -5 & 0 \\ 0 & 1 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 & -\frac{13}{5} & \frac{19}{10} & \frac{16}{5} & 0 \\ 0 & 0 & 0 & 1 & \frac{6}{5} & -\frac{5}{5} & -\frac{7}{5} & 0 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 5 & -\frac{5}{2} & -5 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ -\frac{13}{5} & \frac{19}{10} & \frac{16}{5} & 0 \\ \frac{6}{5} & -\frac{5}{5} & -\frac{7}{5} & 0 \end{bmatrix}$$

EXERCÍCIO 7 (PÁGINA 54)

7.A $$[A \mid I_2] = \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{array}\right] \begin{array}{l} \times(-2) \\ \leftarrow + \end{array}$$

$$= \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & -2 & -2 & 1 \end{array}\right] \begin{array}{l} \\ \times(-\frac{1}{2}) \end{array}$$

$$= \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & 1 & -\frac{1}{2} \end{array}\right] \begin{array}{l} \leftarrow + \\ \times(-3) \end{array}$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & -2 & \frac{3}{2} \\ 0 & 1 & 1 & -\frac{1}{2} \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & \frac{3}{2} \\ 1 & -\frac{1}{2} \end{bmatrix}$$

7.B $$[A \mid I_4] = \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 3 & 1 & 2 & 0 & 1 & 0 & 0 \\ 1 & 2 & -1 & 1 & 0 & 0 & 1 & 0 \\ 5 & 9 & 1 & 6 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \times(-1) \\ \leftarrow + \\ \\ \\ \end{array}$$

$$[A \,|\, I_4] = \begin{array}{c} \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 & -1 & 1 & 0 & 0 \\ 1 & 2 & -1 & 1 & 0 & 0 & 1 & 0 \\ 5 & 9 & 1 & 6 & 0 & 0 & 0 & 1 \end{array}\right] \end{array} \begin{array}{l} \times(-1) \\ + \\ \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & -1 & 0 & 1 & 0 \\ 5 & 9 & 1 & 6 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \times(-5) \\ \\ + \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & -1 & 0 & 1 & 0 \\ 0 & 4 & -4 & 1 & -5 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 4 & -4 & 1 & -5 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} + \\ \times(-1) \\ \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 3 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 4 & -4 & 1 & -5 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \times(-2) \\ + \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 3 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 4 & 1 & 1 & 1 & -2 & 0 \\ 0 & 4 & -4 & 1 & -5 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \times(-4) \\ + \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 3 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 4 & 1 & 1 & 1 & -2 & 0 \\ 0 & 0 & 4 & 1 & -1 & 0 & -4 & 1 \end{array}\right] \begin{array}{l} \\ \\ \times(1/4) \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 3 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1/4 & 1/4 & 1/4 & -1/2 & 0 \\ 0 & 0 & 4 & 1 & -1 & 0 & -4 & 1 \end{array}\right] \begin{array}{l} + \\ \\ \times(-3) \\ \\ \end{array}$$

$$[A \mid I_4] = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1/4 & 1/4 & -3/4 & 3/2 & 0 \\ 0 & 1 & -2 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1/4 & 1/4 & 1/4 & -1/2 & 0 \\ 0 & 0 & 4 & 1 & -1 & 0 & -4 & 1 \end{array}\right] \begin{matrix} \\ \leftarrow + \\ \times(2) \\ \\ \end{matrix}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1/4 & 1/4 & -3/4 & 3/2 & 0 \\ 0 & 1 & 0 & 1/2 & -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 1/4 & 1/4 & 1/4 & -1/2 & 0 \\ 0 & 0 & 4 & 1 & -1 & 0 & -4 & 1 \end{array}\right] \begin{matrix} \\ \\ \times(-4) \\ \leftarrow + \end{matrix}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1/4 & 1/4 & -3/4 & 3/2 & 0 \\ 0 & 1 & 0 & 1/2 & -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 1/4 & 1/4 & 1/4 & -1/2 & 0 \\ 0 & 0 & 0 & 0 & -2 & -1 & -2 & 1 \end{array}\right]$$

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7.C

$$[A \mid I_3] = \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \begin{matrix} \times(-1) \\ \leftarrow + \\ \\ \end{matrix}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \begin{matrix} \times(-1) \\ + \\ \leftarrow \end{matrix}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & -2 & 0 & -1 & 0 & 1 \end{array}\right] \begin{matrix} \leftarrow + \\ \times(-2) \\ \\ \end{matrix}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 3 & -2 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & -2 & 0 & -1 & 0 & 1 \end{array}\right] \begin{matrix} \\ \times(2) \\ \leftarrow + \end{matrix}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 3 & -2 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 2 & -3 & 2 & 1 \end{array}\right] \begin{matrix} \\ \\ \times(1/2) \end{matrix}$$

$$[A \mid I_3] = \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 3 & -2 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 1 & \frac{1}{2} \end{array}\right] \begin{array}{l} \\ + \\ \times(1) \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{2} & -1 & \frac{1}{2} \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 1 & \frac{1}{2} \end{array}\right] \begin{array}{l} \\ + \\ \times(-1) \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{2} & -1 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{3}{2} & 1 & \frac{1}{2} \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} \frac{3}{2} & -1 & \frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{3}{2} & 1 & \frac{1}{2} \end{bmatrix}$$

EXERCÍCIO 8 (PÁGINA 54)

8.A

$$[A \mid I_3] = \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \times(-1) \\ + \\ \\ \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} + \\ \times(-1) \\ \\ \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 2 & -1 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \times(-1) \\ + \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 2 & -1 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & -1 & 1 \end{array}\right] \begin{array}{l} \\ \\ \times(-1) \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 2 & -1 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & -1 \end{array}\right] \begin{array}{l} + \\ \\ \times(1) \end{array}$$

$$[A \mid I_3] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & -1 \end{array}\right]\begin{array}{l} \\ + \\ \times(-2) \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 2 \\ 1 & 0 & -1 \end{bmatrix}$$

