(4) APLICAÇÕES DO ESTUDO DAS DERIVADAS => PÁGINA 232 DO LIVRO MATEMÁTICA 1 PARA OS CURSOS DE ECONOMIA, ADMINISTRAÇÃO (1) MÁXIMOS E MÍNIMOS E CIÊNCIAS CONTÁBEIS (MEDEIROS, 5ª EDIÇÃO, EDITORA ATLAS)

(1) $y = 10x + 5$ $\quad x \in \mathbb{R}$

$y' = (10x+5)'$
$y' = (10x)' + (5)'$
$y' = 10 + 0$
$y' = 10$

$y' > 0 \Rightarrow x \in \mathbb{R}$

E.C. EM $\mathbb{R}$; NÃO TEM PM E NEM Pm

(2) $y = -4x + 5$ $\quad 0 \leq x \leq 1$

$y' = (-4x+5)'$
$y' = (-4x)' + (5)'$
$y' = -4 + 0$
$y' = -4$

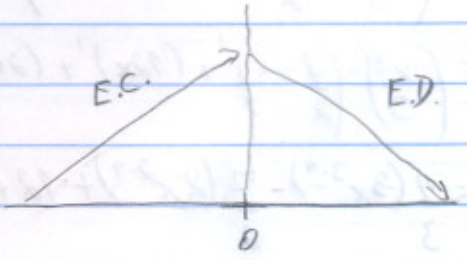
$y' < 0 \Rightarrow x \in \mathbb{R}$

E.D. EM $]0,1[$ $\quad x=0$ É PM E $x=1$ É Pm.

(3) $y = 1 - x^2$ $\quad x \in \mathbb{R}$

$y' = (1-x^2)'$
$y' = (1)' - (x^2)'$
$y' = 0 - (2x^{2-1})$
$y' = -(2x)$
$y' = -2x$

$-2x \neq 0$
$x \neq 0$



$x = -1 \Rightarrow y = -2(-1) = 2 > 0$ $\quad x = 1 \Rightarrow y = -2(1) = -2 < 0$
$x = -2 \Rightarrow y = -2(-2) = 4 > 0$ $\quad x = 2 \Rightarrow y = -2(2) = -4 < 0$
$x = -3 \Rightarrow y = -2(-3) = 6 > 0$ $\quad x = 3 \Rightarrow y = -2(3) = -6 < 0$

(16)

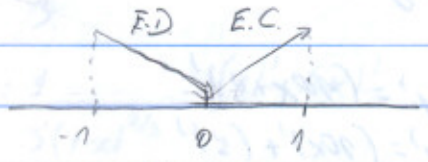
$x=0$ É PM

$y = 1 - x^2$

$y = 1 - (0)^2$

$y = 1$ VALOR MÁXIMO

(4) $y = x^2 + 1, \quad -1 \leq x \leq 1$



$y' = (x^2+1)'$

$y' = (x^2)' + (1)'$

$y' = 2x^{2-1} + 0$

$y' = 2x$

$2x \neq 0$

$x \neq 0$

$x = -0,1 \Rightarrow y = 2(-0,1) = -0,8 < 0$ $\quad x = 0,1 \Rightarrow y = 2(0,1) = 0,8 > 0$

$x = -0,6 \Rightarrow y = 2(-0,6) = -1,2 < 0$ $\quad x = 0,6 \Rightarrow y = 2(0,6) = 1,2 > 0$

$x = -0,8 \Rightarrow y = 2(-0,8) = -1,6 < 0$ $\quad x = 0,8 \Rightarrow y = 2(0,8) = 1,6 > 0$

$x = 0$ É Pm

$y = x^2 + 1$

$y = (0)^2 + 1$

$y = 1$ VALOR MÍNIMO

(5) $y = \frac{x^3}{3} - \frac{7}{2}x^2 + 12x + 20 \quad x \in \mathbb{R}$

$y' = \left(\frac{x^3}{3} - \frac{7}{2}x^2 + 12x + 20\right)'$

$y' = \left(\frac{x^3}{3}\right)' - \left(\frac{7}{2}x^2\right)' + (12x)' + (20)$

