EXERCÍCIOS 3.5 → ÁLGEBRA LINEAR, BERNARD KOLMAN, EDITORA GUANABARA, 3ª EDIÇÃO

EXERCÍCIO 1 (PÁGINA 121)

SEJAM X, Y E Z VETORES QUAISQUER DE $R^2$; SEJAM c E d

ESCALARES QUAISQUER.

α) SE X E Y SÃO ELEMENTOS QUAISQUER DE $\mathbb{R}^2$, ENTÃO $X \oplus Y$ PERTENCE A $\mathbb{R}^2$ (OU SEJA, $\mathbb{R}^2$ É FECHADO SOB A OPERAÇÃO $\oplus$)

a) $X \oplus Y = Y \oplus X$; $\forall$ X E Y EM $\mathbb{R}^2$

b) $X \oplus (Y \oplus Z) = (X \oplus Y) \oplus Z$; $\forall$ X, Y E Z EM $\mathbb{R}^2$

c) $X \oplus 0 = 0 \oplus X = X$; $\forall$ X EM $\mathbb{R}^2$

d) $X \oplus -X = 0$; $\forall$ X EM $\mathbb{R}^2$

β) SE X É QUALQUER ELEMENTO DE $\mathbb{R}^2$ E $c$ É UM NÚMERO REAL QUALQUER, ENTÃO $c \odot X$ PERTENCE A $\mathbb{R}^2$.

e) $c \odot (X \oplus Y) = c \odot X + c \odot Y$, $\forall\, c \in \mathbb{R}$ E X E Y EM $\mathbb{R}^2$

f) $(c+d) \odot X = c \odot X + d \odot X$, $\forall\, c$ E $d \in \mathbb{R}$ E X EM $\mathbb{R}^2$

g) $c \odot (d \odot X) = (cd) \odot X$, $\forall\, c$ E $d \in \mathbb{R}$ E X EM $\mathbb{R}^2$

h) $1 \odot X = X$, $\forall$ X EM $\mathbb{R}^2$

LOGO, $\mathbb{R}^2$ É UM ESPAÇO VETORIAL.

EXERCÍCIO 7 (PÁGINA 121)

a) $X \oplus Y = (x, y, z) \oplus (x', y', z')$

$= (x', y + y', z')$

$Y \oplus X = (x', y', z') \oplus (x, y, z)$
$= (x, y' + y, z)$

LOGO, $X \oplus Y \neq Y \oplus X$ E a) NÃO SE VERIFICA

c) $X \oplus 0 = (x, y, z) \oplus (0, 0, 0)$
$= (0, y + 0, 0)$
$= (0, y, 0)$

$0 \oplus X = (0, 0, 0) \oplus (x, y, z)$
$= (x, 0 + y, z)$
$= (x, y, z)$

LOGO, $X \oplus 0 \neq 0 \oplus X$ E c) NÃO SE VERIFICA.

d) $X \oplus -X = (x, y, z) \oplus (-x, -y, -z)$
$= (-x, y - y, -z)$
$= (-x, 0, -z)$

LOGO, $X \oplus -X \neq 0$ E d) NÃO SE VERIFICA

f) $(c + d) \odot X = (c + d)(x, y, z)$
$= ((c+d)x, (c+d)y, (c+d)z)$

$c \odot X \oplus d \odot X = c(x, y, z) \oplus d(x, y, z)$
$= (cx, cy, cz) \oplus (dx, dy, dz)$
$= (dx, cy + dy, dz)$
$= (dx, (c+d)y, dz)$

LOGO, $(c+d) \odot X \neq c \odot X + d \odot X$ E f) NÃO SE VERIFICA.

EXERCÍCIO 8 (PÁGINA 121)

e) $c \odot (X \oplus Y) = c \odot [(x, y, z) \oplus (x', y', z')]$
$= c \odot (x + x', y + y', z + z')$
$= (x + x', 1, z + z')$

$c \odot X \oplus c \odot Y = c \odot (x, y, z) \oplus c(x', y', z')$
$= (x, 1, z) \oplus (x', 1, z')$
$= (x + x', 1 + 1, z + z')$
$= (x + x', 2, z + z')$

LOGO, $c \odot (X \oplus Y) \neq c \odot X \oplus c \odot Y$ E e) NÃO SE VERIFICA.

f) $(c + d) \odot X = (c + d) \odot (x, y, z)$
$= (x, 1, z)$

$c \odot X \oplus d \odot X = c \odot (x, y, z) \oplus d \odot (x, y, z)$
$= (x, 1, z) \oplus (x, 1, z)$
$= (x + x, 1 + 1, z + z)$
$= (2x, 2, 2z)$

LOGO, $(c + d) \odot X \neq c \odot X \oplus d \odot X$ E f) NÃO SE VERIFICA.

g) $1 \odot X = 1 \odot (x, y, z)$
$= (x, 1, z)$

LOGO, $1 \odot X \neq X$ E g) NÃO SE VERIFICA.

EXERCÍCIO 9 (PÁGINA 121)

É UM ESPAÇO VETORIAL

EXERCÍCIO 11 (PÁGINA 121)

a) NÃO PODE EXISTIR O SIMÉTRICO ADITIVO OU O NEGATIVO DE X POIS $X \leq 0$. E SE EXISTISSE O SIMÉTRICO ADITIVO X ADMITIRIA VALORES POSITIVOS.

B) SE $c < 0$ ENTÃO $c \odot X$ RESULTARIA EM VALORES POSITIVOS PARA X, MAS POR DEFINIÇÃO $X \leq 0$.

EXERCÍCIO 12 (PÁGINA 121)

a) $1 \odot X = 1 \odot (x, y)$
$= (0, 0)$

LOGO, $1 \odot X \neq X$ E a) NÃO SE VERIFICA.

EXERCÍCIO 13 (PÁGINA 121)

É UM ESPAÇO VETORIAL.

EXERCÍCIO 14 (PÁGINA 121)

EXERCÍCIO 15 (PÁGINA 121)

LETRA "b"

EXERCÍCIO 17 (PÁGINA 121)

LETRAS "b" E "c"

EXERCÍCIO 19 (PÁGINA 121)

TEOREMA 2.7 SEJAM P UM ESPAÇO VETORIAL COM OPERAÇÕES $\oplus$ E $\odot$ E $P_2$ UM SUBCONJUNTO NÃO-VAZIO DE P.

