

EXERCÍCIOS 3.8

2, ÁLGEBRA LINEAR, BENJAMIN HOULMAN,
EDITORA GUANABARA, 3ª EDIÇÃO

EXERCÍCIO 1 (PÁGINA 157)

$$\begin{aligned} 1.A \quad (1, -1, 2) \cdot (0, 2, -1) &= [1(0) + (-1)(2) + 2(-1)] \\ &= (0 - 2 - 2) \\ &= -4 \end{aligned}$$

$$\begin{aligned} (0, 2, -1) \cdot (-1, 1, 1) &= [0(-1) + 2(1) + (-1)(1)] \\ &= (0 + 2 - 1) \\ &= 1 \end{aligned}$$

Logo,

O CONJUNTO DE VETORES NÃO É ORTOGONAL.

$$\begin{aligned} 1.B \quad (1, 2, -1, 1) \cdot (0, -1, -2, 0) &= [1(0) + 2(-1) + (-1)(-2) + 1(0)] \\ &= (0 - 2 + 2 + 0) \\ &= 0 \end{aligned}$$

$$\begin{aligned} (0, -1, -2, 0) \cdot (1, 0, 0, -1) &= [0(1) + (-1)(0) + (-2)(0) + 0(-1)] \\ &= (0 + 0 + 0 + 0) \\ &= 0 \end{aligned}$$

Logo,

O CONJUNTO DE VETORES É ORTOGONAL.

$$\begin{aligned} 1.C \quad (0, 1, 0, -1) \cdot (1, 0, 1, 1) &= [0(1) + 1(0) + 0(1) + (-1)(1)] \\ &= (0 + 0 + 0 - 1) \\ &= -1 \end{aligned}$$

$$\begin{aligned}
 (1, 0, 1, 1) \cdot (-1, 1, -1, 2) &= [1(-1) + 0(1) + 1(-1) + 1(2)] \\
 &= (-1 + 0 - 1 + 2) \\
 &= (-2 + 2) \\
 &= 0
 \end{aligned}$$

Logo,

O CONJUNTO DE VETORES NÃO É ORTOGONAL.

EXERCÍCIO 2 (PÁGINA 157)

$$\begin{aligned}
 2.A \left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3} \right) \cdot \left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3} \right) &= \left[\frac{1}{3} \left(\frac{2}{3} \right) + \frac{2}{3} \left(\frac{1}{3} \right) + \frac{2}{3} \left(-\frac{2}{3} \right) \right] \\
 &= \left(\frac{2}{9} + \frac{2}{9} - \frac{4}{9} \right) \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3} \right) \cdot \left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3} \right) &= \left[\frac{2}{3} \left(\frac{2}{3} \right) + \frac{1}{3} \left(-\frac{2}{3} \right) + \left(-\frac{2}{3} \right) \left(\frac{1}{3} \right) \right] \\
 &= \left(\frac{4}{9} - \frac{2}{9} - \frac{2}{9} \right) \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \|X_1\| &= \sqrt{X_1^2 + X_2^2 + X_3^2} \\
 &= \sqrt{\left(\frac{1}{3} \right)^2 + \left(\frac{2}{3} \right)^2 + \left(\frac{2}{3} \right)^2} \\
 &= \sqrt{\frac{1}{9} + \frac{4}{9} + \frac{4}{9}} \\
 &= \sqrt{\frac{9}{9}}
 \end{aligned}$$

$\|X_1\| = \sqrt{1}$
 $= 1$

$$\|X_2\| = \sqrt{x_1^2 + x_2^2 + x_3^2}$$

$$= \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(-\frac{2}{3}\right)^2}$$

$$= \sqrt{\frac{4}{9} + \frac{1}{9} + \frac{4}{9}}$$

$$= \sqrt{\frac{9}{9}}$$

$$= \sqrt{1}$$

$$= 1$$

$$\|X_3\| = \sqrt{x_1^2 + x_2^2 + x_3^2}$$

$$= \sqrt{\left(\frac{2}{3}\right)^2 + \left(-\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2}$$

$$= \sqrt{\frac{4}{9} + \frac{4}{9} + \frac{1}{9}}$$

$$= \sqrt{\frac{9}{9}}$$

$$= \sqrt{1}$$

$$= 1$$

Logo,

ESSE SUBCONJUNTO DE VETORES É ORTONORMAL.

$$\begin{aligned} \textcircled{2.3} \left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right) \cdot \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) &= \left[\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{3}}\right) + 0\left(\frac{1}{\sqrt{3}}\right) + \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{3}}\right)\right] \\ &= \left(\frac{1}{\sqrt{6}} + 0 - \frac{1}{\sqrt{6}}\right) \\ &= 0 \end{aligned}$$

$$\begin{aligned} \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) \cdot (0, 1, 0) &= \left[\frac{1}{\sqrt{3}}(0) + \frac{1}{\sqrt{3}}(1) + \frac{1}{\sqrt{3}}(0)\right] \\ &= \left(0 + \frac{1}{\sqrt{3}} + 0\right) \\ &= \frac{1}{\sqrt{3}} \end{aligned}$$

Logo,

ESSE SUBCONJUNTO DE VETORES NÃO É ORTOGONAL.