8.B

$$[A \mid I_3] = \left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 & 1 & 0 \\ 1 & 3 & 2 & 0 & 0 & 1 \end{array}\right]\begin{array}{l} \times(-1) \\ + \\ \\ \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 1 & 3 & 2 & 0 & 0 & 1 \end{array}\right]\begin{array}{l} \times(-1) \\ \\ + \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \end{array}\right]\begin{array}{l} + \\ \times(-2) \\ \\ \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 4 & 3 & -2 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \end{array}\right]\begin{array}{l} \\ \times(-1) \\ + \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 4 & 3 & -2 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]\begin{array}{l} + \\ \\ \times(-4) \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 2 & -4 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]\begin{array}{l} \\ + \\ \times(1) \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 2 & -4 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 3 & 2 & -4 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

8.C

$$[A \mid I_3] = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \times(-1) \\ \leftarrow + \\ \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & -1 & -1 & -1 & 1 & 0 \end{array}\right] \begin{array}{l} \leftarrow + \\ \times(-2) \\ \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & -2 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & -1 & -1 & -1 & 1 & 0 \end{array}\right] \begin{array}{l} \\ \times(1) \\ \leftarrow + \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & -2 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 & 1 \end{array}\right]$$

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EXERCÍCIO 9 (PÁGINA 54)

9.A

$$[A \mid I_4] = \left[\begin{array}{cccc|cccc} 1 & 2 & -3 & 1 & 1 & 0 & 0 & 0 \\ -1 & 3 & -3 & -2 & 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 5 & 0 & 0 & 1 & 0 \\ 3 & 1 & -2 & 5 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \times(1) \\ \leftarrow + \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 2 & -3 & 1 & 1 & 0 & 0 & 0 \\ 0 & 5 & -6 & -1 & 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 5 & 0 & 0 & 1 & 0 \\ 3 & 1 & -2 & 5 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \times(-2) \\ \\ \leftarrow + \\ \end{array}$$

$$[A \,\vdots\, I_4] = \left[\begin{array}{cccc|cccc} 1 & 2 & -3 & 1 & 1 & 0 & 0 & 0 \\ 0 & 5 & -6 & -1 & 1 & 1 & 0 & 0 \\ 0 & -4 & 7 & 3 & -2 & 0 & 1 & 0 \\ 3 & 1 & -2 & 5 & 0 & 0 & 0 & 1 \end{array}\right]\begin{array}{l} \times(-3) \\ \\ + \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 2 & -3 & 1 & 1 & 0 & 0 & 0 \\ 0 & 5 & -6 & -1 & 1 & 1 & 0 & 0 \\ 0 & -4 & 7 & 3 & -2 & 0 & 1 & 0 \\ 0 & -5 & 7 & 2 & -3 & 0 & 0 & 1 \end{array}\right]\begin{array}{l} \\ \times(1/5) \\ \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 2 & -3 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -6/5 & -1/5 & 1/5 & 1/5 & 0 & 0 \\ 0 & -4 & 7 & 3 & -2 & 0 & 1 & 0 \\ 0 & -5 & 7 & 2 & -3 & 0 & 0 & 1 \end{array}\right]\begin{array}{l} + \\ \times(-2) \\ \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & -3/5 & 7/5 & 3/5 & -2/5 & 0 & 0 \\ 0 & 1 & -6/5 & -1/5 & 1/5 & 1/5 & 0 & 0 \\ 0 & -4 & 7 & 3 & -2 & 0 & 1 & 0 \\ 0 & -5 & 7 & 2 & -3 & 0 & 0 & 1 \end{array}\right]\begin{array}{l} \\ \times(4) \\ + \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & -3/5 & 7/5 & 3/5 & -2/5 & 0 & 0 \\ 0 & 1 & -6/5 & -1/5 & 1/5 & 1/5 & 0 & 0 \\ 0 & 0 & 11/5 & 11/5 & 6/5 & 4/5 & 1 & 0 \\ 0 & -5 & 7 & 2 & -3 & 0 & 0 & 1 \end{array}\right]\begin{array}{l} \\ \times(5) \\ \\ + \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & -3/5 & 7/5 & 3/5 & -2/5 & 0 & 0 \\ 0 & 1 & -6/5 & -1/5 & 1/5 & 1/5 & 0 & 0 \\ 0 & 0 & 11/5 & 11/5 & 6/5 & 4/5 & 1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & -3/5 & 7/5 & 3/5 & -2/5 & 0 & 0 \\ 0 & 1 & -6/5 & -1/5 & 1/5 & 1/5 & 0 & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 & 0 & 1 \\ 0 & 0 & 11/5 & 11/5 & 6/5 & 4/5 & 1 & 0 \end{array}\right]\begin{array}{l} + \\ \\ \times(3/5) \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 2 & 3/5 & 1/5 & 0 & 3/5 \\ 0 & 1 & -6/5 & -1/5 & 1/5 & 1/5 & 0 & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 & 0 & 1 \\ 0 & 0 & 11/5 & 11/5 & 6/5 & 4/5 & 1 & 0 \end{array}\right]\begin{array}{l} \\ + \\ \times(6/5) \\ \\ \end{array}$$

$$[A \mid I_4] = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 2 & 3/3 & 1/3 & 0 & 3/3 \\ 0 & 1 & 0 & 1 & -11/3 & 7/3 & 0 & 6/3 \\ 0 & 0 & 1 & 1 & -2 & 1 & 0 & 1 \\ 0 & 0 & 11/3 & 11/3 & 6/3 & 5/3 & 1 & 0 \end{array}\right] \begin{matrix} \\ \\ \times\left(-11/3\right) \\ \hookleftarrow + \end{matrix}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 2 & 3/3 & 1/3 & 0 & 3/3 \\ 0 & 1 & 0 & 1 & -11/3 & 7/3 & 0 & 6/3 \\ 0 & 0 & 1 & 1 & -2 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 28/3 & -7/3 & 1 & -11/3 \end{array}\right]$$

