* PÁGINA 228 DO LIVRO MATEMÁTICA APLICADA À ECONOMIA E ADMINISTRAÇÃO (LOUIS LEITHOLD, EDITORA HARBRA)

6.2 ANTIDIFERENCIAÇÃO

EXERCÍCIOS 6.2 *

1) $\int 3x^4 dx$

$3\int x^4 dx$

$3 \cdot \frac{x^{4+1}}{4+1} + C$

$\frac{3}{5} x^5 + C$

$d' = \left(\frac{3}{5} x^5 + C\right)'$

$d' = \left(\frac{3}{5} x^5\right)' + (C)'$

$d' = \frac{3}{5}(x^5)' + 0$

$d' = \frac{3}{5}(5x^{5-1})$

$d' = \frac{3}{\cancel{5}_1}(\cancel{5}^1 x^4)$

$d' = 3x^4$

2) $\int (3x^5 - 2x^3) dx$

$\int 3x^5 dx - \int 2x^3 dx$

$3\int x^5 dx - 2\int x^3 dx$

$3 \cdot \frac{x^{5+1}}{5+1} - 2 \cdot \frac{x^{3+1}}{3+1} + C$

$\cancel{3}^1 \cdot \frac{x^6}{\cancel{6}_2} - \cancel{2}^1 \cdot \frac{x^4}{\cancel{4}_2} + C$

$\frac{x^6}{2} - \frac{x^4}{2} + C$

$d' = \left(\frac{x^6}{2} - \frac{x^4}{2} + C\right)'$

$d' = \left(\frac{x^6}{2}\right)' - \left(\frac{x^4}{2}\right)' + (C)'$

$d' = \left(\frac{1}{2} x^6\right)' - \left(\frac{1}{2} x^4\right)' + 0$

$d' = \frac{1}{2}(x^6)' - \frac{1}{2}(x^4)'$

$d' = \frac{1}{2}(6x^{6-1}) - \frac{1}{2}(4x^{4-1})$

$d' = \frac{1}{\cancel{2}_1}(\cancel{6}^3 x^5) - \frac{1}{\cancel{2}_1}(\cancel{4}^2 x^3)$

$d' = 3x^5 - 2x^3$

3) $\int (3 - 2t + t^2) dt$

$\int 3 dt - \int 2t\, dt + \int t^2 dt$

$3\int dt - 2\int t\, dt + \int t^2 dt$

$d' = \left(3t - t^2 + \frac{t^3}{3} + C\right)'$

$d' = (3t)' - (t^2)' + \left(\frac{t^3}{3}\right)' + (C)'$

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$3t - 2\cdot\frac{t^{1+1}}{1+1} + \frac{t^{2+1}}{2+1} + C$

$3t - \cancel{2}\cdot\frac{t^2}{\cancel{2}} + \frac{t^3}{3} + C$

$3t - t^2 + \frac{t^3}{3} + C$

$d' = 3 - 2t^{2-1} + \left(\frac{1}{3}t^3\right)' + 0$

$d' = 3 - 2t + \frac{1}{3}(t^3)'$

$d' = 3 - 2t + \frac{1}{3}(3t^{3-1})$

$d' = 3 - 2t + \frac{1}{\cancel{3}}(\cancel{3}t^2)$

$d' = 3 - 2t + t^2$

(4) $\int (4x^3 - 3x^2 + 6x - 1)\,dx$

$\int 4x^3dx - \int 3x^2dx + \int 6x\,dx - \int 1\,dx$

$4\int x^3dx - 3\int x^2dx + 6\int x\,dx - 1\int dx$

$4\cdot\frac{x^{3+1}}{3+1} - 3\cdot\frac{x^{2+1}}{2+1} + 6\cdot\frac{x^{1+1}}{1+1} - x + C$

$\cancel{4}\cdot\frac{x^4}{\cancel{4}} - \cancel{3}\cdot\frac{x^3}{\cancel{3}} + \cancel{6}^{3}\cdot\frac{x^2}{\cancel{2}} - x + C$

$x^4 - x^3 + 3x^2 - x + C$

$d' = (x^4 - x^3 + 3x^2 - x + C)'$

$d' = (x^4)' - (x^3)' + (3x^2)' - (x)' + (C)'$

$d' = (4x^{4-1}) - (3x^{3-1}) + 3(x^2)' - 1 + 0$

$d' = 4x^3 - 3x^2 + 3(2x^{2-1}) - 1$

$d' = 4x^3 - 3x^2 + 6x - 1$

(5) $\int (8x^4 + 4x^3 - 6x^2 - 4x + 5)\,dx$

$\int 8x^4dx - \int 4x^3dx - \int 6x^2dx - \int 4x\,dx + \int 5\,dx$

$8\int x^4dx - 4\int x^3dx - 6\int x^2dx - 4\int x\,dx + 5\int dx$

$$8.\frac{x^{4+1}}{4+1} - 4.\frac{x^{3+1}}{3+1} - 6.\frac{x^{2+1}}{2+1} - 4.\frac{x^{1+1}}{1+1} + 5x + C$$