SEMDO ASSIM,

TAMBÉM NÃO SERÁ ORTONORMAL.

$$\begin{aligned} 2.C) (0, 2, 2, 1) \cdot (1, 1, -2, 2) &= [0(1) + 2(1) + 2(-2) + 1(2)] \\ &= (0 + 2 - 4 + 2) \\ &= 0 \end{aligned}$$

$$\begin{aligned} (1, 1, -2, 2) \cdot (0, -2, 1, 2) &= [1(0) + 1(-2) + (-2)(1) + 2(2)] \\ &= (0 - 2 - 2 + 4) \\ &= 0 \end{aligned}$$

$$\begin{aligned} \|X_1\| &= \sqrt{X_1^2 + X_2^2 + X_3^2 + X_4^2} \\ &= \sqrt{(0)^2 + 2^2 + 2^2 + 1^2} \\ &= \sqrt{0 + 4 + 4 + 1} \\ &= \sqrt{9} \\ &= 3 \end{aligned}$$

$$\begin{aligned} \|X_2\| &= \sqrt{X_1^2 + X_2^2 + X_3^2 + X_4^2} \\ &= \sqrt{1^2 + 1^2 + (-2)^2 + 2^2} \\ &= \sqrt{1 + 1 + 4 + 4} \\ &= \sqrt{10} \end{aligned}$$

$$\begin{aligned} \|X_3\| &= \sqrt{X_1^2 + X_2^2 + X_3^2 + X_4^2} \\ &= \sqrt{0^2 + (-2)^2 + 1^2 + 2^2} \\ &= \sqrt{0 + 4 + 1 + 4} \\ &= \sqrt{9} \\ &= 3 \end{aligned}$$

LOGO,

APESAR DESSE SUBCONJUNTO DE VETORES SER ORTOGONAL ELE NÃO SERÁ ORTONORMAL, POIS OS VETORES NÃO SÃO UNITÁRIOS.

EXERCÍCIO 3 (PÁGINA 157)

1º PASSO:

$$Y_1 = X_1 = (1, -1, 0)$$

2º PASSO:

$$\begin{aligned} Y_2 &= X_2 - \left(\frac{X_2 \cdot Y_1}{Y_1 \cdot Y_1} \right) Y_1 \\ &= (2, 0, 1) - \left[\frac{(2, 0, 1) \cdot (1, -1, 0)}{(1, -1, 0) \cdot (1, -1, 0)} \right] (1, -1, 0) \\ &= (2, 0, 1) - \left[\frac{2(1) + 0(-1) + 1(0)}{1(1) + (-1)(-1) + 0(0)} \right] (1, -1, 0) \\ &= (2, 0, 1) - \left(\frac{2+0+0}{1+1+0} \right) (1, -1, 0) \\ &= (2, 0, 1) - \left(\frac{2}{2} \right) (1, -1, 0) \\ &= (2, 0, 1) - (1, -1, 0) \\ &= (2, 0, 1) - [1(1), 1(-1), 1(0)] \\ &= (2, 0, 1) - (1, -1, 0) \\ &= [2-1, 0-(-1), 1-0] \\ &= (1, 0+1, 1) \\ &= (1, 1, 1) \end{aligned}$$

3º PASSO:

$$\begin{aligned} Z_1 &= \frac{Y_1}{\|Y_1\|} \\ &= \frac{1}{\|Y_1\|} (Y_1) \end{aligned}$$

$$\begin{aligned} \|Y_1\| &= \sqrt{Y_1^2 + Y_2^2 + Y_3^2} \\ &= \sqrt{1^2 + (-1)^2 + 0^2} \\ &= \sqrt{1+1+0} \\ &= \sqrt{2} \end{aligned}$$

$$Z_1 = \frac{1}{\sqrt{2}} (1, -1, 0)$$

$$= \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0 \right)$$

$$Z_2 = \frac{Y_2}{\|Y_2\|}$$

$$= \frac{1}{\|Y_2\|} (Y_2)$$

$$= \frac{1}{\sqrt{3}} (1, 1, 1)$$

$$= \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right)$$

$$\|Y_2\| = \sqrt{Y_1^2 + Y_2^2 + Y_3^2}$$

$$= \sqrt{1^2 + 1^2 + 1^2}$$

$$= \sqrt{1+1+1}$$

$$= \sqrt{3}$$

$$\left\{ \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0 \right), \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right) \right\}$$

EXERCÍCIO 9 (PÁGINA 157)