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9.B

$$[A \mid I_3] = \left[\begin{array}{ccc|ccc} 3 & 1 & 2 & 1 & 0 & 0 \\ 2 & 1 & 2 & 0 & 1 & 0 \\ 1 & 2 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 0 & 0 & 1 \\ 2 & 1 & 2 & 0 & 1 & 0 \\ 3 & 1 & 2 & 1 & 0 & 0 \end{array}\right] \begin{matrix} \times(-2) \\ \hookleftarrow + \\ \\ \end{matrix}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 0 & 0 & 1 \\ 0 & -3 & -2 & 0 & 1 & -2 \\ 3 & 1 & 2 & 1 & 0 & 0 \end{array}\right] \begin{matrix} \times(-3) \\ + \\ \hookleftarrow \end{matrix}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 0 & 0 & 1 \\ 0 & -3 & -2 & 0 & 1 & -2 \\ 0 & -5 & -4 & 1 & 0 & -3 \end{array}\right] \begin{matrix} \\ \times\left(-1/3\right) \\ \\ \end{matrix}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 0 & 0 & 1 \\ 0 & 1 & 2/3 & 0 & -1/3 & 2/3 \\ 0 & -5 & -4 & 1 & 0 & -3 \end{array}\right] \begin{matrix} \hookleftarrow + \\ \times(-2) \\ \\ \end{matrix}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 2/3 & 0 & 2/3 & -1/3 \\ 0 & 1 & 2/3 & 0 & -1/3 & 2/3 \\ 0 & -5 & -4 & 1 & 0 & -3 \end{array}\right] \begin{matrix} \\ \times(5) \\ \hookleftarrow + \end{matrix}$$

$$[A \mid I_3] = \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{2}{3} & 0 & \frac{2}{3} & -\frac{1}{3} \\ 0 & 1 & \frac{2}{3} & 0 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 0 & -\frac{2}{3} & 1 & -\frac{5}{3} & \frac{1}{3} \end{array}\right] \begin{matrix} \\ \\ \times\left(-\frac{3}{2}\right) \end{matrix}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{2}{3} & 0 & \frac{2}{3} & -\frac{1}{3} \\ 0 & 1 & \frac{2}{3} & 0 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 1 & -\frac{3}{2} & \frac{5}{2} & -\frac{1}{2} \end{array}\right] \begin{matrix} \hookleftarrow \\ + \\ \times\left(-\frac{2}{3}\right) \end{matrix}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & \frac{2}{3} & 0 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 1 & -\frac{3}{2} & \frac{5}{2} & -\frac{1}{2} \end{array}\right] \begin{matrix} \\ \hookleftarrow + \\ \times\left(-\frac{2}{3}\right) \end{matrix}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -\frac{3}{2} & \frac{5}{2} & -\frac{1}{2} \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 1 & -2 & 1 \\ -\frac{3}{2} & \frac{5}{2} & -\frac{1}{2} \end{bmatrix}$$

9.C

$$[A \mid I_3] = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \begin{matrix} \times(-1) \\ \hookleftarrow + \\ \\ \end{matrix}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -1 & -1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \begin{matrix} \times(-1) \\ + \\ \hookleftarrow \end{matrix}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -1 & -1 & 1 & 0 \\ 0 & -1 & -3 & -1 & 0 & 1 \end{array}\right] \begin{matrix} \\ \times(-1) \\ \\ \end{matrix}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & -1 & -3 & -1 & 0 & 1 \end{array}\right] \begin{matrix} \hookleftarrow + \\ \times(-2) \\ \\ \end{matrix}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -1 & 2 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & -1 & -3 & -1 & 0 & 1 \end{array}\right] \begin{matrix} \\ \times(1) \\ \hookleftarrow + \end{matrix}$$

$$[A \,\vdots\, I_3] = \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -1 & 2 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 0 & -2 & 0 & -1 & 1 \end{array}\right] \begin{array}{l} \\ \\ \times(-1/2) \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -1 & 2 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1/2 & -1/2 \end{array}\right] \begin{array}{l} \leftarrow \\ \quad + \\ \times(-1) \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 3/2 & 1/2 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1/2 & -1/2 \end{array}\right] \begin{array}{l} \\ \leftarrow + \\ \times(-1) \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 3/2 & 1/2 \\ 0 & 1 & 0 & 1 & -3/2 & 1/2 \\ 0 & 0 & 1 & 0 & 1/2 & -1/2 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -1 & 3/2 & 1/2 \\ 1 & -3/2 & 1/2 \\ 0 & 1/2 & -1/2 \end{bmatrix}$$

EXERCÍCIO 10 (PÁGINA 54)

10.A

$$[A \,\vdots\, I_3] = \left[\begin{array}{ccc|ccc} 2 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 1 & 0 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 2 & 1 & 3 & 1 & 0 & 0 \end{array}\right] \begin{array}{l} \times(-2) \\ \quad + \\ \leftarrow \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & -3 & 1 & 0 & -2 \end{array}\right] \begin{array}{l} \\ \times(-1) \\ \leftarrow + \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & -5 & 1 & -1 & -2 \end{array}\right] \begin{array}{l} \\ \\ \times(-1/5) \end{array}$$

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$$[A \mid I_3] = \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -\frac{1}{5} & \frac{1}{5} & \frac{2}{5} \end{array}\right] \begin{array}{l} + \\ \\ \times(-3) \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{5} & -\frac{3}{5} & -\frac{1}{5} \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -\frac{1}{5} & \frac{1}{5} & \frac{2}{5} \end{array}\right] \begin{array}{l} \\ + \\ \times(-2) \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{5} & -\frac{3}{5} & -\frac{1}{5} \\ 0 & 1 & 0 & \frac{2}{5} & \frac{3}{5} & -\frac{4}{5} \\ 0 & 0 & 1 & -\frac{1}{5} & \frac{1}{5} & \frac{2}{5} \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} \frac{3}{5} & -\frac{3}{5} & -\frac{1}{5} \\ \frac{2}{5} & \frac{3}{5} & -\frac{4}{5} \\ -\frac{1}{5} & \frac{1}{5} & \frac{2}{5} \end{bmatrix}$$