$y' = \frac{1}{3}(3x^{3-1}) - \frac{7}{2}(2x^{2-1}) + 12 + 0$

$y' = \frac{1}{3_1}(3x^2) - \frac{7}{2_1}(2^1x) + 12$

$y' = x^2 - 7x + 12$

$x^2 - 7x + 12 \neq 0$

$S = 7$
$P = 12$ $\Big\}$ 3 e 4

$x = 2 \Rightarrow (2)^2 - 7(2) + 12 = 4 - 14 + 12 = 2 > 0$

$x = 1 \Rightarrow (1)^2 - 7(1) + 12 = 1 - 7 + 12 = 6 > 0$

$x = 0 \Rightarrow (0)^2 - 7(0) + 12 = 0 - 0 + 12 = 12 > 0$

$x = 3$ É PM

$x = 3{,}2 \Rightarrow (3{,}2)^2 - 7(3{,}2) + 12$

$10{,}24 - 22{,}4 + 12$

$-0{,}16 < 0$

$x = 3{,}4 \Rightarrow (3{,}4)^2 - 7(3{,}4) + 12$

$11{,}56 - 23{,}8 + 12$

$-0{,}24 < 0$

$x = 3{,}6 \Rightarrow (3{,}6)^2 - 7(3{,}6) + 12$

$12{,}96 - 25{,}2 + 12$

$-0{,}24 < 0$

$$y = \frac{x^3}{3} - \frac{7}{2}x^2 + 12x + 20$$

$$y = \frac{(3)^3}{3} - \frac{7}{2}(3)^2 + 12 \cdot 3 + 20$$

$$y = \frac{9^3}{3_1} + \frac{7}{2} \cdot 9 + 36 + 20$$

$$y = 3 + \frac{63}{2} + 56$$

$$y = \frac{63}{2} + 59$$

$$y = \frac{63 + 118}{2}$$

$$y = \frac{181}{2}$$ VALOR MÁXIMO

$x = 4$ É Pm

$x = 5 \Rightarrow (5)^2 - 7(5) + 12$

$25 - 35 + 12$

$2 > 0$

$x = 6 \Rightarrow (6)^2 - 7(6) + 12$

$36 - 42 + 12$

$6 > 0$

$x = 7 \Rightarrow (7)^2 - 7(7) + 12$

$49 - 49 + 12$

$12 > 0$

$$y = \frac{x^3}{3} - \frac{7}{2}x^2 + 12x + 20$$

$$y = \frac{(4)^3}{3} - \frac{7}{2}(4)^2 + 12 \cdot 4 + 20$$

$$y = \frac{64}{3} - \frac{7}{2_1} \cdot \overset{8}{\cancel{16}} + 48 + 20$$

$$y = \frac{64}{3} - 7 \cdot 8 + 68$$

$$y = \frac{64}{3} - 56 + 68$$

$$y = \frac{64}{3} + 12$$

$$y = \frac{64 + 36}{3}$$

$$y = \frac{100}{3}$$ VALOR MÍNIMO

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6) $y = \frac{-x^3}{3} + \frac{5}{2}x^2 + 6x - 5 \quad x \in \mathbb{R}$

$-x^2 + 5x + 6 \neq 0$

$x' = -1 \text{ E } x'' = 6$

$y' = \left(\frac{-x^3}{3} + \frac{5}{2}x^2 + 6x - 5\right)'$

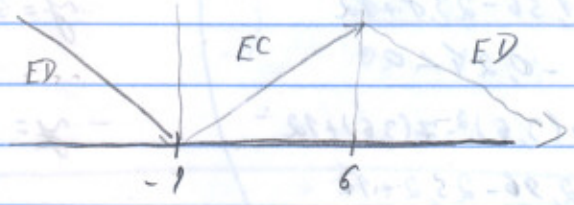
$y' = \left(\frac{-x^3}{3}\right)' + \left(\frac{5}{2}x^2\right)' + (6x)' - (5)'$