SUPONHA QUE

$p(t) = a_0 t^2 + a_1 t + a_2$ E $q(t) = b_0 t^2 + b_1 t + b_2$, EM QUE $a_0 \neq 0$ E $b_0 \neq 0$

ENTÃO,

$p(t) \oplus q(t) = (a_0 + b_0) t^2 + (a_1 + b_1) t + (a_2 + b_2)$

SENDO ASSIM,

$p(t) \oplus q(t)$ PERTENCE A $P_2$

TAMBÉM,

$c \odot p(t) = (c a_0) t^2 + (c a_1) t + (c a_2)$

ENTÃO,

$c\,0_p(t)$ PERTENCE A $P_2$

LOGO,

$P_2$ É UM SUBESPAÇO DE $P$.

EXERCÍCIO 21 (PÁGINA 121)

$X = a(1,2,-3) + a(-2,3,0)$ E $Y = b(1,2,-3) + b(-2,3,0)$

$X + Y = a(1,2,-3) + a(-2,3,0) + b(1,2,-3) + b(-2,3,0)$
$= (a+b)(1,2,-3) + (a+b)(-2,3,0)$

ENTÃO, $X + Y$ PERTENCE A $W$.

$cX = (ca)(1,2,-3) + (ca)(-2,3,0)$

LOGO, $cX$ PERTENCE A $W$.

SENDO ASSIM,

$W$ É UM SUBESPAÇO DE $R^3$.

EXERCÍCIO 23 (PÁGINA 121)

LETRAS "a" E "c"

266

EXERCÍCIOS 3.5 ⇒ ÁLGEBRA LINEAR, BERNARD KOLMAN, EDITORA GUANABARA, 3ª EDIÇÃO

EXERCÍCIO 1 (PÁGINA 130)

1.A
$X_1 = (4, 2, -3)$
$X_2 = (2, 1, -2)$
$X_3 = (-2, 1, 0)$
$X = (1, 1, 1)$

$c_1 X_1 + c_2 X_2 + c_3 X_3 = X$
$c_1(4,2,-3) + c_2(2,1,-2) + c_3(-2,1,0) = (1,1,1)$

$(4c_1 + 2c_2 - 2c_3;\ 2c_1 + c_2 + c_3;\ -3c_1 - 2c_2 + 0c_3) = (1,1,1)$

$$\begin{cases} 4c_1 + 2c_2 - 2c_3 = 1 \\ 2c_1 + c_2 + c_3 = 1 \\ -3c_1 - 2c_2 = 1 \end{cases}$$

$$\left[\begin{array}{ccc|c} 4 & 2 & -2 & 1 \\ 2 & 1 & 1 & 1 \\ -3 & -2 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ -3 & -2 & 0 & 1 \\ 4 & 2 & -2 & 1 \end{array}\right] \times\left(\tfrac{1}{2}\right) \sim \left[\begin{array}{ccc|c} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -3 & -2 & 0 & 1 \\ 4 & 2 & -2 & 1 \end{array}\right] \times(3) \times(-4)$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{3}{2} & \frac{5}{2} \\ 0 & 0 & -4 & -1 \end{array}\right] \times(-2) \sim \left[\begin{array}{ccc|c} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & -3 & -5 \\ 0 & 0 & -4 & -1 \end{array}\right] \times\left(-\tfrac{1}{2}\right) \sim \left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & -3 & -5 \\ 0 & 0 & -4 & -1 \end{array}\right] \times\left(-\tfrac{1}{4}\right)$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & -3 & -5 \\ 0 & 0 & 1 & \frac{1}{4} \end{array}\right] \times(-2) \times(3) \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{5}{2} \\ 0 & 1 & 0 & -\frac{17}{4} \\ 0 & 0 & 1 & \frac{1}{4} \end{array}\right]$$

/ /

267

COMO, $c_1 = \frac{3}{2}$; $c_2 = -\frac{17}{4}$ E $c_3 = \frac{1}{4}$ X É UMA COMBINAÇÃO LINEAR DE $X_1$, $X_2$ E $X_3$.

1.B
$X_1 = (4, 2, -3)$
$X_2 = (2, 1, -2)$
$X_3 = (-2, 1, 0)$
$X = (4, 2, -6)$

$c_1 X_1 + c_2 X_2 + c_3 X_3 = X$

$c_1(4,2,-3) + c_2(2,1,-2) + c_3(-2,1,0) = (4,2,-6)$

$(4c_1 + 2c_2 - 2c_3 ; 2c_1 + c_2 + c_3 ; -3c_1 - 2c_2 + 0c_3) = (4,2,-6)$

$$\begin{cases} 4c_1 + 2c_2 - 2c_3 = 4 \\ 2c_1 + c_2 + c_3 = 2 \\ -3c_1 - 2c_2 \quad = -6 \end{cases}$$

$$\left[\begin{array}{ccc|c} 4 & 2 & -2 & 4 \\ 2 & 1 & 1 & 2 \\ -3 & -2 & 0 & -6 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & 1 & 1 & 2 \\ -3 & -2 & 0 & -6 \\ 4 & 2 & -2 & 4 \end{array}\right] \times(1/2) \sim \left[\begin{array}{ccc|c} 1 & 1/2 & 1/2 & 1 \\ -3 & -2 & 0 & -6 \\ 4 & 2 & -2 & 4 \end{array}\right] \times(3) \times(-4)$$

$$\left[\begin{array}{ccc|c} 1 & 1/2 & 1/2 & 1 \\ 0 & -1/2 & 3/2 & -3 \\ 0 & 0 & -4 & 0 \end{array}\right] \times(-2) \sim \left[\begin{array}{ccc|c} 1 & 1/2 & 1/2 & 1 \\ 0 & 1 & -3 & 6 \\ 0 & 0 & -4 & 0 \end{array}\right] \times(-1/2) \sim \left[\begin{array}{ccc|c} 1 & 0 & 2 & -2 \\ 0 & 1 & -3 & 6 \\ 0 & 0 & -4 & 0 \end{array}\right] \times(-1/4)$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & -2 \\ 0 & 1 & -3 & 6 \\ 0 & 0 & 1 & 0 \end{array}\right] \times(-2) \times(3) \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

COMO, $c_1=-2$; $c_2=6$ E $c_3=0$ X É UMA COMBINAÇÃO LINEAR DE $X_1$, $X_2$ E $X_3$.