1º PASSO:

$$Y_1 = X_1 = (1, 0, 2)$$

2º PASSO:

$$Y_2 = X_2 - \left(\frac{X_2 \cdot Y_1}{Y_1 \cdot Y_1} \right) Y_1$$

$$= (-1, 1, 0) - \left[\frac{(-1, 1, 0) \cdot (1, 0, 2)}{(1, 0, 2) \cdot (1, 0, 2)} \right] (1, 0, 2)$$

$$= (-1, 1, 0) - \left[\frac{-1(1) + 1(0) + 0(2)}{1(1) + 0(0) + 2(2)} \right] (1, 0, 2)$$

$$= (-1, 1, 0) - \left(\frac{-1 + 0 + 0}{1 + 0 + 4} \right) (1, 0, 2)$$

$$\begin{aligned}
 Y_2 &= (-1, 1, 0) - \left(-\frac{1}{5}\right)(1, 0, 2) \\
 &= (-1, 1, 0) + \frac{1}{5}(1, 0, 2) \\
 &= (-1, 1, 0) + \left(\frac{1}{5}, 0, \frac{2}{5}\right) \\
 &= \left(-1 + \frac{1}{5}, 1 + 0, 0 + \frac{2}{5}\right) \\
 &= \left(-\frac{5}{5} + \frac{1}{5}, 1, \frac{2}{5}\right) \\
 &= \left(-\frac{4}{5}, 1, \frac{2}{5}\right)
 \end{aligned}$$

3° PASSO:

$$\begin{aligned}
 Z_1 &= \frac{Y_1}{\|Y_1\|} \\
 &= \frac{1}{\|Y_1\|} (Y_1) \\
 &= \frac{1}{\sqrt{5}} (1, 0, 2)
 \end{aligned}$$

$$\begin{aligned}
 \|Y_1\| &= \sqrt{Y_1^2 + Y_2^2 + Y_3^2} \\
 &= \sqrt{1^2 + 0^2 + 2^2} \\
 &= \sqrt{1 + 0 + 4} \\
 &= \sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 &= \left[\frac{1}{\sqrt{5}}(1), \frac{1}{\sqrt{5}}(0), \frac{1}{\sqrt{5}}(2) \right] \\
 &= \left(\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}} \right)
 \end{aligned}$$

$$\begin{aligned}
 Z_2 &= \frac{Y_2}{\|Y_2\|} \\
 &= \frac{1}{\|Y_2\|} (Y_2)
 \end{aligned}$$

$$\begin{aligned}
 \|Y_2\| &= \sqrt{Y_1^2 + Y_2^2 + Y_3^2} \\
 &= \sqrt{\left(-\frac{4}{5}\right)^2 + 1^2 + \left(\frac{2}{5}\right)^2} \\
 &= \sqrt{\frac{16}{25} + 1 + \frac{4}{25}}
 \end{aligned}$$

$$Z_2 = \frac{1}{\frac{3\sqrt{5}}{3}} \left(\frac{-4}{3}, \frac{1}{3}, \frac{2}{3} \right)$$

$$= \frac{5\sqrt{5}}{3} \left(\frac{-4}{3}, \frac{1}{3}, \frac{2}{3} \right)$$

$$= \left(\frac{-4\sqrt{5}}{3}, \frac{5\sqrt{5}}{3}, \frac{2\sqrt{5}}{3} \right)$$

$$\|Y_2\| = \sqrt{16+25+4}$$

$$= \sqrt{45}$$

$$= \frac{3\sqrt{5}}{3}$$

$$\left\{ \left(\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}} \right), \left(\frac{-4\sqrt{5}}{3}, \frac{5\sqrt{5}}{3}, \frac{2\sqrt{5}}{3} \right) \right\}$$

EXERCÍCIO 6 (PÁGINA 157)

1º PASSO:

$$Y_1 = X_1 = (1, 1, -1, 0)$$

$$\begin{aligned} \|Y_1\| &= \sqrt{X_1^2 + Y_2^2 + Y_3^2 + Y_4^2} \\ &= \sqrt{1^2 + 1^2 + (-1)^2 + 0^2} \\ &= \sqrt{1+1+1+0} \\ &= \sqrt{3} \end{aligned}$$

$$Z_1 = \frac{Y_1}{\|Y_1\|}$$

$$= \frac{1}{\|Y_1\|} (Y_1)$$

$$= \frac{1}{\sqrt{3}} (1, 1, -1, 0)$$

$$= \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, 0 \right)$$

2º PASSO:

$$\begin{aligned}
 Y_2 &= X_2 - \left(\frac{X_2 \cdot Y_1}{Y_1 \cdot Y_1} \right) Y_1 \\
 &= (0, 2, 0, 1) - \left[\frac{(0, 2, 0, 1) \cdot (1, 1, -1, 0)}{(1, 1, -1, 0) \cdot (1, 1, -1, 0)} \right] (1, 1, -1, 0) \\
 &= (0, 2, 0, 1) - \left[\frac{0(1) + 2(1) + 0(-1) + 1(0)}{1(1) + 1(1) + (-1)(-1) + 0(0)} \right] (1, 1, -1, 0) \\
 &= (0, 2, 0, 1) - \left(\frac{0 + 2 + 0 + 0}{1 + 1 + 1 + 0} \right) (1, 1, -1, 0) \\
 &= (0, 2, 0, 1) - \frac{2}{3} (1, 1, -1, 0) \\
 &= (0, 2, 0, 1) - \left(\frac{2}{3}, \frac{2}{3}, -\frac{2}{3}, 0 \right) \\
 &= (0, 2, 0, 1) - \left(\frac{2}{3}, \frac{2}{3}, -\frac{2}{3}, 0 \right) (3) \\
 &= (0, 2, 0, 1) - (2, 2, -2, 0) \\
 &= (0 - 2, 2 - 2, 0 - (-2), 1 - 0) \\
 &= (-2, 0, 2, 1)
 \end{aligned}$$