$y' = -\frac{1}{3}(x^3)' + \frac{5}{2}(x^2)' + 6 - 0$

$y' = -\frac{1}{3}(3x^{3-1}) + \frac{5}{2}(2x^{2-1}) + 6$

$y' = -\frac{1}{3_1}(3x^2) + \frac{5}{2_1}(2x) + 6$

$y' = -x^2 + 5x + 6$



$x = -1$ É Pm

$y = -\frac{x^3}{3} + \frac{5}{2}x^2 + 6x - 5$

$y = -\frac{(-1)^3}{3} + \frac{5}{2}(-1)^2 + 6(-1) - 5$

$y = -\frac{(-1)}{3} + \frac{5}{2}.1 - 6 - 5$

$y = \frac{1}{3} + \frac{5}{2} - 11$

$y = \frac{2 + 15 - 66}{6}$

$y = -\frac{51}{6}$ VALOR MÍNIMO

$x = -2 \Rightarrow -(-2)^2 + 5(-2) + 6$

$-(4) - 10 + 6$

$-8 < 0$

$x = -3 \Rightarrow -(-3)^2 + 5(-3) + 6$

$-(9) - 15 + 6$

$-18 < 0$

$x = -4 \Rightarrow -(-4)^2 + 5(-4) + 6$

$-(16) - 20 + 6$

$-30 < 0$

$x = 0 \Rightarrow -(0)^2 + 5(0) + 6$

$0 + 0 + 6$

$6 > 0$

$x = 2 \Rightarrow -(2)^2 + 5(2) + 6$

$-(4) + 10 + 6$

$12 > 0$

$x = 4 \Rightarrow -(4)^2 + 5(4) + 6$

$-(16) + 20 + 6$

$10 > 0$

$x = 6$ É PM

$y = -\frac{x^3}{3} + \frac{5}{2}x^2 + 6x - 5$

$y = -\frac{(6)^3}{3} + \frac{5}{2}(6)^2 + 6(6) - 5$

$y = -\frac{(216)}{3} + \frac{5}{2} \cdot 36 + 36 - 5$

$y = -72 + 90 + 31$

$y = 49$

$x = 7 \Rightarrow -(7)^2 + 5(7) + 6$
$-(49) + 35 + 6$
$-8 < 0$

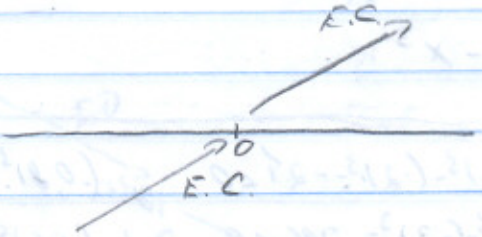
$x = 8 \Rightarrow -(8)^2 + 5(8) + 6$
$-(64) + 40 + 6$
$-18 < 0$

(7) $y = x^3 + 4 \quad x \in \mathbb{R}$

$y' = (x^3 + 4)'$
$y' = (x^3)' + (4)'$
$y' = 3x^{3-1} + 0$
$y' = 3x^2$

$3x^2 \neq 0$
$x \neq 0$

$x = -1 \Rightarrow 3(-1)^2 = 3 > 0$ $\quad x = 1 \Rightarrow 3(1)^2 = 3 > 0$
$x = -2 \Rightarrow 3(-2)^2 = 12 > 0$ $\quad x = 2 \Rightarrow 3(2)^2 = 12 > 0$
$x = -3 \Rightarrow 3(-3)^2 = 27 > 0$ $\quad x = 3 \Rightarrow 3(3)^2 = 27 > 0$

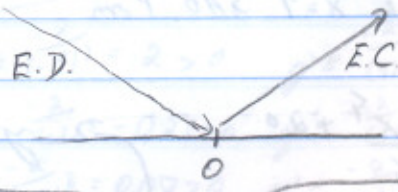


E.C. EM $\mathbb{R}$; NÃO TEM PM E NEM Pm

(8) $y = x^4 \quad x \in \mathbb{R}$

$y' = (x^4)'$
$y' = 4x^{4-1}$
$y' = 4x^3$

$4x^3 \neq 0$
$x \neq 0$



$x = -1 \Rightarrow 4(-1)^3 = -4 < 0$ $\quad x = 1 \Rightarrow 4(1)^3 = 4 > 0$
$x = -2 \Rightarrow 4(-2)^3 = -32 < 0$ $\quad x = 2 \Rightarrow 4(2)^3 = 32 > 0$
$x = -3 \Rightarrow 4(-3)^3 = -108 < 0$ $\quad x = 3 \Rightarrow 4(3)^3 = 108 > 0$