1.C $X_1=(4,2,-3)$
$X_2=(2,1,-2)$
$X_3=(-2,1,0)$
$X=(-2,-1,1)$

$c_1X_1+c_2X_2+c_3X_3=X$

$c_1(4,2,-3)+c_2(2,1,-2)+c_3(-2,1,0)=(-2,-1,1)$

$(4c_1+2c_2-2c_3;\ 2c_1+c_2+c_3;\ -3c_1-2c_2+0c_3)=(-2,-1,1)$

$$\begin{cases}4c_1+2c_2-2c_3=-2\\2c_1+c_2+c_3=-1\\-3c_1-2c_2=1\end{cases}$$

$$\left[\begin{array}{ccc|c}4&2&-2&-2\\2&1&1&-1\\-3&-2&0&1\end{array}\right]\sim\left[\begin{array}{ccc|c}2&1&1&-1\\-3&-2&0&1\\4&2&-2&-2\end{array}\right]x(1/2)\sim\left[\begin{array}{ccc|c}1&1/2&1/2&-1/2\\-3&-2&0&1\\4&2&-2&-2\end{array}\right]x(3)\,x(-4)$$

$$\left[\begin{array}{ccc|c}1&1/2&1/2&-1/2\\0&-1/2&3/2&-1/2\\0&0&-4&0\end{array}\right]x(-2)\sim\left[\begin{array}{ccc|c}1&1/2&1/2&-1/2\\0&1&-3&1\\0&0&-4&0\end{array}\right]x(-1/2)\sim\left[\begin{array}{ccc|c}1&0&2&-1\\0&1&-3&1\\0&0&-4&0\end{array}\right]x(-1/4)$$

$$\left[\begin{array}{ccc|c}1&0&2&-1\\0&1&-3&1\\0&0&1&0\end{array}\right]x(-2)\,x(3)\sim\left[\begin{array}{ccc|c}1&0&0&-1\\0&1&0&1\\0&0&1&0\end{array}\right]$$

COMO, $c_1=-1$; $c_2=1$ E $c_3=0$ X É UMA COMBINAÇÃO LINEAR DE $X_1$, $X_2$ E $X_3$

1-D $X_1 = (4, 2, -3)$

$X_2 = (2, 1, -2)$

$X_3 = (-2, 1, 0)$

$X = (-1, 2, 3)$

$c_1 X_1 + c_2 X_2 + c_3 X_3 = X$

$c_1(4,2,-3) + c_2(2,1,-2) + c_3(-2,1,0) = (-1,2,3)$

$(4c_1 + 2c_2 - 2c_3;\ 2c_1 + c_2 + c_3;\ -3c_1 - 2c_2 + 0c_3) = (-1, 2, 3)$

$$\begin{cases} 4c_1 + 2c_2 - 2c_3 = -1 \\ 2c_1 + c_2 + c_3 = 2 \\ -3c_1 - 2c_2 = 3 \end{cases}$$

$$\left[\begin{array}{ccc|c} 4 & 2 & -2 & -1 \\ 2 & 1 & 1 & 2 \\ -3 & -2 & 0 & 3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & 1 & 1 & 2 \\ -3 & -2 & 0 & 3 \\ 4 & 2 & -2 & -1 \end{array}\right] \times(1/2) \sim \left[\begin{array}{ccc|c} 1 & 1/2 & 1/2 & 1 \\ -3 & -2 & 0 & 3 \\ 4 & 2 & -2 & -1 \end{array}\right] \times(3) \times(-4)\ +\ +$$

$$\left[\begin{array}{ccc|c} 1 & 1/2 & 1/2 & 1 \\ 0 & -1/2 & 3/2 & 6 \\ 0 & 0 & -4 & -5 \end{array}\right] \times(-2) \sim \left[\begin{array}{ccc|c} 1 & 1/2 & 1/2 & 1 \\ 0 & 1 & -3 & -12 \\ 0 & 0 & -4 & -5 \end{array}\right] + \times(-1/2) \sim \left[\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & -3 & -12 \\ 0 & 0 & -4 & -5 \end{array}\right] \times(-1/4)$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & -3 & -12 \\ 0 & 0 & 1 & 5/4 \end{array}\right] + + \times(-2) \times(3) \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 9/2 \\ 0 & 1 & 0 & -35/4 \\ 0 & 0 & 1 & 5/4 \end{array}\right]$$

COMO, $c_1 = \frac{9}{2}$, $c_2 = \frac{-35}{4}$ E $c_3 = \frac{5}{4}$ X É UMA COMBINAÇÃO LINEAR DE $X_1$, $X_2$ E $X_3$.

EXERCÍCIO 3 (PÁGINA 130)

3.A $p_1(t) = t^2 + 2t + 1$

$p_2(t) = t^2 + 3$

$p_3(t) = t - 1$

$p(t) = t^2 + t + 2$

$c_1 p_1(t) + c_2 p_2(t) + c_3 p_3(t) = p(t)$

$c_1(1, 2, 1) + c_2(1, 0, 3) + c_3(0, 1, -1) = (1, 1, 2)$

$(c_1 + c_2 + 0c_3 ; 2c_1 + 0c_2 + c_3 ; c_1 + 3c_2 - c_3) = (1, 1, 2)$

$$\begin{cases} c_1 + c_2 = 1 \\ 2c_1 + c_3 = 1 \\ c_1 + 3c_2 - c_3 = 2 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 2 & 0 & 1 & 1 \\ 1 & 3 & -1 & 2 \end{array}\right] \begin{array}{l} \times(-2)\ \times(-1) \\ + \quad + \\ \end{array} \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & -2 & 1 & -1 \\ 0 & 2 & -1 & 1 \end{array}\right] \times\left(-\tfrac{1}{2}\right) \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 2 & -1 & 1 \end{array}\right] \begin{array}{l} + \\ \times(-1)\times(-2) \\ + \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} c_1 + \cancel{\frac{1}{2}c_3} = \frac{1}{2} \\ c_2 - \cancel{\frac{1}{2}c_3} = \frac{1}{2} \end{cases} \qquad \begin{array}{l} c_1 = \frac{1}{2} \\ c_2 = \frac{1}{2} \end{array}$$

$c_1 + \frac{1}{2}c_3 = \frac{1}{2} \Rightarrow \frac{1}{2} + \frac{1}{2}c_3 = \frac{1}{2} \Rightarrow \frac{1}{2}c_3 = \frac{1}{2} - \frac{1}{2} = \frac{1}{2}c_3 = 0 \Rightarrow c_3 = 0$

/ /

271

COMO, $c_1 = \frac{1}{2}$; $c_2 = \frac{1}{2}$ E $c_3 = 0$ $p(t)$ É UMA COMBINAÇÃO LINEAR DE $p_1(t)$, $p_2(t)$ E $p_3(t)$.