$$8.\frac{x^{5}}{5} - \cancel{4}^{1}.\frac{x^{4}}{\cancel{4}_{1}} - \cancel{6}^{2}.\frac{x^{3}}{\cancel{3}_{1}} - \cancel{4}^{2}.\frac{x^{2}}{\cancel{2}_{1}} + 5x + C$$

$$\frac{8}{5}x^5 - x^4 - 2x^3 - 2x^2 + 5x + C$$

6) $\int (2 + 3x^2 - 8x^3)\,dx$

$$\int 2\,dx + \int 3x^2\,dx - \int 8x^3\,dx$$

$$2\int dx + 3\int x^2\,dx - 8\int x^3\,dx$$

$$2x + 3.\frac{x^{2+1}}{2+1} - 8.\frac{x^{3+1}}{3+1} + C$$

$$2x + \cancel{3}^{1}.\frac{x^{3}}{\cancel{3}_{1}} - \cancel{8}^{2}.\frac{x^{4}}{\cancel{4}_{1}} + C$$

$$2x + x^3 - 2x^4 + C$$

7) $\int \sqrt{x}\,(x+1)\,dx$

$$\int x^{1/2}(x+1)\,dx$$

$$\int (x^{3/2} + x^{1/2})\,dx$$

$$\int x^{3/2}\,dx + \int x^{1/2}\,dx$$

$$\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$\frac{x^{5/2}}{5/2} + \frac{x^{3/2}}{3/2} + C$$

$$\frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} + C$$

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08 $\int (ax^2 + bx + c)\,dx$

$\int ax^2\,dx + \int bx\,dx + \int c\,dx$

$a\int x^2\,dx + b\int x\,dx + c\int dx$

$a \cdot \frac{x^{2+1}}{2+1} + b \cdot \frac{x^{1+1}}{1+1} + c \cdot x + C$

$a \cdot \frac{x^3}{3} + b \cdot \frac{x^2}{2} + cx + C$

$\frac{a}{3} x^3 + \frac{b}{2} x^2 + cx + C$

09 $\int (x^{3/2} - x)\,dx$

$\int x^{3/2}\,dx - \int x\,dx$

$\frac{x^{3/2+1}}{3/2+1} - \frac{x^{1+1}}{1+1} + C$

$\frac{x^{5/2}}{5/2} - \frac{x^2}{2} + C$

$\frac{2}{5} x^{5/2} - \frac{x^2}{2} + C$

10 $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) dx$

$\int \sqrt{x}\,dx - \int \frac{1}{\sqrt{x}}\,dx$

$\int x^{1/2}\,dx - \int \frac{1}{x^{1/2}}\,dx$

$\int x^{1/2}\,dx - \int x^{-1/2}\,dx$

$\rightarrow \frac{x^{1/2+1}}{1/2+1} - \frac{x^{-1/2+1}}{-1/2+1} + C$

$\frac{x^{3/2}}{3/2} - \frac{x^{1/2}}{1/2} + C$

$\frac{2}{3} x^{3/2} - 2x^{1/2} + C$

11) $\int \left(\frac{2}{x^3} + \frac{3}{x^2} + 5 \right) dx$

$\int \frac{2}{x^3} dx + \int \frac{3}{x^2} dx + \int 5 dx$

$\int 2x^{-3} dx + \int 3x^{-2} dx + \int 5 dx$

$2\int x^{-3} dx + 3\int x^{-2} dx + 5\int dx$

$2 \cdot \frac{x^{-3+1}}{-3+1} + 3 \cdot \frac{x^{-2+1}}{-2+1} + 5x + C$

$2 \cdot \frac{x^{-2}}{-2} + 3 \cdot \frac{x^{-1}}{-1} + 5x + C$

$-x^{-2} - 3x^{-1} + 5x + C$

$-\frac{1}{x^2} - \frac{3}{x} + 5x + C$

12) $\int \left(3 - \frac{1}{x^4} + \frac{1}{x^2} \right) dx$

$\int 3 dx - \int \frac{1}{x^4} dx + \int \frac{1}{x^2} dx$

$3\int dx - \int x^{-4} dx + \int x^{-2} dx$

$3x - \frac{x^{-4+1}}{-4+1} + \frac{x^{-2+1}}{-2+1} + C$

$3x - \frac{x^{-3}}{-3} + \frac{x^{-1}}{-1} + C$

$3x + \frac{x^{-3}}{3} - x^{-1} + C$

$3x + \frac{1}{3x^3} - \frac{1}{x} + C$

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(13) $\int \frac{x^2+4x-4}{\sqrt{x}}\,dx$

$\int\left(\frac{x^2}{\sqrt{x}}+\frac{4x}{\sqrt{x}}-\frac{4}{\sqrt{x}}\right)dx$

$\int \frac{x^2}{\sqrt{x}}\,dx+\int \frac{4x}{\sqrt{x}}\,dx-\int \frac{4}{\sqrt{x}}\,dx$

$\int \frac{x^2}{x^{1/2}}\,dx+\int \frac{4x}{x^{1/2}}\,dx-\int \frac{4}{x^{1/2}}\,dx$