10.3

$$[A \mid I_4] = \left[\begin{array}{cccc|cccc} 1 & -1 & 2 & 3 & 1 & 0 & 0 & 0 \\ 4 & 1 & 2 & 0 & 0 & 1 & 0 & 0 \\ 2 & -1 & 3 & 1 & 0 & 0 & 1 & 0 \\ 4 & 2 & 1 & -5 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{cccc|cccc} 1 & -1 & 2 & 3 & 1 & 0 & 0 & 0 \\ 2 & -1 & 3 & 1 & 0 & 0 & 1 & 0 \\ 4 & 1 & 2 & 0 & 0 & 1 & 0 & 0 \\ 4 & 2 & 1 & -5 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \times(-2) \\ + \\ \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & -1 & 2 & 3 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -5 & -2 & 0 & 1 & 0 \\ 4 & 1 & 2 & 0 & 0 & 1 & 0 & 0 \\ 4 & 2 & 1 & -5 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \times(-4) \\ \\ + \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & -1 & 2 & 3 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -5 & -2 & 0 & 1 & 0 \\ 0 & 5 & -6 & -12 & -4 & 1 & 0 & 0 \\ 4 & 2 & 1 & -5 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \times(-4) \\ \\ + \\ \\ \end{array}$$

$$[A \mid I_4] = \left[\begin{array}{cccc|cccc} 1 & -1 & 2 & 3 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -5 & -2 & 0 & 1 & 0 \\ 0 & 5 & -6 & -12 & -4 & 1 & 0 & 0 \\ 0 & 6 & -7 & -17 & -4 & 0 & 0 & 1 \end{array}\right] \begin{matrix} + \\ \times(1) \\ \\ \\ \end{matrix}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & -2 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -5 & -2 & 0 & 1 & 0 \\ 0 & 5 & -6 & -12 & -4 & 1 & 0 & 0 \\ 0 & 6 & -7 & -17 & -4 & 0 & 0 & 1 \end{array}\right] \begin{matrix} \\ \times(-5) \\ + \\ \\ \end{matrix}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & -2 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -5 & -2 & 0 & 1 & 0 \\ 0 & 0 & -1 & 13 & 6 & 1 & -5 & 0 \\ 0 & 6 & -7 & -17 & -4 & 0 & 0 & 1 \end{array}\right] \begin{matrix} \\ \times(-6) \\ + \\ \\ \end{matrix}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & -2 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -5 & -2 & 0 & 1 & 0 \\ 0 & 0 & -1 & 13 & 6 & 1 & -5 & 0 \\ 0 & 0 & -1 & 13 & 8 & 0 & -6 & 1 \end{array}\right] \begin{matrix} \\ \\ \times(-1) \\ \\ \end{matrix}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & -2 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -5 & -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -13 & -6 & -1 & 5 & 0 \\ 0 & 0 & -1 & 13 & 8 & 0 & -6 & 1 \end{array}\right] \begin{matrix} \\ + \\ \times(-1) \\ \\ \end{matrix}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 11 & 5 & 1 & -4 & 0 \\ 0 & 1 & -1 & -5 & -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -13 & -6 & -1 & 5 & 0 \\ 0 & 0 & -1 & 13 & 8 & 0 & -6 & 1 \end{array}\right] \begin{matrix} \\ + \\ \times(1) \\ \\ \end{matrix}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 11 & 5 & 1 & -4 & 0 \\ 0 & 1 & 0 & -18 & -8 & -1 & 6 & 0 \\ 0 & 0 & 1 & -13 & -6 & -1 & 5 & 0 \\ 0 & 0 & -1 & 13 & 8 & 0 & -6 & 1 \end{array}\right] \begin{matrix} \\ \\ \times(1) \\ + \end{matrix}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 11 & 5 & 1 & -4 & 0 \\ 0 & 1 & 0 & -18 & -8 & -1 & 6 & 0 \\ 0 & 0 & 1 & -13 & -6 & -1 & 5 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 & -1 & 1 \end{array}\right]$$

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10.C

$$[A \,\vdots\, I_3] = \left[\begin{array}{ccc|ccc} 2 & 1 & -2 & 1 & 0 & 0 \\ 3 & 4 & 6 & 0 & 1 & 0 \\ 7 & 6 & 2 & 0 & 0 & 1 \end{array}\right] \times \left(\tfrac{1}{2}\right)$$

$$= \left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & -1 & \frac{1}{2} & 0 & 0 \\ 3 & 4 & 6 & 0 & 1 & 0 \\ 7 & 6 & 2 & 0 & 0 & 1 \end{array}\right] \times(-3) \; +$$

$$= \left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & -1 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{5}{2} & 9 & -\frac{3}{2} & 1 & 0 \\ 7 & 6 & 2 & 0 & 0 & 1 \end{array}\right] \times(-7) \; +$$

$$= \left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & -1 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{5}{2} & 9 & -\frac{3}{2} & 1 & 0 \\ 0 & \frac{5}{2} & 9 & -\frac{7}{2} & 0 & 1 \end{array}\right] \times\left(\tfrac{2}{5}\right)$$

$$= \left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & -1 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{18}{5} & -\frac{3}{5} & \frac{2}{5} & 0 \\ 0 & \frac{5}{2} & 9 & -\frac{7}{2} & 0 & 1 \end{array}\right] \; + \; \times\left(-\tfrac{1}{2}\right)$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{14}{5} & \frac{4}{5} & -\frac{1}{5} & 0 \\ 0 & 1 & \frac{18}{5} & -\frac{3}{5} & \frac{2}{5} & 0 \\ 0 & \frac{5}{2} & 9 & -\frac{7}{2} & 0 & 1 \end{array}\right] \times\left(-\tfrac{5}{2}\right) \; +$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{14}{5} & \frac{4}{5} & -\frac{1}{5} & 0 \\ 0 & 1 & \frac{18}{5} & -\frac{3}{5} & \frac{2}{5} & 0 \\ 0 & 0 & 0 & -2 & -1 & 1 \end{array}\right]$$