$$Z_2 = \frac{Y_2}{\|Y_2\|}$$

$$= \frac{1}{\|Y_2\|} (Y_2)$$

$$= \frac{1}{3} (-2, 0, 2, 1)$$

$$= \left(-\frac{2}{3}, 0, \frac{2}{3}, \frac{1}{3} \right)$$

$$\begin{aligned}
 \|Y_2\| &= \sqrt{Y_1^2 + Y_2^2 + Y_3^2 + Y_4^2} \\
 &= \sqrt{(-2)^2 + 0^2 + 2^2 + 1^2} \\
 &= \sqrt{4 + 0 + 4 + 1} \\
 &= \sqrt{9}
 \end{aligned}$$

$$= 3$$

3º PASSO:

$$Y_3 = X_3 - \left(\frac{X_3 \cdot Y_1}{Y_1 \cdot Y_1} \right) Y_1 - \left(\frac{X_3 \cdot Y_2}{Y_2 \cdot Y_2} \right) Y_2$$

$$= (-1, 0, 0, 1) - \left[\frac{(-1, 0, 0, 1) \cdot (1, 1, -1, 0)}{(1, 1, -1, 0) \cdot (1, 1, -1, 0)} \right] (1, 1, -1, 0) - \left[\frac{(-1, 0, 0, 1) \cdot (-2, 0, 2, 1)}{(-2, 0, 2, 1) \cdot (-2, 0, 2, 1)} \right] (-2, 0, 2, 1)$$

$$= (-1, 0, 0, 1) - \left[\frac{(-1)(1) + 0(1) + 0(-1) + 1(0)}{1(1) + 1(1) + (-1)(-1) + 0(0)} \right] (1, 1, -1, 0) - \left[\frac{(-1)(-2) + 0(0) + 0(2) + 1(1)}{(-2)(-2) + 0(0) + 0(2) + 1(1)} \right] (-2, 0, 2, 1)$$

$$= (-1, 0, 0, 1) - \left(\frac{-1 + 0 + 0 + 0}{1 + 1 + 1 + 0} \right) (1, 1, -1, 0) - \left(\frac{2 + 0 + 0 + 1}{4 + 0 + 4 + 1} \right) (-2, 0, 2, 1)$$

$$= (-1, 0, 0, 1) - \left(\frac{-1}{3} \right) (1, 1, -1, 0) - \left(\frac{3}{9} \right) (-2, 0, 2, 1)$$

$$= (-1, 0, 0, 1) + \frac{1}{3} (1, 1, -1, 0) - \left(\frac{1}{3} \right) (-2, 0, 2, 1)$$

$$= (-1, 0, 0, 1) + \left(\frac{1}{3}, \frac{1}{3}, -\frac{1}{3}, 0 \right) - \left(\frac{-2}{3}, 0, \frac{2}{3}, \frac{1}{3} \right)$$

$$= \left(-1 + \frac{1}{3} + \frac{2}{3}, 0 + \frac{1}{3} + 0, 0 - \frac{1}{3} - \frac{2}{3}, 1 + 0 - \frac{1}{3} \right)$$

$$= \left(-1 + \frac{3}{3}, \frac{1}{3}, -\frac{3}{3}, \frac{3-1}{3} \right)$$

$$= \left(-1 + 1, \frac{1}{3}, -1, \frac{2}{3} \right)$$

$$= \left(0, \frac{1}{3}, -1, \frac{2}{3} \right) (3)$$

$$= (0, 1, -3, 2)$$

$$Z_3 = \frac{Y_3}{\|Y_3\|}$$

$$\|Y_3\| = \sqrt{Y_1^2 + Y_2^2 + Y_3^2 + Y_4^2}$$

$$= \sqrt{0^2 + 1^2 + (-3)^2 + 2^2}$$

$$Z_3 = \frac{1}{\|Y_3\|} (Y_3)$$

$$\|Y_3\| = \sqrt{0+1+9+4}$$

$$= \sqrt{14}$$

$$= \frac{1}{\sqrt{14}} (0, 1, -3, 2)$$

$$= \left(0, \frac{1}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{2}{\sqrt{14}}\right)$$

$$\left\{ \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, 0 \right), \left(-\frac{2}{3}, 0, \frac{2}{3}, \frac{1}{3} \right), \left(0, \frac{1}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{2}{\sqrt{14}} \right) \right\}$$

EXERCÍCIO 7 (PÁGINA 157)

7.A 1.º PASSO:

$$Y_1 = X_1 = (1, 2)$$

2.º PASSO

$$Y_2 = X_2 - \left(\frac{X_2 \cdot Y_1}{Y_1 \cdot Y_1} \right) Y_1$$

$$= (-3, 4) - \left[\frac{(-3, 4) \cdot (1, 2)}{(1, 2) \cdot (1, 2)} \right] (1, 2)$$

$$= (-3, 4) - \left[\frac{-3(1) + 4(2)}{1(1) + 2(2)} \right] (1, 2)$$

$$= (-3, 4) - \left(\frac{-3 + 8}{1 + 4} \right) (1, 2)$$

$$= (-3, 4) - \left(\frac{5}{5} \right) (1, 2)$$

$$= (-3, 4) - (1)(1, 2)$$

$$= (-3, 4) - [1(1), 1(2)]$$

$$\begin{aligned}
 Y_2 &= (-3, 4) - (1, 2) \\
 &= (-3-1, 4-2) \\
 &= (-4, 2)
 \end{aligned}$$