$\int x^{3/2}dx+\int 4x^{1/2}dx-\int 4x^{-1/2}dx$

$\int x^{3/2}dx+4\int x^{1/2}dx-4\int x^{-1/2}dx$

$\frac{x^{3/2+1}}{3/2+1}+4.\frac{x^{1/2+1}}{1/2+1}-4.\frac{x^{-1/2+1}}{-1/2+1}+C$

$\frac{x^{5/2}}{5/2}+4.\frac{x^{3/2}}{3/2}-4.\frac{x^{1/2}}{1/2}+C$

$\frac{2}{5}x^{5/2}+4.\frac{2}{3}x^{3/2}-4.2x^{1/2}+C$

$\frac{2}{5}x^{5/2}+\frac{8}{3}x^{3/2}-8x^{1/2}+C$

(14) $\int \frac{y^4+2y^2-1}{\sqrt{y}}\,dy$

$\int\left(\frac{y^4}{\sqrt{y}}+\frac{2y^2}{\sqrt{y}}-\frac{1}{\sqrt{y}}\right)dy$

$\int \frac{y^4}{\sqrt{y}}\,dy+\int \frac{2y^2}{\sqrt{y}}\,dy-\int \frac{1}{\sqrt{y}}\,dy$

$\int \frac{y^4}{y^{1/2}}\,dy+\int \frac{2y^2}{y^{1/2}}\,dy-\int \frac{1}{y^{1/2}}\,dy$

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$\int y^{7/2}dy + \int 2y^{3/2}dy - \int y^{-1/2}dy$

$\int y^{7/2}dy + 2\int y^{3/2}dy - \int y^{-1/2}dy$

$\frac{y^{7/2+1}}{7/2+1} + 2 \cdot \frac{y^{3/2+1}}{3/2+1} - \frac{y^{-1/2+1}}{-1/2+1} + C$

$\frac{y^{9/2}}{9/2} + 2 \cdot \frac{y^{5/2}}{5/2} - \frac{y^{1/2}}{1/2} + C$

$\frac{2}{9}y^{9/2} + 2 \cdot \frac{2}{5}y^{5/2} - 2y^{1/2} + C$

$\frac{2}{9}y^{9/2} + \frac{4}{5}y^{5/2} - 2y^{1/2} + C$

(15) $\int \left(\sqrt{2x} - \frac{1}{\sqrt{2x}}\right)dx$

$\int \sqrt{2x}\,dx - \int \frac{1}{\sqrt{2x}}\,dx$

$\int (2x)^{1/2}dx - \int \frac{1}{(2x)^{1/2}}\,dx$

$\int (2x)^{1/2}dx - \int (2x)^{-1/2}dx$

$\frac{(2x)^{1/2+1}}{1/2+1} - \frac{(2x)^{-1/2+1}}{-1/2+1} + C$

$\frac{(2x)^{3/2}}{3/2} - \frac{(2x)^{1/2}}{1/2} + C$

$\frac{2}{3}(2x)^{3/2} - 2(2x)^{1/2} + C$

(16) $\int \frac{27t^3 - 1}{\sqrt[3]{t}}\,dt$

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$\int \frac{27t^3}{\sqrt[3]{t}} dt - \int \frac{1}{\sqrt[3]{t}} dt$

$\int \frac{27t^3}{t^{1/3}} dt - \int \frac{1}{t^{1/3}} dt$

$\int 27t^{8/3} dt - \int t^{-1/3} dt$

$27\int t^{8/3} dt - \int t^{-1/3} dt$

$27 \cdot \frac{t^{8/3+1}}{8/3+1} - \frac{t^{-1/3+1}}{-1/3+1} + C$

$27 \cdot \frac{t^{11/3}}{11/3} - \frac{t^{2/3}}{2/3} + C$

$27 \cdot \frac{3}{11} t^{11/3} - \frac{3}{2} t^{2/3} + C$

$\frac{81}{11} t^{11/3} - \frac{3}{2} t^{2/3} + C$

17) $\int \sqrt{1-4y}\, dy$ $\qquad u = 1-4y$ $\qquad \frac{du}{dy} = -4$

$\int \sqrt{u} \left(-\frac{du}{4}\right)$ $\qquad du = -4dy$

$dy = \frac{-du}{4}$

$\int -\frac{1}{4} u^{1/2} du$

$-\frac{1}{4} \int u^{1/2} du \Rightarrow -\frac{1}{4} \frac{u^{1/2+1}}{1/2+1} + C \Rightarrow -\frac{1}{4} \frac{u^{3/2}}{3/2} \Rightarrow -\frac{1}{4_2} \cdot \frac{\cancel{2}}{3} u^{3/2} \Rightarrow -\frac{1}{6} u^{3/2} \Rightarrow -\frac{1}{6}(1-4y)^{3/2}$