3.B
$p_1(t) = t^2 + 2t + 1$
$p_2(t) = t^2 + 3$
$p_3(t) = t - 1$
$p(t) = 2t^2 + 2t + 3$

$c_1 p_1(t) + c_2 p_2(t) + c_3 p_3(t) = p(t)$
$c_1(1,2,1) + c_2(1,0,3) + c_3(0,1,-1) = (2,2,3)$
$(c_1 + c_2 + 0c_3;\ 2c_1 + 0c_2 + c_3;\ c_1 + 3c_2 - c_3) = (2,2,3)$

$$\begin{cases} c_1 + c_2 = 2 \\ 2c_1 + c_3 = 2 \\ c_1 + 3c_2 - c_3 = 3 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 2 & 0 & 1 & 2 \\ 1 & 3 & -1 & 3 \end{array}\right] x(-2)\,x(-1) \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & -2 & 1 & -2 \\ 0 & 2 & -1 & 1 \end{array}\right] x\left(-\tfrac{1}{2}\right) \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & -\tfrac{1}{2} & 1 \\ 0 & 2 & -1 & 1 \end{array}\right] x(-1)\,x(-2)$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \tfrac{1}{2} & 1 \\ 0 & 1 & -\tfrac{1}{2} & 1 \\ 0 & 0 & 0 & -1 \end{array}\right]$$

ESSE SISTEMA LINEAR NÃO POSSUI SOLUÇÃO. LOGO, $p(t)$ NÃO É UMA COMBINAÇÃO LINEAR DE $p_1(t)$, $p_2(t)$ E $p_3(t)$

3.C
$p_1(t) = t^2 + 2t + 1$
$p_2(t) = t^2 + 3$

$p_3(t) = t - 1$
$p(t) = -t^2 + t - 4$

$c_1 p_1(t) + c_2 p_2(t) + c_3 p_3(t) = p(t)$
$c_1(1,2,1) + c_2(1,0,3) + c_3(0,1,-1) = (-1,1,-4)$
$(c_1 + c_2 + 0c_3 ; 2c_1 + 0c_2 + c_3 ; c_1 + 3c_2 - c_3) = (-1,1,-4)$

$$\begin{cases} c_1 + c_2 = -1 \\ 2c_1 + c_3 = 1 \\ c_1 + 3c_2 - c_3 = -4 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & -1 \\ 2 & 0 & 1 & 1 \\ 1 & 3 & -1 & -4 \end{array}\right] \times(-2)\times(-1) \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & -1 \\ 0 & -2 & 1 & 3 \\ 0 & 2 & -1 & -3 \end{array}\right] \times\left(-\tfrac{1}{2}\right) \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & -1 \\ 0 & 1 & -\frac{1}{2} & -\frac{3}{2} \\ 0 & 2 & -1 & -3 \end{array}\right] \times(-1)\times(-2)$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & -\frac{3}{2} \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} c_1 + \frac{1}{2}c_3 = \frac{1}{2} \\ c_2 - \frac{1}{3}c_3 = -\frac{3}{2} \end{cases} \qquad c_1 = \frac{1}{2} \qquad c_2 = -\frac{3}{2}$$

$c_1 + \frac{1}{2}c_3 = \frac{1}{2} \Rightarrow \frac{1}{2} + \frac{1}{2}c_3 = \frac{1}{2} \Rightarrow \frac{1}{2}c_3 = \frac{1}{2} - \frac{1}{2} \Rightarrow \frac{1}{2}c_3 = 0 \Rightarrow c_3 = 0$

COMO, $c_1 = \frac{1}{2}$ ; $c_2 = -\frac{3}{2}$ E $c_3 = 0$ $p(t)$ É UMA COMBINAÇÃO LINEAR DE $p_1(t)$, $p_2(t)$ E $p_3(t)$.

3.D $p_1(t) = t^2 + 2t + 1$
$p_2(t) = t^2 + 3$
$p_3(t) = t - 1$
$p(t) = -2t^2 + 3t + 1$

$c_1 p_1(t) + c_2 p_2(t) + c_3 p_3(t) = p(t)$
$c_1(1,2,1) + c_2(1,0,3) + c_3(0,1,-1) = (-2,3,1)$

$(c_1 + c_2 + 0c_3 ; 2c_1 + 0c_2 + c_3 ; c_1 + 3c_2 - c_3) = (-2,3,1)$

$$\begin{cases} c_1 + c_2 = -2 \\ 2c_1 + c_3 = 3 \\ c_1 + 3c_2 - c_3 = 1 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & -2 \\ 2 & 0 & 1 & 3 \\ 1 & 3 & -1 & 1 \end{array}\right] \begin{array}{l} \times(-2)\times(-1) \\ + \\ + \end{array} \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & -2 \\ 0 & -2 & 1 & 7 \\ 0 & 2 & -1 & 3 \end{array}\right] \times\left(-\tfrac{1}{2}\right) \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & -2 \\ 0 & 1 & -\frac{1}{2} & -\frac{7}{2} \\ 0 & 2 & -1 & 3 \end{array}\right] \begin{array}{l} + \\ \times(-1)\times(-2) \\ + \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & \frac{3}{2} \\ 0 & 1 & -\frac{1}{2} & -\frac{7}{2} \\ 0 & 0 & 0 & 10 \end{array}\right]$$

ESSE SISTEMA LINEAR NÃO POSSUI SOLUÇÃO. LOGO, $p(t)$ NÃO É UMA COMBINAÇÃO LINEAR DE $p_1(t)$, $p_2(t)$ E $p_3(t)$.