Logo,

A BASE ORTOGONAL É $\{(1, 2), (-4, 2)\}$

$$\begin{aligned}
 \textcircled{7.3} \quad Z_1 &= \frac{Y_1}{\|Y_1\|} \\
 &= \frac{1}{\|Y_1\|} (Y_1) \\
 &= \frac{1}{\sqrt{5}} (1, 2) \\
 &= \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right)
 \end{aligned}$$

$$\begin{aligned}
 \|Y_1\| &= \sqrt{Y_1^2 + Y_2^2} \\
 &= \sqrt{1^2 + 2^2} \\
 &= \sqrt{1+4} \\
 &= \sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 Z_2 &= \frac{Y_2}{\|Y_2\|} \\
 &= \frac{1}{\|Y_2\|} (Y_2) \\
 &= \frac{1}{2\sqrt{5}} (-4, 2) \\
 &= \left(\frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right)
 \end{aligned}$$

$$\begin{aligned}
 \|Y_2\| &= \sqrt{Y_1^2 + Y_2^2} \\
 &= \sqrt{(-4)^2 + 2^2} \\
 &= \sqrt{16+4} \\
 &= \sqrt{20} \\
 &= \sqrt{4(5)} \\
 &= 2\sqrt{5}
 \end{aligned}$$

Logo,

A BASE ORTOGONAL É $\left\{ \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right), \left(\frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right) \right\}$

EXERCÍCIO 9 (PÁGINA 157)

EXERCÍCIO 10 (PÁGINA 137)

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 0 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix} \xrightarrow{\sim} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & 0 & 1 \\ 2 & 2 & 2 \end{bmatrix} \xrightarrow{\begin{matrix} x(-1) \times (-2) \\ \leftarrow + \\ + \sim \end{matrix}} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{\begin{matrix} \leftarrow + \\ x(-1) \sim \end{matrix}}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{\begin{matrix} \leftarrow + \\ x(-2) \times (1) \end{matrix}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

LOGO,

O VETOR $(2, 2, 2)$ É L.D. (LINEARMENTE DEPENDENTE), POR ISSO, É DESCONTADO.

1º PASSO:

$$Y_1 = X_1 = (1, 1, 1)$$

$$Z_1 = \frac{Y_1}{\|Y_1\|}$$

$$= \frac{1}{\|Y_1\|} (Y_1)$$

$$= \frac{1}{\sqrt{3}} (1, 1, 1)$$

$$= \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right)$$

$$\begin{aligned} \|Y_1\| &= \sqrt{Y_1^2 + Y_2^2 + Y_3^2} \\ &= \sqrt{1^2 + 1^2 + 1^2} \\ &= \sqrt{1+1+1} \\ &= \sqrt{3}
 \end{aligned}$$

328

X X

2.º PASSO:

$$\begin{aligned}
 Y_2 &= X_2 - \left(\frac{X_2 \cdot Y_1}{Y_1 \cdot Y_1} \right) Y_1 \\
 &= (0, 0, 1) - \left[\frac{(0, 0, 1) \cdot (1, 1, 1)}{(1, 1, 1) \cdot (1, 1, 1)} \right] (1, 1, 1) \\
 &= (0, 0, 1) - \left[\frac{0(1) + 0(1) + 1(1)}{1(1) + 1(1) + 1(1)} \right] (1, 1, 1) \\
 &= (0, 0, 1) - \left(\frac{0 + 0 + 1}{1 + 1 + 1} \right) (1, 1, 1) \\
 &= (0, 0, 1) - \left(\frac{1}{3} \right) (1, 1, 1) \\
 &= (0, 0, 1) - \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right) \\
 &= \left(0 - \frac{1}{3}, 0 - \frac{1}{3}, 1 - \frac{1}{3} \right) \\
 &= \left(-\frac{1}{3}, -\frac{1}{3}, \frac{3-1}{3} \right) \\
 &= \left(-\frac{1}{3}, -\frac{1}{3}, \frac{2}{3} \right) (3) \\
 &= (-1, -1, 2)
 \end{aligned}$$

$$\begin{aligned}
 Z_2 &= \frac{Y_2}{\|Y_2\|} \\
 &= \frac{1}{\|Y_2\|} (Y_2) \\
 &= \frac{1}{\sqrt{6}} (-1, -1, 2) \\
 &= \left(-\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}} \right)
 \end{aligned}$$

$$\begin{aligned}
 \|Y_2\| &= \sqrt{Y_1^2 + Y_2^2 + Y_3^2} \\
 &= \sqrt{(-1)^2 + (-1)^2 + 2^2} \\
 &= \sqrt{1+1+4} \\
 &= \sqrt{6}
 \end{aligned}$$