18) $\int \sqrt[3]{3x-4}\, dx$ $\qquad u = 3x-4$ $\qquad \frac{du}{dx} = 3$

$\int \sqrt[3]{u} \frac{du}{3}$ $\qquad du = 3dx$

$dx = \frac{du}{3}$

$\int \frac{1}{3} u^{1/3} du$

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$\frac{1}{3}\int u^{1/3}\,du$

$\frac{1}{3}\,\frac{u^{1/3+1}}{1/3+1}+C$

$\frac{1}{3}\,\frac{u^{4/3}}{4/3}+C$

$\frac{1}{\cancel{3}_1}\cdot\frac{\cancel{3}^1}{4}u^{4/3}+C$

$\frac{1}{4}u^{4/3}+C$

$\frac{1}{4}(3x-4)^{4/3}+C$

19) $\int \sqrt[3]{6-2x}\,dx$ $\quad u=6-2x \quad \frac{du}{dx}=-2$

$du=-2\,dx$

$dx=\frac{du}{-2}$

$\int \sqrt[3]{u}\,\frac{du}{-2}$

$\int -\frac{1}{2}u^{1/3}\,du$

$-\frac{1}{2}\int u^{1/3}\,du$

$-\frac{1}{2}\,\frac{u^{1/3+1}}{1/3+1}+C$

$-\frac{1}{2}\,\frac{u^{4/3}}{4/3}+C$

$-\frac{1}{2}\cdot\frac{3}{4}u^{4/3}+C$

$-\frac{3}{8}u^{4/3}+C$

$-\frac{3}{8}(6-2x)^{4/3}+C$

20) $\int \sqrt{5r+1}\,dr$ $\quad u=5r+1 \quad \frac{du}{dr}=5$

$du=5\,dr$

$dr=\frac{du}{5}$

$\int \sqrt{u}\,\frac{du}{5}$

$\int \frac{1}{5}u^{1/2}\,du$

$\frac{1}{5}\int u^{1/2}\,du$

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$\frac{1}{5}\frac{u^{1/2+1}}{1/2+1}+C$ $\to$ $\frac{1}{5}\frac{2}{3}u^{3/2}+C$ $\to$ $\frac{2}{15}(5n+1)^{3/2}+C$

$\frac{1}{5}\cdot\frac{u^{3/2}}{3/2}+C$ $\to$ $\frac{2}{15}u^{3/2}+C$

(21) $\int x\sqrt{x^2-9}\,dx$ $\qquad u=x^2-9$ $\qquad \frac{du}{dx}=2x$

$\int\sqrt{u}\,\frac{du}{2}$ $\qquad du=2x\,dx$

$\int\frac{1}{2}u^{1/2}du$ $\qquad x\,dx=\frac{du}{2}$

$\frac{1}{2}\int u^{1/2}du$ $\to$ $\frac{1}{\cancel{2}_1}\cdot\frac{\cancel{2}^1}{3}u^{3/2}+C$

$\frac{1}{2}\frac{u^{1/2+1}}{1/2+1}+C$ $\to$ $\frac{1}{3}u^{3/2}+C$

$\frac{1}{2}\frac{u^{3/2}}{3/2}+C$ $\to$ $\frac{1}{3}(x^2-9)^{3/2}+C$

(22) $\int 3x\sqrt{4-x^2}\,dx$ $\qquad u=4-x^2$ $\qquad \frac{du}{dx}=-2x$

$3\int x\sqrt{4-x^2}\,dx$ $\qquad du=-2x\,dx$

$3\int\sqrt{u}\,\frac{du}{-2}$ $\qquad x\,dx=\frac{du}{-2}$

$3\int-\frac{1}{2}u^{1/2}du$ $\to$ $-\frac{\cancel{3}^1}{\cancel{2}_1}\cdot\frac{\cancel{2}^1}{\cancel{3}_1}u^{3/2}+C$

$3\left(-\frac{1}{2}\right)\int u^{1/2}du$ $\to$ $-u^{3/2}+C$

$-(4-x^2)^{3/2}+C$

$-\frac{3}{2}\frac{u^{1/2+1}}{1/2+1}+C$

$-\frac{3}{2}\frac{u^{3/2}}{3/2}+C$

23) $\int x^2 \sqrt{x^3-1}\,dx$ $\qquad u = x^3-1 \qquad \frac{du}{dx} = 3x^2$

$du = 3x^2\,dx$

$x^2\,dx = \frac{du}{3}$

$\int \sqrt{u}\,\frac{du}{3}$

$\int \frac{1}{3} u^{1/2}\,du$

$\frac{1}{3}\int u^{1/2}\,du$

$\frac{1}{3}\frac{u^{1/2+1}}{1/2+1} + C$

$\frac{1}{3}\frac{u^{3/2}}{3/2} + C$

$\frac{1}{3}\cdot\frac{2}{3} u^{3/2} + C$

$\frac{2}{9} u^{3/2} + C$

$\frac{2}{9}(x^3-1)^{3/2} + C$

24) $\int x(2x^2+1)^6\,dx$ $\qquad u = 2x^2+1 \qquad \frac{du}{dx} = 4x$

$du = 4x\,dx$

$x\,dx = \frac{du}{4}$

$\int (u)^6 \frac{du}{4}$

$\int \frac{1}{4} u^6\,du$

$\frac{1}{4}\int u^6\,du$

$\frac{1}{4}\frac{u^{6+1}}{6+1} + C$

$\frac{1}{4}\frac{u^7}{7} + C$

$\frac{1}{28} u^7 + C$

$\frac{1}{28}(2x^2+1)^7 + C$

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(23) $\int 5x\sqrt[3]{(9-4x^2)^2}\,dx$