EXERCÍCIO 4 (PÁGINA 130)

4.A $A_1 = \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}$ ; $A_2 = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$ ; $A_3 = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$ E $A = \begin{bmatrix} 5 & 1 \\ -1 & 9 \end{bmatrix}$

$c_1 A_1 + c_2 A_2 + c_3 A_3 = A$

$$c_1\begin{bmatrix}1 & -1\\0 & 3\end{bmatrix} + c_2\begin{bmatrix}1 & 1\\0 & 2\end{bmatrix} + c_3\begin{bmatrix}2 & 2\\-1 & 1\end{bmatrix} = \begin{bmatrix}5 & 1\\-1 & 9\end{bmatrix}$$

$$\begin{bmatrix}c_1 & -c_1\\0 & 3c_1\end{bmatrix} + \begin{bmatrix}c_2 & c_2\\0 & 2c_2\end{bmatrix} + \begin{bmatrix}2c_3 & 2c_3\\-c_3 & c_3\end{bmatrix} = \begin{bmatrix}5 & 1\\-1 & 9\end{bmatrix}$$

$$\begin{cases} c_1 + c_2 + 2c_3 = 5\\ -c_1 + c_2 + 2c_3 = 1\\ 0 + 0 - c_3 = -1\\ 3c_1 + 2c_2 + c_3 = 9\end{cases}$$

$$\left[\begin{array}{ccc|c}1 & 1 & 2 & 5\\-1 & 1 & 2 & 1\\0 & 0 & -1 & -1\\3 & 2 & 1 & 9\end{array}\right] x(1)\ x(-3) \sim \left[\begin{array}{ccc|c}1 & 1 & 2 & 5\\0 & 2 & 4 & 6\\0 & 0 & -1 & -1\\0 & -1 & -5 & -6\end{array}\right] x(1/2) \sim$$

$$\left[\begin{array}{ccc|c}1 & 1 & 2 & 5\\0 & 1 & 2 & 3\\0 & 0 & -1 & -1\\0 & -1 & -5 & -6\end{array}\right] x(-1)\ x(1) \sim \left[\begin{array}{ccc|c}1 & 0 & 0 & 2\\0 & 1 & 2 & 3\\0 & 0 & -1 & -1\\0 & 0 & -3 & -3\end{array}\right] x(-1) \sim \left[\begin{array}{ccc|c}1 & 0 & 0 & 2\\0 & 1 & 2 & 3\\0 & 0 & 1 & 1\\0 & 0 & -3 & -3\end{array}\right] x(-2)\ x(3)$$

$$\left[\begin{array}{ccc|c}1 & 0 & 0 & 2\\0 & 1 & 0 & 1\\0 & 0 & 1 & 1\\0 & 0 & 0 & 0\end{array}\right]$$

COMO $c_1 = 2$; $c_2 = 1$ E $c_3 = 1$ A É UMA COMBINAÇÃO LINEAR DE $A_1$, $A_2$ E $A_3$.

/ /

275

4.B $A_1=\begin{bmatrix}1&-1\\0&3\end{bmatrix}$; $A_2=\begin{bmatrix}1&1\\0&2\end{bmatrix}$; $A_3=\begin{bmatrix}2&2\\-1&1\end{bmatrix}$ e $A=\begin{bmatrix}-3&-1\\3&2\end{bmatrix}$

$c_1A_1+c_2A_2+c_3A_3=A$

$$c_1\begin{bmatrix}1&-1\\0&3\end{bmatrix}+c_2\begin{bmatrix}1&1\\0&2\end{bmatrix}+c_3\begin{bmatrix}2&2\\-1&1\end{bmatrix}=\begin{bmatrix}-3&-1\\3&2\end{bmatrix}$$

$$\begin{bmatrix}c_1&-c_1\\0&3c_1\end{bmatrix}+\begin{bmatrix}c_2&c_2\\0&2c_2\end{bmatrix}+\begin{bmatrix}2c_3&2c_3\\-c_3&c_3\end{bmatrix}=\begin{bmatrix}-3&-1\\3&2\end{bmatrix}$$

$$\begin{cases}c_1+c_2+2c_3=-3\\-c_1+c_2+2c_3=-1\\0+0-c_3=3\\3c_1+2c_2+c_3=2\end{cases}$$

$$\left[\begin{array}{ccc|c}1&1&2&-3\\-1&1&2&-1\\0&0&-1&3\\3&2&1&2\end{array}\right]\times(1)\times(-3)\sim\left[\begin{array}{ccc|c}1&1&2&-3\\0&2&4&-8\\0&0&-1&3\\0&-1&-5&11\end{array}\right]\times(1/2)\sim$$

$$\left[\begin{array}{ccc|c}1&1&2&-3\\0&1&2&-2\\0&0&-1&3\\0&-1&-5&11\end{array}\right]\times(-1)\times(1)\sim\left[\begin{array}{ccc|c}1&0&0&-1\\0&1&2&-2\\0&0&-1&3\\0&0&-3&9\end{array}\right]\times(-1)\sim$$

$$\left[\begin{array}{ccc|c}1&0&0&-1\\0&1&2&-2\\0&0&1&-3\\0&0&-3&9\end{array}\right]\times(-2)\times(3)\sim\left[\begin{array}{ccc|c}1&0&0&-1\\0&1&0&4\\0&0&1&-3\\0&0&0&0\end{array}\right]$$

COMO, $c_1 = -1$; $c_2 = 4$ E $c_3 = -3$ A É UMA COMBINAÇÃO LINEAR DE $A_1$, $A_2$ E $A_3$.