$x = 0$ É Pm

$y = x^4$
$y = (0)^4$
$y = 0$ VALOR MÍNIMO

(9) $y = \frac{x^6}{6} - \frac{x^4}{4} + 20 \quad x \in \mathbb{R}$

$y' = \left(\frac{x^6}{6} - \frac{x^4}{4} + 20\right)'$

$y' = \left(\frac{x^6}{6}\right)' - \left(\frac{x^4}{4}\right)' + (20)'$

$y' = \frac{1}{6}(x^6)' - \frac{1}{4}(x^4)' + 0$

$y' = \frac{1}{6}(6x^{6-1}) - \frac{1}{4}(4x^{4-1})$

$y' = \frac{1}{6_1}(\overset{1}{6}x^5) - \frac{1}{4_1}(\overset{1}{4}x^3)$

$y' = x^5 - x^3$

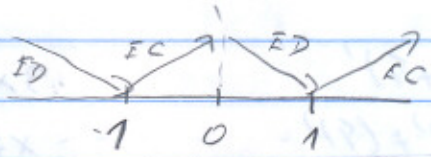
$x^5 - x^3 = x^3(x^2 - 1)$

$x^3(x^2-1) \neq 0$

$x^3 \neq 0 \qquad x^2 - 1 \neq 0$

$x \neq 0 \qquad x^2 \neq 1$

$x \neq \pm 1$



$x = -2 \Rightarrow (-2)^5 - (-2)^3 = -24 < 0 \qquad y' = (-0,8)^5 - (-0,8)^3 = 0,18 > 0 \qquad y' = (0,4)^5 - (0,4)^3 = -0,05 < 0$

$x = -3 \Rightarrow (-3)^5 - (-3)^3 = -216 < 0 \qquad y' = (-0,6)^5 - (-0,6)^3 = 0,13 > 0 \qquad y' = (0,6)^5 - (0,6)^3 = -0,13 < 0$

$x = -4 \Rightarrow (-4)^5 - (-4)^3 = -960 < 0 \qquad y' = (-0,4)^5 - (-0,4)^3 = 0,05 > 0 \qquad y' = (0,8)^5 - (0,8)^3 = -0,18 < 0$

$x = -1$ E $x = 1$ SÃO Pm

$y = \frac{x^6}{6} - \frac{x^4}{4} + 20$

$y = \frac{(-1)^6}{6} - \frac{(-1)^4}{4} + 20$

$y = \frac{1}{6} - \frac{1}{4} + 20$

$y = \frac{2 - 3 + 240}{12}$

$y = \frac{239}{12}$ VALOR MÍNIMO

MESMO VALOR QUANDO SUBSTITUIRMOS POR "1"

$x = 0$ É PM

$y = \frac{x^6}{6} - \frac{x^4}{4} + 20$

$y = \frac{(0)^6}{6} - \frac{(0)^4}{4} + 20$

$y = \frac{0}{6} - \frac{0}{4} + 20$

$y = 20$ VALOR MÁXIMO

$x = (2)^5 - (2)^3 = 24 > 0$

$x = (3)^5 - (3)^3 = 216 > 0$

$x = (4)^5 - (4)^3 = 960 > 0$

10) $y = \frac{1}{x} \quad x \neq 0$

$y' = \left(\frac{1}{x}\right)' \qquad \frac{-1}{x^2} \neq 0$

$y' = (x^{-1})'$

$y' = (-1x^{-1-1})$

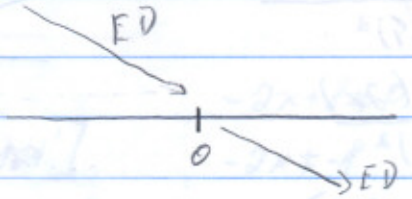
$y' = -x^{-2}$

$y' = -\frac{1}{x^2}$

$y' = \frac{-1}{(-1)^2} = -1 < 0$ $\quad y' = \frac{-1}{(1)^2} = -1 < 0$

$y' = \frac{-1}{(-2)^2} = -0{,}25 < 0$ $\quad y' = \frac{-1}{(2)^2} = -0{,}25 < 0$