$u = 9-4x^2$ $\frac{du}{dx} = -8x$

$du = -8x\,dx$

$x\,dx = \frac{du}{-8}$

$5\int x\sqrt[3]{(9-4x^2)^2}\,dx$

$5\int \sqrt[3]{u^2}\,\frac{du}{-8}$

$5\int -\frac{1}{8}u^{1/3}\,du$

$5\left(-\frac{1}{8}\right)\int u^{1/3}\,du$

$-\frac{5}{8}\cdot\frac{u^{1/3+1}}{1/3+1}+C$

$-\frac{5}{8}\cdot\frac{u^{4/3}}{4/3}+C$

$-\frac{5}{8}\cdot\frac{3}{4}u^{4/3}+C$

$\rightarrow -\frac{15}{32}u^{4/3}+C$

$-\frac{15}{32}(9-4x^2)^{4/3}+C$

(26) $\int \frac{x\,dx}{\sqrt[3]{x^2+1}}$

$u = x^2+1$ $\frac{du}{dx} = 2x$

$du = 2x\,dx$

$x\,dx = \frac{du}{2}$

$\int \frac{\frac{du}{2}}{\sqrt[3]{u}}$

$\int \frac{\frac{du}{2}}{u^{1/3}}$

$\int \frac{du}{2}\cdot u^{-1/3}$

$\int \frac{1}{2}\,du\,u^{-1/3}$

$\frac{1}{2}\int du\,u^{-1/3}$

$\frac{1}{2}\frac{u^{-1/3+1}}{-1/3+1}+C$

$\rightarrow \frac{1}{2}\frac{u^{2/3}}{2/3}+C$

$\frac{1}{2}\cdot\frac{3}{2}u^{2/3}+C$

$\frac{3}{4}u^{2/3}+C$

$\frac{3}{4}(x^2+1)^{2/3}+C$

27 $\int \frac{y^3}{(1-2y^4)^5}\,dy$

$u = 1-2y^4 \qquad \frac{du}{dy} = -8y^3$

$du = -8y^3\,dy$

$y^3\,dy = \frac{du}{-8}$

$\int \frac{\frac{du}{-8}}{u^5}$

$\int \frac{du}{-8}\, u^{-5}$

$\int -\frac{1}{8}\, u^{-5}\,du$

$-\frac{1}{8} \int u^{-5}\,du$

$-\frac{1}{8}\, \frac{u^{-5+1}}{-5+1}$

$\to -\frac{1}{8}\, \frac{u^{-4}}{-4}$

$\frac{1}{32}\, u^{-4}$

$\frac{1}{32}\,(1-2y^4)^{-4}$

$\frac{1}{32(1-2y^4)^4}$

28 $\int (x^2-4x+4)^{4/3}\,dx$

$\int [(x-2)^2]^{4/3}\,dx \qquad u = x-2 \qquad \frac{du}{dx} = 1$

$\int (x-2)^{8/3}\,dx \qquad du = dx$

$\int u^{8/3}\,du$

$\frac{u^{8/3+1}}{8/3+1} + C$

$\frac{u^{11/3}}{11/3} + C$

$\frac{3}{11}\, u^{11/3} + C$

$\frac{3}{11}\,(x-2)^{11/3} + C$

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(29) $\int \frac{s\,ds}{\sqrt{3s^2+1}}$