4.C $A_1 = \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}$; $A_2 = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$; $A_3 = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$ E $A = \begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix}$

$$c_1 A_1 + c_2 A_2 + c_3 A_3 = A$$

$$c_1 \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix} + c_2 \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} + c_3 \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} c_1 & -c_1 \\ 0 & 3c_1 \end{bmatrix} + \begin{bmatrix} c_2 & c_2 \\ 0 & 2c_2 \end{bmatrix} + \begin{bmatrix} 2c_3 & 2c_3 \\ -c_3 & c_3 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix}$$

$$\begin{cases} c_1 + c_2 + 2c_3 = 3 \\ -c_1 + c_2 + 2c_3 = -2 \\ 0 + 0 - c_3 = 3 \\ 3c_1 + 2c_2 + c_3 = 2 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ -1 & 1 & 2 & -2 \\ 0 & 0 & -1 & 3 \\ 3 & 2 & 1 & 2 \end{array}\right] \begin{array}{l} \times(1)\ \times(-3) \\ + \\ + \\ \end{array} \sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 2 & 4 & 1 \\ 0 & 0 & -1 & 3 \\ 0 & -1 & -3 & -7 \end{array}\right] \times(1/2) \sim$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 1 & 2 & 1/2 \\ 0 & 0 & -1 & 3 \\ 0 & -1 & -3 & -7 \end{array}\right] \begin{array}{l} + \\ \times(-1)\times(1) \\ + \\ \end{array} \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5/2 \\ 0 & 1 & 2 & 1/2 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & -3 & -13/2 \end{array}\right] \times(-1) \sim$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5/2 \\ 0 & 1 & 2 & 1/2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & -3 & -13/2 \end{array}\right] \begin{array}{l} \\ + \\ \times(-2)\times(3) \\ + \end{array} \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5/2 \\ 0 & 1 & 0 & 13/2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & -31/2 \end{array}\right]$$

ESSE SISTEMA LINEAR NÃO POSSUI SOLUÇÃO, LOGO, A NÃO É UMA COMBINAÇÃO LINEAR DE $A_1$, $A_2$ E $A_3$

4.1 $A_1 = \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$, $A_3 = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$ E $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$$c_1 A_1 + c_2 A_2 + c_3 A_3 = A$$

$$c_1 \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix} + c_2 \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} + c_3 \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} c_1 & -c_1 \\ 0 & 3c_1 \end{bmatrix} + \begin{bmatrix} c_2 & c_2 \\ 0 & 2c_2 \end{bmatrix} + \begin{bmatrix} 2c_3 & 2c_3 \\ -c_3 & c_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$\begin{cases} c_1 + c_2 + 2c_3 = 1 \\ -c_1 + c_2 + 2c_3 = 0 \\ 0 + 0 - c_3 = 2 \\ 3c_1 + 2c_2 + c_3 = 1 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ -1 & 1 & 2 & 0 \\ 0 & 0 & -1 & 2 \\ 3 & 2 & 1 & 1 \end{array}\right] \times(1) \times(-3) \sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 2 & 4 & 1 \\ 0 & 0 & -1 & 2 \\ 0 & -1 & -5 & -2 \end{array}\right] \times(1/2) \sim$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 2 & 1/2 \\ 0 & 0 & -1 & 2 \\ 0 & -1 & -5 & -2 \end{array}\right] \times(-1) \times(1) \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1/2 \\ 0 & 1 & 2 & 1/2 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & -3 & -3/2 \end{array}\right] \times(-1) \sim$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1/2 \\ 0 & 1 & 2 & 1/2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & -3 & -3/2 \end{array}\right] \times(-2) \times(3) \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 9/2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & -15/2 \end{array}\right]$$

ESSE SISTEMA LINEAR NÃO POSSUI SOLUÇÃO. LOGO, A NÃO É UMA COMBINAÇÃO LINEAR DE $A_1$, $A_2$ E $A_3$.

EXERCÍCIO 6 (PÁGINA 130)

6.A
$p_1(t) = t^2 - t$
$p_2(t) = t^2 - 2t + 1$
$p_3(t) = -t^2 + 1$
$p(t) = 3t^2 - 3t + 1$

$c_1 p_1(t) + c_2 p_2(t) + c_3 p_3(t) = p(t)$
$c_1(1,-1,0) + c_2(1,-2,1) + c_3(-1,0,1) = (3,-3,1)$

$(c_1 + c_2 - c_3; -c_1 - 2c_2 + 0c_3; 0c_1 + c_2 + c_3) = (3,-3,1)$

$$\begin{cases} c_1 + c_2 - c_3 = 3 \\ -c_1 - 2c_2 = -3 \\ c_2 + c_3 = 1 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ -1 & -2 & 0 & -3 \\ 0 & 1 & 1 & 1 \end{array}\right] \begin{array}{l} \times(1) \\ \hookleftarrow + \\ \phantom{0} \end{array} \sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & -1 & -1 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right] \times(-1) \sim$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right] \begin{array}{l} \hookleftarrow + \\ \times(-1) \\ \hookleftarrow + \end{array} \sim \left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

ESSE SISTEMA LINEAR NÃO POSSUI SOLUÇÃO. LOGO, $p(t)$ NÃO É UMA COMBINAÇÃO LINEAR DE $p_1(t)$, $p_2(t)$ E $p_3(t)$, OU SEJA, $p(t)$ NÃO PERTENCE A $\{p_1(t), p_2(t), p_3(t)\}$

/ /

* $p(t)$ NÃO PERTENCE A $\{p_1(t), p_2(t), p_3(t)\}$

279

6.B $p_1(t) = t^2 - t$
$p_2(t) = t^2 - 2t + 1$
$p_3(t) = -t^2 + 1$
$p(t) = t^2 - t + 1$

$c_1 p_1(t) + c_2 p_2(t) + c_3 p_3(t) = p(t)$
$c_1(1, -1, 0) + c_2(1, -2, 1) + c_3(-1, 0, 1) = (1, -1, 1)$

$(c_1 + c_2 - c_3 ; -c_1 - 2c_2 + 0c_3 ; 0c_1 + c_2 + c_3) = (1, -1, 1)$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ -1 & -2 & 0 & -1 \\ 0 & 1 & 1 & 1 \end{array}\right] \times(1) \sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right] \times(-1) \sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right] \times(-1) \sim$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

ESSE SISTEMA LINEAR NÃO POSSUI SOLUÇÃO, LOGO, $p(t)$ NÃO É UMA COMBINAÇÃO LINEAR DE $p_1(t)$, $p_2(t)$ E $p_3(t)$, OU SEJA, *

6.C $p_1(t) = t^2 - t$
$p_2(t) = t^2 - 2t + 1$
$p_3(t) = -t^2 + 1$
$p(t) = t + 1$

$c_1 p_1(t) + c_2 p_2(t) + c_3 p_3(t) = p(t)$
$c_1(1, -1, 0) + c_2(1, -2, 1) + c_3(-1, 0, 1) = (0, 1, 1)$