$y' = \frac{-1}{(-3)^2} = -0{,}11 < 0$ $\quad y' = \frac{-1}{(3)^2} = -0{,}11 < 0$



E.D EM $\mathbb{R}$; NÃO TEM PM E NEM Pm

11) $y = \frac{1}{x^2} \quad x \neq 0$

$y' = \left(\frac{1}{x^2}\right)' \qquad \frac{-2}{x^3} \neq 0$

$y' = (x^{-2})'$

$y' = -2x^{-2-1}$

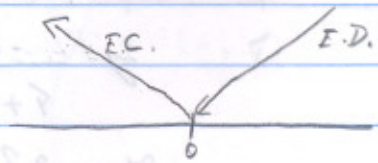
$y' = -2x^{-3}$

$y' = -\frac{2}{x^3}$

$y' = \frac{-2}{(-1)^3} = 2 > 0$ $\quad y' = \frac{-2}{(1)^3} = -2 < 0$

$y' = \frac{-2}{(-2)^3} = 0{,}25 > 0$ $\quad y' = \frac{-2}{(2)^3} = -0{,}25 < 0$

$y' = \frac{-2}{(-3)^3} = 0{,}07 > 0$ $\quad y' = \frac{-2}{(3)^3} = -0{,}07 < 0$



NÃO TEM PM E NEM Pm

12) $y = \frac{x}{x^2+1}$ $\quad x \in \mathbb{R}$

$y' = \left(\frac{x}{x^2+1}\right)'$

$y' = \frac{(x)'(x^2+1) - x(x^2+1)'}{(x^2+1)^2}$

$y' = \frac{1(x^2+1) - x[(x^2)' + (1)']}{(x^2+1)^2}$

$y' = \frac{x^2+1 - x[(2x^{2-1}) + 0]}{(x^2+1)^2}$

$y' = \frac{x^2+1 - x(2x+0)}{(x^2+1)^2}$

$y' = \frac{x^2+1-x(2x)}{(x^2+1)^2}$

$y' = \frac{x^2+1-2x^2}{(x^2+1)^2}$

$y' = \frac{1-x^2}{(x^2+1)^2}$

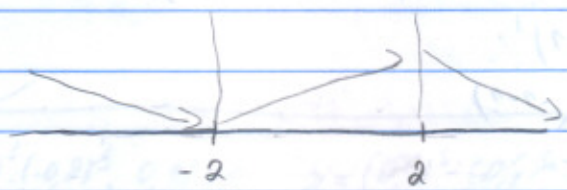
$\frac{1-x^2}{(x^2+1)^2} \neq 0$

$1-x^2 \neq 0 \qquad (x^2+1)^2 \neq 0$

$-x^2 \neq -1$

$x^2 \neq 1$

$x \neq \pm 2$



$x = -2$ É Pm

$y = \frac{x}{x^2+1}$

$y = \frac{-2}{(-2)^2+1}$

$y = \frac{-2}{4+1}$

$y = -\frac{2}{5} \Rightarrow y = -\frac{1}{1}$ VALOR MÍNIMO

$x=2$ É PM

$$y = \frac{x}{x^2+4}$$

$$y = \frac{2}{(2)^2+4}$$

$$y = \frac{2}{4+4}$$

$$y = \frac{2}{8} \Rightarrow y = \frac{1}{4}$$ VALOR MÁXIMO

(13) $$y = \frac{1}{x^2-8x+12}$$

$$y' = \left(\frac{1}{x^2-8x+12}\right)'$$

$$y' = \frac{(1)'(x^2-8x+12)-(1)(x^2-8x+12)'}{(x^2-8x+12)^2}$$

$$y' = \frac{0(x^2-8x+12)-(1)[(x^2)'-(8x)'+(12)']}{(x^2-8x+12)^2}$$

$$y' = \frac{0-1[(2x^{2-1})-8+0]}{(x^2-8x+12)^2}$$