3º PASSO:

$$\begin{aligned}
 Y_3 &= X_3 - \left(\frac{X_3 \cdot Y_1}{Y_1 \cdot Y_1} \right) Y_1 - \left(\frac{X_3 \cdot Y_2}{Y_2 \cdot Y_2} \right) Y_2 \\
 &= (1, 2, 3) - \left[\frac{(1, 2, 3) \cdot (1, 1, 1)}{(1, 1, 1) \cdot (1, 1, 1)} \right] (1, 1, 1) - \left[\frac{(1, 2, 3) \cdot (0, 0, 1)}{(0, 0, 1) \cdot (0, 0, 1)} \right] (0, 0, 1) \\
 &= (1, 2, 3) - \left[\frac{2(1) + 2(1) + 3(1)}{1(1) + 1(1) + 1(1)} \right] (1, 1, 1) - \left[\frac{2(0) + 2(0) + 3(1)}{0(0) + 0(0) + 1(1)} \right] (0, 0, 1) \\
 &= (1, 2, 3) - \left(\frac{1+2+3}{1+1+1} \right) (1, 1, 1) - \left(\frac{0+0+3}{0+0+1} \right) (0, 0, 1) \\
 &= (1, 2, 3) - \left(\frac{6}{3} \right) (1, 1, 1) - \left(\frac{3}{1} \right) (0, 0, 1) \\
 &= (1, 2, 3) - 2(1, 1, 1) - 3(0, 0, 1) \\
 &= (1, 2, 3) - [2(1), 2(1), 2(1)] - [3(0), 3(0), 3(1)] \\
 &= (1, 2, 3) - (2, 2, 2) - (0, 0, 3) \\
 &= (1-2-0, 2-2-0, 3-2-3) \\
 &= (-1, 0, -2)
 \end{aligned}$$

$$\begin{aligned}
 Z_3 &= \frac{Y_3}{\|Y_3\|} \\
 &= \frac{1}{\|Y_3\|} (Y_3) \\
 &= \frac{1}{\sqrt{5}} (-1, 0, -2) \\
 &= \left(\frac{-1}{\sqrt{5}}, 0, \frac{-2}{\sqrt{5}} \right)
 \end{aligned}$$

$$\begin{aligned}
 \|Y_3\| &= \sqrt{Y_1^2 + Y_2^2 + Y_3^2} \\
 &= \sqrt{(-1)^2 + 0^2 + (-2)^2} \\
 &= \sqrt{1+0+4} \\
 &= \sqrt{5}
 \end{aligned}$$

Logo,

$\left\{ \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right), \left(\frac{-1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}} \right), \left(\frac{-1}{\sqrt{5}}, 0, \frac{-2}{\sqrt{5}} \right) \right\}$ É BASE ORTOGONAL DE $W = [S]$ DE $\mathbb{R}^3$

EXERCÍCIO 13 (PÁGINA 157)

$$X = (2, -3, 1)$$

$$X = \kappa_1 Z_1 + \kappa_2 Z_2 + \kappa_3 Z_3$$

$$\kappa_1 = X \cdot Z_1$$

$$= (2, -3, 1) \cdot \left(\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}} \right)$$

$$= \left[2 \left(\frac{1}{\sqrt{5}} \right) + (-3)(0) + 1 \left(\frac{2}{\sqrt{5}} \right) \right]$$

$$= \left(\frac{2}{\sqrt{5}} + 0 + \frac{2}{\sqrt{5}} \right)$$

$$= \frac{4}{\sqrt{5}}$$

$$\kappa_2 = X \cdot Z_2$$

$$= (2, -3, 1) \cdot \left(-\frac{2}{\sqrt{5}}, 0, \frac{1}{\sqrt{5}} \right)$$

$$= \left[2 \left(-\frac{2}{\sqrt{5}} \right) + (-3)(0) + 1 \left(\frac{1}{\sqrt{5}} \right) \right]$$

$$= \left(-\frac{4}{\sqrt{5}} + 0 + \frac{1}{\sqrt{5}} \right)$$

$$= -\frac{3}{\sqrt{5}}$$

$$\kappa_3 = X \cdot Z_3$$

$$= (2, -3, 1) \cdot (0, 1, 0)$$

$$= [2(0) + (-3)(1) + 1(0)]$$

$$= (0 - 3 + 0)$$

$$= -3$$

$$X = \frac{4}{\sqrt{5}} z_1 - \frac{3}{\sqrt{5}} z_2 - 3 z_3$$

$$= \frac{4}{\sqrt{5}} \left(\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}} \right) - \frac{3}{\sqrt{5}} \left(\frac{-2}{\sqrt{5}}, 0, \frac{1}{\sqrt{5}} \right) - 3(0, 1, 0)$$

$$= \left(\frac{4}{5}, 0, \frac{8}{5} \right) + \left(\frac{6}{5}, 0, -\frac{3}{5} \right) + (0, -3, 0)$$

$$= \left(\frac{4}{5} + \frac{6}{5} + 0, 0 + 0 + (-3), \frac{8}{5} - \frac{3}{5} + 0 \right)$$