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EXERCÍCIO 11 (PÁGINA 54)

11.A

$$\begin{cases} x + 2y + 3z = 0 \\ 2y + 2z = 0 \\ x + 2y + 3z = 0 \end{cases} \sim \begin{cases} 2y + 2z = 0 \\ x + 2y + 3z = 0 \\ x + 2y + 3z = 0 \end{cases}$$

$x + 2y + 3z = 0$
$-2y - 2z = 0$ $\qquad x + z = 0$
$x + 0 + z = 0$

$x + 2y + 3z = 0$
$-2y - 2z = 0$ $\qquad x + z = 0$
$x + 0 + z = 0$

$z = r$
$r \Rightarrow$ QUALQUER NÚMERO REAL

$x + z = 0$ $\qquad -2y - 2z = 0$ $\qquad y = \frac{2r}{-2}$
$x + r = 0$ $\qquad -2y - 2r = 0$
$x = -r$ $\qquad -2y = 2r$ $\qquad = -r$

$x = -r$, $y = -r$ E $z = r$

ESSE SISTEMA LINEAR POSSUI INFINITAS SOLUÇÕES.

11.B $\begin{cases} 2x + y - z = 0 \\ x - 2y - 3z = 0 \\ -3x - y + 2z = 0 \end{cases} \sim \begin{cases} x - 2y - 3z = 0 \\ 2x + y - z = 0 \\ -3x - y + 2z = 0 \end{cases}$

$2x + y - z = 0$ $\qquad x + 3y + 2z = 0$ $\qquad 5y + 5z = 0$
$-x + 2y + 3z = 0$ $\qquad -x + 2y + 3z = 0$ $\qquad 5(y + z) = 0$
$x + 3y + 2z = 0$ $\qquad 0 + 5y + 5z = 0$ $\qquad y + z = \frac{0}{5}$
$= 0$

$-3x - y + 2z = 0$ | $-2x - 3y - z = 0$ | $-x - 5y - 4z = 0$
$x - 2y - 3z = 0$ | $x - 2y - 3z = 0$ | $x - 2y - 3z = 0$
$-2x - 3y - z = 0$ | $-x - 5y - 4z = 0$ | $0 - 7y - 7z = 0$

$-7y - 7z = 0$
$-7(y + z) = 0$
$y + z = \frac{0}{-7}$
$= 0$

$z = r$
$r \Rightarrow$ QUALQUER NÚMERO REAL

$y + z = 0$
$y + r = 0$
$y = -r$

$x - 2y - 3z = 0$
$x - 2(-r) - 3(r) = 0$
$x + 2r - 3r = 0$
$x - r = 0$
$x = r$

$x = r$, $y = -r$ E $z = r$

ESSE SISTEMA LINEAR POSSUI INFINITAS SOLUÇÕES.

EXERCÍCIO 12 (PÁGINA 54)

12.A
$$\begin{cases} x + y + 2z = 0 \\ 2x + y + z = 0 \\ 3x - y + z = 0 \end{cases}$$

$$[A \mid 0] = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 2 & 1 & 1 & 0 \\ 3 & -1 & 1 & 0 \end{array}\right] \begin{array}{l} \times(-2) \\ + \\ \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & -1 & -3 & 0 \\ 3 & -1 & 1 & 0 \end{array}\right] \begin{array}{l} \times(-3) \\ \\ + \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & -1 & -3 & 0 \\ 0 & -4 & -5 & 0 \end{array}\right] \begin{array}{l} \\ \times(-1) \\ \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & -4 & -5 & 0 \end{array}\right] \begin{array}{l} + \\ \times(-1) \\ \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & -4 & -5 & 0 \end{array}\right] \begin{array}{l} \\ \times(4) \\ + \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 7 & 0 \end{array}\right] \begin{array}{l} \\ \\ \times(1/7) \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} + \\ \\ \times(1) \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} \\ + \\ \times(-3) \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{cases} x + 0y + 0z = 0 \\ 0x + y + 0z = 0 \\ 0x + 0y + z = 0 \end{cases} \qquad \begin{array}{l} x = 0 \\ y = 0 \\ z = 0 \end{array}$$

ESSE SISTEMA LINEAR SÓ POSSUI A SOLUÇÃO TRIVIAL.

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12-B
$$\begin{cases} x - y + z = 0 \\ 2x + y = 0 \\ 2x - 2y + 2z = 0 \end{cases}$$

$$[A \mid 0] = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 2 & 1 & 0 & 0 \\ 2 & -2 & 2 & 0 \end{array}\right] \begin{array}{l} \times(-2) \\ + \\ \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 3 & -2 & 0 \\ 2 & -2 & 2 & 0 \end{array}\right] \begin{array}{l} \times(-2) \\ + \\ \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 3 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \times\left(\tfrac{1}{3}\right)$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & -2/3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} + \\ \times(1) \\ \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 1/3 & 0 \\ 0 & 1 & -2/3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} x + 0y + 1/3\, z = 0 \\ 0x + y - 2/3\, z = 0 \\ 0x + 0y + 0z = 0 \end{cases} \qquad \begin{array}{l} x + 1/3\, z = 0 \\ y - 2/3\, z = 0 \end{array}$$

$z = r$

r = QUALQUER NÚMERO REAL

$x + 1/3\, z = 0 \qquad y - 2/3\, z = 0$

$x + 1/3\, r = 0 \qquad y - 2/3\, r = 0$

$x = -\frac{1}{3} r \qquad y = \frac{2}{3} r$

$x = -\frac{1}{3} r$, $y = \frac{2}{3} r$ E $z = r$

ESSE SISTEMA LINEAR POSSUI INFINITAS SOLUÇÕES, OU SEJA, POSSUI SOLUÇÃO NÃO-TRIVIAL.