$u = 3s^2+1 \quad \frac{du}{ds} = 6s$

$du = 6s\,ds$

$s\,ds = \frac{du}{6}$

$\int \frac{\frac{du}{6}}{\sqrt{u}}$

$\int \frac{\frac{du}{6}}{u^{1/2}}$

$\int u^{-1/2} \frac{du}{6}$

$\int \frac{1}{6} u^{-1/2} du$

$\frac{1}{6} \int u^{-1/2} du$

$\frac{1}{6} \frac{u^{-1/2+1}}{-1/2+1} + C$

$\frac{1}{6} \frac{u^{1/2}}{1/2} + C$

$\frac{1}{\cancel{6}_3} \cancel{2} u^{1/2} + C$

$\frac{1}{3} u^{1/2} + C$

$\frac{1}{3} (3s^2+1)^{1/2} + C$

(30) $\int \frac{2r\,dr}{(1-r)^{2/3}}$

$u = 1-r \quad \frac{du}{dr} = -1$

$r = 1-u$

$du = -dr$

$dr = -du$

$\int \frac{2(1-u)(-du)}{u^{2/3}}$

$2\int \frac{(1-u)(-du)}{u^{2/3}}$

$2\int (-1)(1-u) u^{-2/3} du$

$2(-1)\int (1-u) u^{-2/3} du$

$-2\int (u^{-2/3} - u^{1/3}) du$

$-2\left(\int u^{-2/3} du - \int u^{1/3} du\right)$

$-2\left(\frac{u^{-2/3+1}}{-2/3+1} - \frac{u^{1/3+1}}{1/3+1}\right) + C$

$-2\left(\frac{u^{1/3}}{1/3} - \frac{u^{4/3}}{4/3}\right) + C$

$-2\left(3u^{1/3} - \frac{3}{4} u^{4/3}\right) + C$

$-6u^{1/3} + \frac{3}{2} u^{4/3} + C$

$-6(1-r)^{1/3} + \frac{3}{2}(1-r)^{4/3} + C$

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31) $\int \frac{x\,dx}{\sqrt{x+3}}$ $\quad u = x+3 \quad x = u-3 \qquad \frac{du}{dx} = 1 \quad du = dx$

$\int \frac{(u-3)\,du}{\sqrt{u}}$

$\int \frac{(u-3)\,du}{u^{1/2}}$

$\int (u-3)u^{-1/2}\,du$

$\int (u^{1/2} - 3u^{-1/2})\,du$

$\int u^{1/2}du - \int 3u^{-1/2}du$

$\int u^{1/2}du - 3\int u^{-1/2}du$

$\frac{u^{1/2+1}}{1/2+1} - 3\frac{u^{-1/2+1}}{-1/2+1} + C$

$\frac{u^{3/2}}{3/2} - 3\frac{u^{1/2}}{1/2} + C$

$\frac{2}{3}u^{3/2} - 3.2\,u^{1/2} + C$

$\frac{2}{3}u^{3/2} - 6u^{1/2} + C$

$\frac{2}{3}(x+3)^{3/2} - 6(x+3)^{1/2} + C$

32) $\int \sqrt{\frac{1}{x} - 1}\,\frac{dx}{x^2}$ $\quad u = \frac{1}{x} - 1 \qquad \frac{du}{dx} = \frac{-1}{x^2} \quad du = -\frac{dx}{x^2} \quad \frac{dx}{x^2} = -du$

$\int \sqrt{u}\,(-du)$

$\int -1\,u^{1/2}du$

$-1\int u^{1/2}du$

$-1\,\frac{u^{1/2+1}}{1/2+1} + C$

$-1\,\frac{u^{3/2}}{3/2} + C$

$-1.\frac{2}{3}u^{3/2} + C$

$-\frac{2}{3}u^{3/2} + C$

$-\frac{2}{3}\left(\frac{1}{x} - 1\right)^{3/2} + C$

33) $\int \sqrt{1+\frac{1}{3x}}\,\frac{dx}{x^2}$ $\qquad u = 1+\frac{1}{3x}$ $\qquad \frac{du}{dx} = -\frac{3}{x^2}$

$du = -\frac{3dx}{x^2}$

$\int \sqrt{u}\,\frac{du}{-3}$

$\frac{dx}{x^2} = \frac{du}{-3}$

$\int -\frac{1}{3} u^{1/2} du$

$-\frac{1}{3}\int u^{1/2} du$

$-\frac{1}{3}\,\frac{u^{1/2+1}}{1/2+1} + C$

$-\frac{1}{3}\,\frac{u^{3/2}}{3/2} + C$

$-\frac{1}{3}\cdot\frac{2}{3} u^{3/2} + C$

$-\frac{2}{9} u^{3/2} + C$

$-\frac{2}{9}\left(1+\frac{1}{3x}\right)^{3/2} + C$

34) $\int x^4 \sqrt{3x^5-5}\,dx$ $\qquad u = 3x^5-5$ $\qquad \frac{du}{dx} = 15x^4$

$\int \sqrt{u}\,\frac{du}{15}$

$du = 15x^4 dx$

$x^4 dx = \frac{du}{15}$

$\int \frac{1}{15} u^{1/2} du$

$\frac{1}{15}\int u^{1/2} du$

$\frac{1}{15}\,\frac{u^{1/2+1}}{1/2+1} + C$

$\frac{1}{15}\,\frac{u^{3/2}}{3/2} + C$

$\frac{1}{15}\cdot\frac{2}{3} u^{3/2} + C$

$\frac{2}{45} u^{3/2} + C$

$\frac{2}{45}(3x^5-5)^{3/2} + C$

35 $\int \sqrt{3-x}\, x^2 dx$ $\qquad u = 3-x \qquad \frac{du}{dx} = -1$

$x = 3-u \qquad du = -dx$

$dx = -du$

$\int \sqrt{u}\,(3-u)^2(-du)$

$\int -1\, u^{1/2}(9-6u+u^2)\,du$

$-1\int u^{1/2}(9-6u+u^2)\,du$

$-\int(9u^{1/2}-6u^{3/2}+u^{5/2})\,du$

$-\left(\int 9u^{1/2}du - \int 6u^{3/2}du + \int u^{5/2}du\right)$

$-\left(9\int u^{1/2}du - 6\int u^{3/2}du + \int u^{5/2}du\right)$

$-\left(9 \cdot \frac{u^{1/2+1}}{1/2+1} - 6 \cdot \frac{u^{3/2+1}}{3/2+1} + \frac{u^{5/2+1}}{5/2+1}\right) + C$