$$= \left(\frac{10}{5} + 0, -3, \frac{5}{5} + 0 \right)$$

$$= (2 + 0, -3, 1 + 0)$$

$$= (2, -3, 1)$$

EXERCÍCIO 14 (PÁGINA 157)

$$\text{proj}_W X = Z = (X \cdot X_1) X_1 + (X \cdot X_2) X_2$$

$$= [(1, 2, -1) \cdot (0, 1, 0)](0, 1, 0) + \left[(1, 2, -1) \cdot \left(\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}} \right) \right] \left(\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}} \right)$$

$$= [1(0) + 2(1) + (-1)(0)](0, 1, 0) + \left[1\left(\frac{1}{\sqrt{5}}\right) + 2(0) + (-1)\left(\frac{2}{\sqrt{5}}\right) \right] \left(\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}} \right)$$

$$= (0 + 2 + 0)(0, 1, 0) + \left(\frac{1}{\sqrt{5}} + 0 - \frac{2}{\sqrt{5}} \right) \left(\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}} \right)$$

$$= 2(0, 1, 0) + \left(-\frac{1}{\sqrt{5}} \right) \left(\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}} \right)$$

$$= [2(0), 2(1), 2(0)] + \left[-\frac{1}{\sqrt{5}} \left(\frac{1}{\sqrt{5}} \right), \left(-\frac{1}{\sqrt{5}} \right)(0), \left(-\frac{1}{\sqrt{5}} \right) \left(\frac{2}{\sqrt{5}} \right) \right]$$

$$= (0, 2, 0) + \left(-\frac{1}{5}, 0, -\frac{2}{5} \right)$$

$$= \left(0 - \frac{1}{5}, 2 + 0, 0 - \frac{2}{5} \right)$$

$$= \left(-\frac{1}{5}, 2, -\frac{2}{5} \right)$$

2º PASSO:

$$X = Z + Y$$

$$Y = X - Z$$

$$= (1, 2, -1) - \left(-\frac{1}{5}, 2, -\frac{2}{5}\right)$$

$$= \left(1 + \frac{1}{5}, 2 - 2, -1 + \frac{2}{5}\right)$$

$$= \left(\frac{5+1}{5}, 0, -\frac{5+2}{5}\right)$$

$$= \left(\frac{6}{5}, 0, -\frac{3}{5}\right)$$

EXERCÍCIO 15 (PÁGINA 157)

1º PASSO:

$$\begin{aligned} \text{proj}_W X = Z &= (X \cdot X_1)X_1 + (X \cdot X_2)X_2 + (X \cdot X_3)X_3 \\ &= \left[(1, 0, 2, 3) \cdot \left(\frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}\right)\right] \left(\frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}\right) + \left[(1, 0, 2, 3) \cdot (0, 0, 1, 0)\right] \\ &\quad (0, 0, 1, 0) + \left[(1, 0, 2, 3) \cdot \left(\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}\right)\right] \left(\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}\right) \\ &= \left[1\left(\frac{1}{\sqrt{2}}\right) + 0(0) + 2(0) + 3\left(-\frac{1}{\sqrt{2}}\right)\right] \left(\frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}\right) + [1(0) + 0(0) + \\ &\quad + 2(1) + 3(0)](0, 0, 1, 0) + \left[1\left(\frac{1}{\sqrt{2}}\right) + 0(0) + 2(0) + 3\left(\frac{1}{\sqrt{2}}\right)\right] \left(\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}\right) \\ &= \left(\frac{1}{\sqrt{2}} + 0 + 0 - \frac{3}{\sqrt{2}}\right) \left(\frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}\right) + (0 + 0 + 2 + 0)(0, 0, 1, 0) + \\ &\quad + \left(\frac{1}{\sqrt{2}} + 0 + 0 + \frac{3}{\sqrt{2}}\right) \left(\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}\right) \end{aligned}$$

$$\begin{aligned}
 \text{proj}_W X = Z &= -\frac{2}{\sqrt{2}} \left(\frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}} \right) + 2(0, 0, 1, 0) + \frac{1}{\sqrt{2}} \left(\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}} \right) \\
 &= \left(-\frac{2}{2}, 0, 0, \frac{2}{2} \right) + (0, 0, 2, 0) + \left(\frac{1}{2}, 0, 0, \frac{1}{2} \right) \\
 &= (-1, 0, 0, 1) + (0, 0, 2, 0) + (0.5, 0, 0, 0.5) \\
 &= (-1+0+0.5, 0+0+0, 0+2+0, 1+0+0.5) \\
 &= (-0.5, 0, 2, 1.5)
 \end{aligned}$$