12.C $\begin{cases} 2x - y + 5z = 0 \\ 3x + 2y - 3z = 0 \\ x - y + 4z = 0 \end{cases} \sim \begin{cases} x - y + 4z = 0 \\ 3x + 2y - 3z = 0 \\ 2x - y + 5z = 0 \end{cases}$

$\sim \begin{cases} x - y + 4z = 0 \\ 2x - y + 5z = 0 \\ 3x + 2y - 3z = 0 \end{cases}$

$$[A \,\vdots\, 0] = \left[\begin{array}{ccc|c} 1 & -1 & 4 & 0 \\ 2 & -1 & 5 & 0 \\ 3 & 2 & -3 & 0 \end{array}\right] \begin{array}{l} \times(-2) \\ \leftarrow + \\ \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 4 & 0 \\ 0 & 1 & -3 & 0 \\ 3 & 2 & -3 & 0 \end{array}\right] \begin{array}{l} \times(-3) \\ + \\ \leftarrow \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 4 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 5 & -15 & 0 \end{array}\right] \begin{array}{l} \leftarrow + \\ \times(1) \\ \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 5 & -15 & 0 \end{array}\right] \begin{array}{l} \\ \times(-5) \\ \leftarrow + \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} x + 0y + z = 0 \\ 0x + y - 3z = 0 \\ 0x + 0y + 0z = 0 \end{cases} \qquad \begin{matrix} x + z = 0 \\ y - 3z = 0 \end{matrix}$$

$z = r$

$r \Rightarrow$ QUALQUER NÚMERO REAL.

$x + z = 0 \qquad y - 3z = 0$

$x + r = 0 \qquad y - 3r = 0$

$x = -r \qquad y = 3r$

$x = -r$, $y = 3r$ E $z = r$

ESSE SISTEMA LINEAR POSSUI INFINITAS SOLUÇÕES, OU SEJA, POSSUI SOLUÇÃO NÃO-TRIVIAL.

EXERCÍCIO 13 (PÁGINA 54)

$$[A^{-1} \mid I_2] = \left[\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 1 & 4 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{cc|cc} 1 & 4 & 0 & 1 \\ 2 & 3 & 1 & 0 \end{array}\right] \begin{matrix} \times(-2) \\ +\end{matrix}$$

$$= \left[\begin{array}{cc|cc} 1 & 4 & 0 & 1 \\ 0 & -5 & 1 & -2 \end{array}\right] \begin{matrix} \\ \times(-\frac{1}{5})\end{matrix}$$

$$= \left[\begin{array}{cc|cc} 1 & 4 & 0 & 1 \\ 0 & 1 & -1/5 & 2/5 \end{array}\right] \begin{matrix} + \\ \times(-4)\end{matrix}$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & 4/5 & -3/5 \\ 0 & 1 & -1/5 & 2/5 \end{array}\right]$$

$$A = \begin{bmatrix} 4/5 & -3/5 \\ -1/5 & 2/5 \end{bmatrix}$$

EXERCÍCIO 12 (PÁGINA 54)

$$[A^{-1} \mid I_2] = \left[\begin{array}{cc|cc} 3 & 4 & 1 & 0 \\ -1 & -1 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{cc|cc} -1 & -1 & 0 & 1 \\ 3 & 4 & 1 & 0 \end{array}\right] \times(-1)$$

$$= \left[\begin{array}{cc|cc} 1 & 1 & 0 & -1 \\ 3 & 4 & 1 & 0 \end{array}\right] \begin{array}{l} \times(-3) \\ \leftarrow + \end{array}$$

$$= \left[\begin{array}{cc|cc} 1 & 1 & 0 & -1 \\ 0 & 1 & 1 & 3 \end{array}\right] \begin{array}{l} \leftarrow + \\ \times(-1) \end{array}$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & -1 & -4 \\ 0 & 1 & 1 & 3 \end{array}\right]$$

$$A = \begin{bmatrix} -1 & -4 \\ 1 & 3 \end{bmatrix}$$

EXERCÍCIO 13 (PÁGINA 54)

$$AA^{-1} = I_n$$

CONTUDO, SE 'A' POSSUI UMA LINHA OU UMA COLUN DE ZEROS IMPLICA QUE SEU PRODUTO POR QUALQUER OUTR MATRIZ TERÁ COMO RESULTADO UMA NOVA MATRIZ COM UMA LINHA OU UMA COLUNA DE ZEROS. LOGO,

'A' É UMA MATRIZ SINGULAR, UMA VEZ QUE:

AB NUNCA RESULTARÁ NUMA MATRIZ I_n

EXERCÍCIO 16 (PÁGINA 54)

$A A^{-1} = I_3$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 2 & a \end{bmatrix} \times \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x_{11}+x_{21} & x_{12}+x_{22} & x_{13}+x_{23} \\ x_{11} & x_{12} & x_{13} \\ x_{11}+2x_{21}+ax_{31} & x_{12}+2x_{22}+ax_{32} & x_{13}+2x_{23}+ax_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$x_{11}+x_{21}=1$

$x_{12}+x_{22}=0$

$x_{13}+x_{23}=0$

$x_{11}=0$

$x_{12}=1$

$x_{13}=0$

$x_{11}+2x_{21}+ax_{31}=0$

$x_{12}+2x_{22}+ax_{32}=0$

$x_{13}+2x_{23}+ax_{33}=1$

$x_{11}+x_{21}=1$	$x_{12}+x_{22}=0$	$x_{13}+x_{23}=0$
$0+x_{21}=1$	$1+x_{22}=0$	$0+x_{23}=0$
$x_{21}=1$	$x_{22}=-1$	$x_{23}=0$

$x_{11}+2x_{21}+ax_{31}=0$	$x_{12}+2x_{22}+ax_{32}=0$
$0+2(1)+ax_{31}=0$	$1+2(-1)+ax_{32}=0$
$0+2+ax_{31}=0$	$1-2+ax_{32}=0$
$ax_{31}=-2$	$ax_{32}=1$