$(c_1 + c_2 - c_3 ; -c_1 - 2c_2 + 0c_3 ; 0c_1 + c_2 + c_3) = (0, 1, 1)$

$$\begin{cases} c_1 + c_2 - c_3 = 0 \\ -c_1 - 2c_2 = 1 \\ c_2 + c_3 = 1 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ -1 & -2 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{array}\right] \begin{array}{l} \times(1) \\ \leftarrow + \\ \phantom{0} \end{array} \sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -1 & -1 & 1 \\ 0 & 1 & 1 & 1 \end{array}\right] \begin{array}{l} \phantom{0} \\ \times(-1) \\ \phantom{0} \end{array} \sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & 1 & 1 \end{array}\right] \begin{array}{l} \leftarrow + \\ \times(-1) \\ \leftarrow + \end{array} \sim$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 2 \end{array}\right]$$

ESSE SISTEMA LINEAR NÃO POSSUI SOLUÇÃO. LOGO, $p(t)$ NÃO É UMA COMBINAÇÃO LINEAR DE $p_1(t)$, $p_2(t)$ E $p_3(t)$, OU SEJA, $p(t)$ NÃO PERTENCE A $\{p_1(t), p_2(t), p_3(t)\}$

6.7

$p_1(t) = t^2 - t$

$p_2(t) = t^2 - 2t + 1$

$p_3(t) = -t^2 + 1$

$p(t) = 2t^2 - t - 1$

$c_1 p_1(t) + c_2 p_2(t) + c_3 p_3(t) = p(t)$

$c_1(1, -1, 0) + c_2(1, -2, 1) + c_3(-1, 0, 1) = (2, -1, -1)$

$(c_1 + c_2 - c_3 ; -c_1 - 2c_2 + 0c_3 ; 0c_1 + c_2 + c_3) = (2, -1, -1)$

$$\begin{cases} c_1 + c_2 - c_3 = 2 \\ -c_1 - 2c_2 = -1 \\ c_2 + c_3 = -1 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ -1 & -2 & 0 & -1 \\ 0 & 1 & 1 & -1 \end{array}\right] \begin{array}{l} \times(1) \\ \hookleftarrow + \\ \phantom{x} \end{array} \sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & -1 & -1 & 1 \\ 0 & 1 & 1 & -1 \end{array}\right] \times(-1) \sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & 1 & -1 \end{array}\right] \begin{array}{l} \hookleftarrow + \\ \times(-1) \\ \hookleftarrow + \end{array} \sim$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

ESSE SISTEMA LINEAR POSSUI INFINITAS SOLUÇÕES. LOGO, $p(t)$ É UMA COMBINAÇÃO LINEAR DE $p_1(t)$, $p_2(t)$ E $p_3(t)$, OU SEJA, $p(t)$ PERTENCE A $\{p_1(t), p_2(t), p_3(t)\}$.

EXERCÍCIO 7 (PÁGINA 130)

7.A $X_1 = (1,2)$

$X_2 = (-1,1)$

$X = (a,b)$

$c_1X_1 + c_2X_2 = X$

$c_1(1,2) + c_2(-1,1) = (a,b)$

$(c_1 - c_2; 2c_1 + c_2) = (a,b)$

$$\begin{cases} c_1 - \cancel{c_2} = a \\ 2c_1 + \cancel{c_2} = b \end{cases} \qquad \begin{array}{l} c_1 = a \\ 2c_1 = b \Rightarrow c_1 = \dfrac{b}{2} \end{array}$$

ESSE SISTEMA LINEAR NÃO POSSUI SOLUÇÃO. LOGO, S NÃO GERA V OU V NÃO É GERADO POR S.

7.B $X_1=(0,0)$
$X_2=(1,1)$
$X_3=(-2,-2)$
$X=(a,b)$

$c_1X_1+c_2X_2+c_3X_3=X$
$c_1(0,0)+c_2(1,1)+c_3(-2,-2)=(a,b)$

$(0c_1+c_2-2c_3;\ 0c_1+c_2-2c_3)=(a,b)$

$$\begin{cases} c_2-2c_3=a \\ c_2-2c_3=b \end{cases}$$

ESSE SISTEMA LINEAR NÃO POSSUI SOLUÇÃO. LOGO, S NÃO GERA V OU V NÃO É GERADO POR S.

7.C $X_1=(1,3)$
$X_2=(2,-3)$
$X_3=(0,2)$
$X=(a,b)$

$c_1X_1+c_2X_2+c_3X_3=X$
$c_1(1,3)+c_2(2,-3)+c_3(0,2)=(a,b)$

$(c_1+2c_2+0c_3;\ 3c_1-3c_2+2c_3)=(a,b)$

$$\begin{cases} c_1+2c_2=a \\ 3c_1-3c_2+2c_3=b \end{cases}$$

ESSE SISTEMA LINEAR TERÁ INFINITAS SOLUÇÕES, LOGO, S GERA V OU V É GERADO POR S. POIS m (NÚMERO DE EQUAÇÕES É MENOR QUE n (NÚMERO DE INCÓGNITAS).

7.1 $X_1 = (2,4)$
$X_2 = (-1,2)$
$X = (a,b)$

$c_1 X_1 + c_2 X_2 = X$
$c_1(2,4) + c_2(-1,2) = (a,b)$

$(2c_1 - c_2 ; 4c_1 + 2c_2) = (a,b)$

$$\begin{cases} 2c_1 - c_2 = a \\ 4c_1 + 2c_2 = b \end{cases}$$

$$\left[\begin{array}{cc|c} 2 & -1 & a \\ 4 & 2 & b \end{array}\right] \times(1/2) \sim \left[\begin{array}{cc|c} 1 & -\frac{1}{2} & \frac{a}{2} \\ 4 & 2 & b \end{array}\right] \times(-4) \hookleftarrow + \sim \left[\begin{array}{cc|c} 1 & -\frac{1}{2} & \frac{a}{2} \\ 0 & 4 & -2a+b \end{array}\right] \times(1/4) \sim$$

$$\left[\begin{array}{cc|c} 1 & -\frac{1}{2} & \frac{a}{2} \\ 0 & 1 & \frac{-2a+b}{4} \end{array}\right] \overset{\hookleftarrow +}{\times(1/2)} \sim \left[\begin{array}{cc|c} 1 & 0 & \frac{2a+b}{8} \\ 0 & 1 & \frac{-2a+b}{4} \end{array}\right]$$

ESSE SISTEMA LINEAR POSSUI INFINITAS SOLUÇÕES. LOGO, S GERA V OU V É GERADO POR S.