2º PASSO:

$$X = Z + Y$$

$$Y = X - Z$$

$$= (1, 0, 2, 3) - (-0.5, 0, 2, 1.5)$$

$$= (1 - (-0.5), 0 - 0, 2 - 2, 3 - 1.5)$$

$$= (1.5, 0, 0, 1.5)$$

EXERCÍCIO 16 (PÁGINA 158)

$$\begin{aligned}
 \text{proj}_W X = Z &= (X \cdot X_1)X_1 + (X \cdot X_2)X_2 \\
 &= [(-1, 0, 1) \cdot (0, 1, 0)](0, 1, 0) + [(-1, 0, 1) \cdot \left(\frac{1}{\sqrt{3}}, 0, \frac{2}{\sqrt{3}} \right)] \left(\frac{1}{\sqrt{3}}, 0, \frac{2}{\sqrt{3}} \right) \\
 &= [(-1)(0) + 0(1) + 1(0)](0, 1, 0) + \left[(-1)\left(\frac{1}{\sqrt{3}}\right) + 0(0) + 1\left(\frac{2}{\sqrt{3}}\right) \right] \left(\frac{1}{\sqrt{3}}, 0, \frac{2}{\sqrt{3}} \right) \\
 &= (0+0+0)(0, 1, 0) + \left(-\frac{1}{\sqrt{3}} + 0 + \frac{2}{\sqrt{3}} \right) \left(\frac{1}{\sqrt{3}}, 0, \frac{2}{\sqrt{3}} \right) \\
 &= 0(0, 1, 0) + \frac{1}{\sqrt{3}} \left(\frac{1}{\sqrt{3}}, 0, \frac{2}{\sqrt{3}} \right) \\
 &= [0(0), 0(1), 0(0)] + \left[\frac{1}{\sqrt{3}} \left(\frac{1}{\sqrt{3}} \right), \frac{1}{\sqrt{3}}(0), \frac{1}{\sqrt{3}} \left(\frac{2}{\sqrt{3}} \right) \right] \\
 &= (0, 0, 0) + \left(\frac{1}{3}, 0, \frac{2}{3} \right)
 \end{aligned}$$

334

1 / 1

$$\text{proj}_W X = Z = \left(0 + \frac{1}{5}, 0 + 0, 0 + \frac{2}{5}\right) = \left(\frac{1}{5}, 0, \frac{2}{5}\right)$$

$$\begin{aligned} X - \text{proj}_W X &= (-1, 0, 1) - \left(\frac{1}{5}, 0, \frac{2}{5}\right) = \left(-1 - \frac{1}{5}, 0 - 0, 1 - \frac{2}{5}\right) \\ &= \left(-\frac{6}{5}, 0, \frac{3}{5}\right) \\ \|X - \text{proj}_W X\| &= \sqrt{\left(-\frac{6}{5}\right)^2 + 0^2 + \left(\frac{3}{5}\right)^2} = \sqrt{\frac{36}{25} + 0 + \frac{9}{25}} = \sqrt{\frac{45}{25}} = \sqrt{\frac{9}{5}} = \frac{3}{\sqrt{5}} \end{aligned}$$

EXERCÍCIO 17 (PÁGINA 158)

$$\begin{aligned} \text{proj}_W X &= Z = (X \cdot X_1)X_1 + (X \cdot X_2)X_2 + (X \cdot X_3)X_3 \\ &= \left[(1, 2, -1, 0) \cdot \left(\frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}\right) \right] \left(\frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}\right) + \left[(1, 2, -1, 0) \cdot (0, 0, 1, 0) \right] (0, 0, 1, 0) \\ &\quad + \left[(1, 2, -1, 0) \cdot \left(\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}\right) \right] \left(\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}\right) \\ &= \left[1\left(\frac{1}{\sqrt{2}}\right) + 2(0) + (-1)(0) + 0\left(-\frac{1}{\sqrt{2}}\right) \right] \left(\frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}\right) + [1(0) + 2(0) + (-1)(1) + 0(0)] (0, 0, 1, 0) \\ &\quad + \left[1\left(\frac{1}{\sqrt{2}}\right) + 2(0) + (-1)\left(\frac{1}{\sqrt{2}}\right) + 0(0) \right] \left(\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}\right) \\ &= \left(\frac{1}{\sqrt{2}} + 0 + 0 + 0\right) \left(\frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}\right) + (0 + 0 - 1 + 0) (0, 0, 1, 0) + (0 + 0 - 1 + 0) \left(\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}\right) \end{aligned}$$

$$+\left(\frac{1}{\sqrt{2}}+0+0+0\right)\left(\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}\right)$$

$$\begin{aligned}\text{proj}_w X = Z &= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}\right) + (-1)(0, 0, 1, 0) + \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}\right) \\ &= \left(\frac{1}{2}, 0, 0, -\frac{1}{2}\right) + (0, 0, -1, 0) + \left(\frac{1}{2}, 0, 0, \frac{1}{2}\right) \\ &= \left(\frac{1}{2}+0+\frac{1}{2}, 0+0+0, 0+(-1)+0, -\frac{1}{2}+0+\frac{1}{2}\right) \\ &= (1, 0, -1, 0)\end{aligned}$$

$$\begin{aligned}X - \text{proj}_w X &= (1, 2, -1, 0) - (1, 0, -1, 0) \\ &= (1-1, 2-0, -1+1, 0-0) \\ &= (0, 2, 0, 0)\end{aligned}$$

$$\begin{aligned}\|X - \text{proj}_w X\| &= \sqrt{0^2 + 2^2 + 0^2 + 0^2} \\ &= \sqrt{0+4+0+0} \\ &= \sqrt{4} \\ &= 2\end{aligned}$$