$x_{13} + 2x_{23} + a x_{33} = 1$

$0 + 2(0) + a x_{33} = 1$

$0 + 0 + a x_{33} = 1$

$a x_{33} = 1$

EXERCÍCIO 17 (PÁGINA 54)

17.A $\begin{bmatrix} 2 & 1 & 3 \\ 3 & 2 & -1 \\ 2 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 30 \\ 20 \\ 10 \end{bmatrix}$

$\begin{bmatrix} 2x_1 + x_2 + 3x_3 \\ 3x_1 + 2x_2 - x_3 \\ 2x_1 + x_2 + x_3 \end{bmatrix} = \begin{bmatrix} 30 \\ 20 \\ 10 \end{bmatrix}$

$\begin{cases} 2x_1 + x_2 + 3x_3 = 30 \\ 3x_1 + 2x_2 - x_3 = 20 \\ 2x_1 + x_2 + x_3 = 10 \end{cases}$

$[A \,\vdots\, B] = \left[\begin{array}{ccc|c} 2 & 1 & 3 & 30 \\ 3 & 2 & -1 & 20 \\ 2 & 1 & 1 & 10 \end{array}\right]$

$= \left[\begin{array}{ccc|c} 2 & 1 & 3 & 30 \\ 2 & 1 & 1 & 10 \\ 3 & 2 & -1 & 20 \end{array}\right] \times (1/2)$

$= \left[\begin{array}{ccc|c} 1 & 1/2 & 3/2 & 15 \\ 2 & 1 & 1 & 10 \\ 3 & 2 & -1 & 20 \end{array}\right] \begin{array}{l} \times(-2) \\ \leftarrow + \\ \end{array}$

$= \left[\begin{array}{ccc|c} 1 & 1/2 & 3/2 & 15 \\ 0 & 0 & -2 & -20 \\ 3 & 2 & -1 & 20 \end{array}\right] \begin{array}{l} \times(-3) \\ + \\ \leftarrow \end{array}$

$$[A \mid B] = \left[\begin{array}{ccc|c} 1 & 1/2 & 3/2 & 15 \\ 0 & 0 & -2 & -20 \\ 0 & 1/2 & -11/2 & -23 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 1/2 & 3/2 & 15 \\ 0 & 1/2 & -11/2 & -23 \\ 0 & 0 & -2 & -20 \end{array}\right] \begin{array}{l} \\ \times(2) \\ \\ \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 1/2 & 3/2 & 15 \\ 0 & 1 & -11 & -50 \\ 0 & 0 & -2 & -20 \end{array}\right] \begin{array}{l} + \\ \times(-1/2) \\ \\ \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 7 & 40 \\ 0 & 1 & -11 & -50 \\ 0 & 0 & -2 & -20 \end{array}\right] \begin{array}{l} \\ \\ \times(-1/2) \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 7 & 40 \\ 0 & 1 & -11 & -50 \\ 0 & 0 & 1 & 10 \end{array}\right] \begin{array}{l} + \\ \\ \times(-7) \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & -30 \\ 0 & 1 & -11 & -50 \\ 0 & 0 & 1 & 10 \end{array}\right] \begin{array}{l} \\ + \\ \times(11) \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & -30 \\ 0 & 1 & 0 & 60 \\ 0 & 0 & 1 & 10 \end{array}\right]$$

$$\begin{cases} x_1 + 0x_2 + 0x_3 = -30 \\ 0x_1 + x_2 + 0x_3 = 60 \\ 0x_1 + 0x_2 + x_3 = 10 \end{cases} \qquad \begin{array}{l} x_1 = -30 \\ x_2 = 60 \\ x_3 = 10 \end{array}$$

$$\begin{bmatrix} -30 \\ 60 \\ 10 \end{bmatrix}$$

17.B
$$\begin{bmatrix} 2 & 1 & 3 \\ 3 & 2 & -1 \\ 2 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 8 \\ 14 \end{bmatrix}$$

$$\begin{cases} 2x_1 + x_2 + 3x_3 = 12 \\ 3x_1 + 2x_2 - x_3 = 8 \\ 2x_1 + x_2 + x_3 = 14 \end{cases}$$

$$[A \vdots B] = \left[\begin{array}{ccc|c} 2 & 1 & 3 & 12 \\ 3 & 2 & -1 & 8 \\ 2 & 1 & 1 & 14 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 2 & 1 & 3 & 12 \\ 2 & 1 & 1 & 14 \\ 3 & 2 & -1 & 8 \end{array}\right] \begin{array}{l} \times(1/2) \\ \\ \\ \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 1/2 & 3/2 & 6 \\ 2 & 1 & 1 & 14 \\ 3 & 2 & -1 & 8 \end{array}\right] \begin{array}{l} \times(-2) \\ + \\ \\ \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 1/2 & 3/2 & 6 \\ 0 & 0 & -2 & 2 \\ 3 & 2 & -1 & 8 \end{array}\right] \begin{array}{l} \times(-3) \\ + \\ \\ \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 1/2 & 3/2 & 6 \\ 0 & 0 & -2 & 2 \\ 0 & 1/2 & -11/2 & -10 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 1/2 & 3/2 & 6 \\ 0 & 1/2 & -11/2 & -10 \\ 0 & 0 & -2 & 2 \end{array}\right] \begin{array}{l} \\ \times(2) \\ \\ \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 1/2 & 3/2 & 6 \\ 0 & 1 & -11 & -20 \\ 0 & 0 & -2 & 2 \end{array}\right] \begin{array}{l} + \\ \times(-1/2) \\ \\ \end{array}$$

$$[A \,|\, B] = \left[\begin{array}{ccc|c} 1 & 0 & 7 & 16 \\ 0 & 1 & -11 & -20 \\ 0 & 0 & -2 & 2 \end{array}\right] \times\left(-\tfrac{1}{2}\right)$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 7 & 16 \\ 0 & 1 & -11 & -20 \\ 0 & 0 & 1 & -1 \end{array}\right] \times(-7)\ +$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 23 \\ 0 & 1 & -11 & -20 \\ 0 & 0 & 1 & -1 \end{array}\right] \times(11)\ +$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 23 \\ 0 & 1 & 0 & -31 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