EXERCÍCIO 11 (PÁGINA 130)

$p_1(t) = t^3 + 2t + 1$
$p_2(t) = t^2 - t + 2$
$p_3(t) = -t^3 + t^2 - 5t + 2$
$p(t) = (a,b,c,d)$

284

$c_1p_1(t) + c_2p_2(t) + c_3p_3(t) = p(t)$

$c_1(1,0,2,1) + c_2(0,1,-1,2) + c_3(-1,1,-5,2) = (a,b,c,d)$

$(c_1 + 0c_2 - c_3;\ 0c_1 + c_2 + c_3;\ 2c_1 - c_2 - 5c_3;\ c_1 + 2c_2 + 2c_3) = (a,b,c,d)$

$$\begin{cases} c_1 - c_3 = a \\ c_2 + c_3 = b \\ 2c_1 - c_2 - 5c_3 = c \\ c_1 + 2c_2 + 2c_3 = d \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & a \\ 0 & 1 & 1 & b \\ 2 & -1 & -5 & c \\ 1 & 2 & 2 & d \end{array}\right] \begin{array}{l} \times(-2)\ \times(-1) \\ + \\ + \\ \end{array} \sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & a \\ 0 & 1 & 1 & b \\ 0 & -1 & -3 & -2a+c \\ 0 & 2 & 3 & -a+d \end{array}\right] \begin{array}{l} \times(-2) \\ + \\ + \end{array} \sim$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & a \\ 0 & 1 & 1 & b \\ 0 & 0 & -2 & -2a+b+c \\ 0 & 0 & 1 & -a-2b+d \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & a \\ 0 & 1 & 1 & b \\ 0 & 0 & 1 & -a-2b+d \\ 0 & 0 & -2 & -2a+b+c \end{array}\right] \begin{array}{l} + \quad + \\ \times(-1)\ \times(2) \\ + \end{array} \sim$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2b+d \\ 0 & 1 & 0 & a+3b-d \\ 0 & 0 & 1 & -a-2b+d \\ 0 & 0 & 0 & -4a-3b+c+2d \end{array}\right]$$

SE $-4a-3b+c+2d \neq 0$ O SISTEMA LINEAR NÃO TERÁ SOLUÇÃO PORTANTO $S = \{p_1(t), p_2(t), p_3(t)\}$ NÃO GERA $P_3$ OU $P_3$ NÃO É GERADO POR $S = \{p_1(t), p_2(t), p_3(t)\}$

EXERCÍCIO 12 (PÁGINA 130)

12.A $X_1 = (4, 2, 1)$
$X_2 = (2, 6, -5)$
$X_3 = (1, -2, 3)$

$$\begin{vmatrix} 4 & 2 & 1 \\ 2 & 6 & -2 \\ 1 & -5 & 3 \end{vmatrix} = -\begin{vmatrix} 1 & -5 & 3 \\ 2 & 6 & -2 \\ 4 & 2 & 1 \end{vmatrix} \begin{matrix} \times(-2) & \times(-4) \\ + & \\ & + \end{matrix} = -\begin{vmatrix} 1 & -5 & 3 \\ 0 & 16 & -8 \\ 0 & 22 & -11 \end{vmatrix} \times\left(\tfrac{1}{16}\right) =$$

$$= -16\begin{vmatrix} 1 & -5 & 3 \\ 0 & 1 & -\frac{1}{2} \\ 0 & 22 & -11 \end{vmatrix} \begin{matrix} \times(-22) \\ + \end{matrix} = -16\begin{vmatrix} 1 & -5 & 3 \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 \end{vmatrix} = -16\left[1(1)(0)\right] = 0$$

LOGO, $X_1$; $X_2$ E $X_3$ SÃO LINEARMENTE DEPENDENTES.

$c_1 X_1 + c_2 X_2 = X_3$

$c_1(4, 2, 1) + c_2(2, 6, -5) = (1, -2, 3)$

$(4c_1 + 2c_2;\ 2c_1 + 6c_2;\ c_1 - 5c_2) = (1, -2, 3)$

$$\begin{cases} 4c_1 + 2c_2 = 1 \\ 2c_1 + 6c_2 = -2 \\ c_1 - 5c_2 = 3 \end{cases}$$

$$\left[\begin{array}{cc|c} 4 & 2 & 1 \\ 2 & 6 & -2 \\ 1 & -5 & 3 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -5 & 3 \\ 2 & 6 & -2 \\ 4 & 2 & 1 \end{array}\right] \begin{matrix} \times(-2) & \times(-4) \\ + & \\ & + \end{matrix} \sim \left[\begin{array}{cc|c} 1 & -5 & 3 \\ 0 & 16 & -8 \\ 0 & 22 & -11 \end{array}\right]$$

$$\begin{cases} c_1 - 5c_2 = 3 \\ 16c_2 = -8 \\ 22c_2 = -11 \end{cases}$$

$16c_2 = -8$
$c_2 = -\frac{8}{16}$
$= -\frac{1}{2}$

$22c_2 = -11$
$c_2 = -\frac{11}{22}$
$= -\frac{1}{2}$

$c_1 - 5c_2 = 3$
$c_1 - 5\left(-\frac{1}{2}\right) = 3$
$c_1 + \frac{5}{2} = 3$

$c_1 = 3 - \frac{5}{2}$
$= \frac{6-5}{2}$
$= \frac{1}{2}$

286

12.B $x_1 = (1, 2, -1)$
$x_2 = (3, 2, 5)$

12.C

(12.1) $X_1 = (1, 2, 3)$

$X_2 = (1, 1, 1)$

$X_3 = (1, 0, 1)$

$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} \begin{array}{l} \times(-1) \\ + \\ + \end{array} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & -2 & -2 \end{vmatrix} \times(-1) = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -2 & -2 \end{vmatrix} \begin{array}{l} \\ \times(2) \\ + \end{array} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{vmatrix} =$$

$= 1(1)(2) = 2$

$\det \neq 0$

LOGO, $X_1$, $X_2$ E $X_3$ SÃO LINEARMENTE INDEPENDENTES