$$y' = \frac{-1(2x-8)}{(x^2-8x+12)^2}$$

$$y' = \frac{-2x+8}{(x^2-8x+12)^2}$$

$$\frac{-2x+8}{(x^2-8x+12)^2} \neq 0$$

$-2x+8 \neq 0$

$-2x \neq -8$

$x \neq 4$

$(x^2-8x+12)^2 \neq 0$

$S=8$ $P=12$ $\Big\}$ 2×6

+ + + + + | − − − − −

4

+ | − | +

2 6

E.C. (+) | E.D. (−) | E.C. (+) | E.D. (−)

2 4 6

$y' > 0 \Rightarrow x < 2$ OU $4 < x < 6$

$y' < 0 \Rightarrow x > 6$ OU $2 < x < 4$

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$x=4$ É P_m $\quad y = \dfrac{1}{x^2-8x+12}$

$$y = \frac{1}{(4)^2-8\cdot 4+12}$$

$$y = \frac{1}{16-32+12}$$

$y = -\frac{1}{4}$ VALOR MÍNIMO

14 $y = \sqrt{x} \qquad x \geq 0$

$y' = (\sqrt{x})'$ $\qquad \frac{1}{2\sqrt{x}} \neq 0$

$y' = (x^{1/2})'$

$y' = \left(\frac{1}{2} x^{1/2-1}\right)$

$y' = \frac{1}{2} x^{-1/2}$

$y' = \frac{1}{2 x^{1/2}}$

$y' = \frac{1}{2\sqrt{x}}$

+ + + | + + + +

0

$y' > 0 \Rightarrow x \in \mathbb{R}^+$

E.C. EM $\mathbb{R}^+$

15 $y = \sqrt{4-x^2} \qquad -2 \leq x \leq 2$

$y' = \left(\sqrt{4-x^2}\right)'$

$y' = \left[(4-x^2)^{1/2}\right]'$

$y' = \frac{1}{2}(4-x^2)^{1/2-1}(4-x^2)'$

$y' = \frac{1}{2}(4-x^2)^{-1/2}\left[(4)'-(x^2)'\right]$

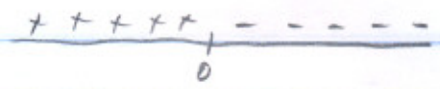
$y' = \frac{1}{2}\frac{1}{(4-x^2)^{1/2}}(0-2x)$

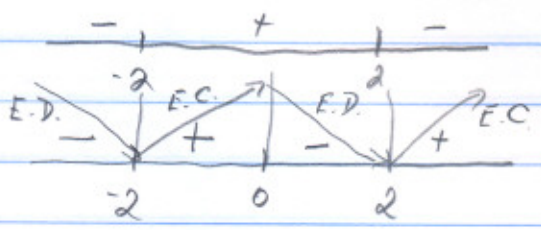
$y' = \frac{1}{2}\frac{1}{1\sqrt{4-x^2}}(-2x)$

$y' = \frac{-x}{\sqrt{4-x^2}}$

$\frac{-x}{\sqrt{1-x^2}} \neq 0$

$x \neq 0$





$x = 0$ É PM

$y = \sqrt{1-x^2}$
$y = \sqrt{1-(0)^2}$
$y = \sqrt{1-0}$
$y = \sqrt{1}$
$y = 2$ VALOR MÁXIMO

$x = -2$ E $x = 2$ SÃO Pm

$y = \sqrt{1-x^2}$
$y = \sqrt{1-(-2)^2}$
$y = \sqrt{1-4}$
$y = \sqrt{0}$
$y = 0$ VALOR MÍNIMO

16) $y = e^{2x}$ $\quad x \in \mathbb{R}$

$y' = (e^{2x})'$
$y' = e^{2x}(2x)'$
$y' = e^{2x} \cdot 2$
$y' = 2e^{2x}$

$2e^{2x} \neq 0$