$-\left(9 \cdot \frac{u^{3/2}}{3/2} - 6 \cdot \frac{u^{5/2}}{5/2} + \frac{u^{7/2}}{7/2}\right) + C$

$-\left(\overset{3}{\cancel{9}} \cdot \frac{2}{\cancel{3}_1} u^{3/2} - 6 \cdot \frac{2}{5} u^{5/2} + \frac{2}{7} u^{7/2}\right) + C$

$-\left(6u^{3/2} - \frac{12}{5}u^{5/2} + \frac{2}{7}u^{7/2}\right) + C$

$-6(3-x)^{3/2} + \frac{12}{5}(3-x)^{5/2} - \frac{2}{7}(3-x)^{7/2} + C$

36 $\int (x^3+3)^{1/4} x^5 dx$ $\qquad u = x^3+3 \qquad \frac{du}{dx} = 3x^2$

$x^3 = u-3$

$du = 3x^2 dx$

$x^2 dx = \frac{du}{3}$

$\int (x^3+3)^{1/4} x^2 x^3 dx$

$\int u^{1/4}(u-3)\frac{du}{3}$

$\int \frac{1}{3} u^{1/4}(u-3)\,du$

$\frac{1}{3}\int u^{1/4}(u-3)\,du$

$\frac{1}{3}\int(u^{5/4} - 3u^{1/4})\,du$

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$$\frac{1}{3}\left(\int u^{5/4}\,du - \int 3u^{1/4}\,du\right)$$

$$\frac{1}{3}\left(\int u^{5/4}\,du - 3\int u^{1/4}\,du\right)$$

$$\frac{1}{3}\left(\frac{u^{5/4+1}}{5/4+1} - 3\,\frac{u^{1/4+1}}{1/4+1}\right)+C$$

$$\frac{1}{3}\left(\frac{u^{9/4}}{9/4} - 3\,\frac{u^{5/4}}{5/4}\right)+C$$

$$\frac{1}{3}\left(\frac{4}{9}u^{9/4} - 3\cdot\frac{4}{5}u^{5/4}\right)+C$$

$$\frac{1}{3}\left(\frac{4}{9}u^{9/4} - \frac{12}{5}u^{5/4}\right)+C$$

$$\frac{4}{27}u^{9/4} - \frac{12}{15}u^{5/4}+C$$

$$\frac{4}{27}(x^3+3)^{9/4} - \frac{\cancel{12}^{\,4}}{\cancel{15}_{\,5}}(x^3+3)^{5/4}+C$$

$$\frac{4}{27}(x^3+3)^{9/4} - \frac{4}{5}(x^3+3)^{5/4}+C$$

37 $$\int \frac{(x^2+2x)\,dx}{\sqrt{x^3+3x^2+1}}$$

$u = x^3+3x^2+1$

$$\frac{du}{dx} = 3x^2+6x$$

$$\frac{du}{dx} = 3(x^2+2x)$$

$$du = 3(x^2+2x)\,dx$$

$$(x^2+2x)\,dx = \frac{du}{3}$$

$$\int \frac{\frac{du}{3}}{\sqrt{u}}$$

$$\int \frac{\frac{du}{3}}{u^{1/2}}$$

$$\int u^{-1/2}\,\frac{du}{3}$$

$$\int \frac{1}{3}u^{-1/2}\,du$$

$$\frac{1}{3}\int u^{-1/2}\,du$$

$$\frac{1}{3}\,\frac{u^{-1/2+1}}{-1/2+1}+C$$

$$\frac{1}{3}\,\frac{u^{1/2}}{1/2}+C$$

$$\frac{1}{3}\cdot 2\,u^{1/2}+C$$

$$\frac{2}{3}u^{1/2}+C$$

$$\frac{2}{3}(x^3+3x^2+1)^{1/2}+C$$

38) $\int x(x^2+1)\sqrt{4-2x^2-x^4}\,dx$

$u = 4-2x^2-x^4$

$\frac{du}{dx} = -4x-4x^3$

$\frac{du}{dx} = -4x(1+x^2)$

$du = -4x(x^2+1)dx$

$x(x^2+1)dx = \frac{du}{-4}$

$\int \sqrt{u}\,\frac{du}{-4}$

$\int -\frac{1}{4} u^{1/2} du$

$-\frac{1}{4}\int u^{1/2} du$

$-\frac{1}{4}\frac{u^{1/2+1}}{1/2+1}+C$

$-\frac{1}{4}\frac{u^{3/2}}{3/2}+C$

$-\frac{1}{\cancel{4}_2}\cdot\frac{\cancel{2}}{3} u^{3/2}+C$

$-\frac{1}{6} u^{3/2}+C$

$-\frac{1}{6}(4-2x^2-x^4)^{3/2}+C$

39) $\int \frac{x(3x^2+1)dx}{(3x^4+2x^2+1)^2}$

$u = 3x^4+2x^2+1$

$\frac{du}{dx} = 12x^3+4x$

$\frac{du}{dx} = 4x(3x^2+1)$

$du = 4x(3x^2+1)dx$

$x(3x^2+1)dx = \frac{du}{4}$

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$\int \frac{\frac{du}{4}}{u^2}$