EXERCÍCIO 18 (PÁGINA 54)

18.A $[A \,|\, I_2] = \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{array}\right] \times(-2)\ +$

$$= \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right] \times(-3)\ +$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & 7 & -3 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

18.B $A^T = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$

$$[A^T \,|\, I_2] = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 7 & 0 & 1 \end{array}\right] \times(-3)\ +$$

$$[A^T \vdots I_2] = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & -3 & 1 \end{array}\right] \begin{array}{l} \leftarrow + \\ \times(-2) \end{array}$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & 7 & -2 \\ 0 & 1 & -3 & 1 \end{array}\right]$$

$$(A^T)^{-1} = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

EXERCÍCIO 19 (PÁGINA 54) SIM

SE A É UMA MATRIZ SIMÉTRICA E COMO O PRODUTO AA^{-1} RESULTA EM UMA MATRIZ TAMBÉM SIMÉTRICA (I_n), ENTÃO, ESSE RESULTADO SÓ SERÁ POSSÍVEL SE A A^{-1} FOR UMA MATRIZ TAMBÉM SIMÉTRICA.

EXERCÍCIO 20 (PÁGINA 54)

20.A $(A+B)^{-1} = A^{-1} + B^{-1}$

$A + B = C$ $\qquad$ $A^{-1} + B^{-1} = D$ $\qquad$ NÃO SÃO IGUAIS

$(A+B)^{-1} = C^{-1}$

20.B $(kA)^{-1} = \frac{1}{k} A^{-1}$

NÃO SÃO IGUAIS.

EXERCÍCIO 21 (PÁGINA 55)

$$\left[\begin{array}{cc|c} \lambda-1 & 2 & 0 \\ 2 & \lambda-1 & 0 \end{array}\right]$$

EXERCÍCIO 22 (PÁGINA 55)

22.A) A + B

22.B

22.C

EXERCÍCIO 23 (PÁGINA 55)

$$[D \mid I_3] = \left[\begin{array}{ccc|ccc} 4 & 0 & 0 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 & 0 & 1 \end{array}\right] \times \left(\tfrac{1}{4}\right)$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & -2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 & 0 & 1 \end{array}\right] \times \left(-\tfrac{1}{2}\right)$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 0 \\ 0 & 0 & 3 & 0 & 0 & 1 \end{array}\right] \times \left(\tfrac{1}{3}\right)$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1/3 \end{array}\right]$$

$$D^{-1} = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & -1/2 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$$

EXERCÍCIO 24 (PÁGINA 55)

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$= \begin{bmatrix} 2 & 5 \\ 3 & -2 \end{bmatrix} \times \begin{bmatrix} 3 & 2 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2(3)+5(1) & 2(2)+5(3) \\ 3(3)+(-2)(1) & 3(2)+(-2)(3) \end{bmatrix}$$

$$= \begin{bmatrix} 6+5 & 4+15 \\ 9-2 & 6-6 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 19 \\ 7 & 0 \end{bmatrix}$$

EXERCÍCIO 23 (PÁGINA 55)

$AX = B$

$$[A^{-1} \mid I_2] = \left[\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 4 & 1 & 0 & 1 \end{array}\right] \times \left(\tfrac{1}{2}\right)$$

$$= \left[\begin{array}{cc|cc} 1 & \frac{3}{2} & \frac{1}{2} & 0 \\ 4 & 1 & 0 & 1 \end{array}\right] \begin{array}{l} \times(-4) \\ + \end{array}$$

$$= \left[\begin{array}{cc|cc} 1 & \frac{3}{2} & \frac{1}{2} & 0 \\ 0 & -5 & -2 & 1 \end{array}\right] \times \left(-\tfrac{1}{5}\right)$$

$$= \left[\begin{array}{cc|cc} 1 & \frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 1 & \frac{2}{5} & -\frac{1}{5} \end{array}\right] \begin{array}{l} + \\ \times\left(-\frac{3}{2}\right) \end{array}$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & -\frac{1}{10} & \frac{3}{10} \\ 0 & 1 & \frac{2}{5} & -\frac{1}{5} \end{array}\right]$$

$$AX = B \Rightarrow \begin{bmatrix} -\frac{1}{10} & \frac{3}{10} \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix} \times \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} -\frac{1}{10}x_1 + \frac{3}{5}x_2 \\ \frac{2}{5}x_1 - \frac{1}{5}x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$\begin{cases} -\frac{1}{10}X_1 + \frac{3}{10}X_2 = 5 \\ \frac{2}{5}X_1 - \frac{1}{5}X_2 = 3 \quad \times\left(\frac{3}{2}\right) \end{cases} \sim \begin{cases} -\frac{1}{10}X_1 + \frac{3}{10}X_2 = 5 \\ \frac{6}{10}X_1 - \frac{3}{10}X_2 = \frac{9}{2} \end{cases}$$

$$\frac{1}{2}x_1 = \frac{19}{2}$$

$$\frac{1}{2}X_1 = \frac{19}{2}$$

$$x_1 = \cancel{2}\left(\frac{19}{\cancel{2}_1}\right)$$

$$= 19$$

$$-\frac{1}{10}X_1 + \frac{3}{10}X_2 = 5$$

$$-\frac{1}{10}(19) + \frac{3}{10}X_2 = 5$$

$$-\frac{19}{10} + \frac{3}{10}X_2 = 5$$

$$\frac{3}{10}X_2 = 5 + \frac{19}{10}$$

$$\frac{3}{10}X_2 = \frac{50+19}{10}$$

$$\frac{3}{10}X_2 = \frac{69}{10}$$

$$X_2 = \frac{\cancel{10}^{\,1}}{\cancel{3}_{\,1}}\left(\frac{\cancel{69}^{\,23}}{\cancel{10}_{\,1}}\right)$$

$$= 23$$

$$X = \begin{bmatrix} 19 \\ 23 \end{bmatrix}$$