$\int u^{-2} \frac{du}{4}$

$\int \frac{1}{4} u^{-2} du$

$\frac{1}{4} \int u^{-2} du$

$\frac{1}{4} \frac{u^{-2+1}}{-2+1} + C$

$\frac{1}{4} \frac{u^{-1}}{-1} + C$

$-\frac{1}{4} u^{-1} + C$

$-\frac{1}{4} (3x^4 + 2x^2 + 1)^{-1} + C$

$\frac{-1}{4(3x^4+2x^2+1)} + C$

40) $\int \sqrt{3+s}\,(s+1)^2\, ds$

$u = 3 + s \qquad \frac{du}{ds} = 1$

$s = u - 3$

$du = ds$

$\int \sqrt{u}\,(u-3+1)^2\, du$

$\int u^{1/2} (u-2)^2\, du$

$\int u^{1/2} (u^2 - 4u + 4)\, du$

$\int (u^{5/2} - 4u^{3/2} + 4u^{1/2})\, du$

$\int u^{5/2} du - \int 4u^{3/2} du + \int 4u^{1/2} du$

$\int u^{5/2} du - 4\int u^{3/2} du + 4\int u^{1/2} du$

$\frac{u^{5/2+1}}{5/2+1} - 4\frac{u^{3/2+1}}{3/2+1} + 4\frac{u^{1/2+1}}{1/2+1} + C$

$\frac{u^{7/2}}{7/2} - 4\frac{u^{5/2}}{5/2} + 4\frac{u^{3/2}}{3/2} + C$

$\frac{2}{7} u^{7/2} - 4\frac{2}{5} u^{5/2} + 4 \cdot \frac{2}{3} u^{3/2} + C$

$\frac{2}{7} (3+s)^{7/2} - \frac{8}{5} (3+s)^{5/2} + \frac{8}{3} (3+s)^{3/2} + C$

41) $\int \frac{y+3}{(3-y)^{2/3}}\,dy$

$u = 3-y$ $\quad \frac{du}{dy} = -1$

$y = 3-u$ $\quad du = -dy$

$dy = -du$

$\int \frac{3-u+3}{u^{2/3}}(-du)$

$\int -1\,\frac{6-u}{u^{2/3}}\,du$

$-1\int (6-u)u^{-2/3}\,du$

$-\int (6u^{-2/3}-u^{1/3})\,du$

$-(\int 6u^{-2/3}\,du - \int u^{1/3}\,du)$

$-(6\int u^{-2/3}\,du - \int u^{1/3}\,du)$

$-\left(6\frac{u^{-2/3+1}}{-2/3+1} - \frac{u^{1/3+1}}{1/3+1}\right)+C$

$-\left(6\frac{u^{1/3}}{1/3} - \frac{u^{4/3}}{4/3}\right)+C$

$-\left(6\cdot 3u^{1/3} - \frac{3}{4}u^{4/3}\right)+C$

$-\left(18u^{1/3} - \frac{3}{4}u^{4/3}\right)+C \rightarrow -18(3-y)^{1/3}+\frac{3}{4}(3-y)^{4/3}+C$

42) $\int (2t^2+1)^{1/3}t^3\,dt$

$u = 2t^2+1$ $\quad \frac{du}{dt} = 4t$

$2t^2 = u-1$

$t^2 = \frac{u-1}{2}$ $\quad du = 4t\,dt$

$t\,dt = \frac{du}{4}$

$\int (2t^2+1)^{1/3}\,t\,t^2\,dt$

$\int u^{1/3}\left(\frac{u-1}{2}\right)\frac{du}{4}$

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$\int \frac{1}{8} u^{1/3}(u-1)\,du$

$\frac{1}{8}\int u^{1/3}(u-1)\,du$

$\frac{1}{8}\int (u^{4/3}-u^{1/3})\,du$

$\frac{1}{8}\left(\int u^{4/3}du - \int u^{1/3}du\right)$

$\frac{1}{8}\left(\frac{u^{4/3+1}}{4/3+1} - \frac{u^{1/3+1}}{1/3+1}\right)+C$

$\frac{1}{8}\left(\frac{u^{7/3}}{7/3} - \frac{u^{4/3}}{4/3}\right)+C$

$\frac{1}{8}\left(\frac{3}{7}u^{7/3} - \frac{3}{4}u^{4/3}\right)+C$

$\frac{3}{56}u^{7/3} - \frac{3}{32}u^{4/3}+C$

$\frac{3}{56}(2t^2+1)^{7/3} - \frac{3}{32}(2t^2+1)^{4/3}+C$

43) $\int \frac{(r^{1/3}+2)^4}{\sqrt[3]{r^2}}\,dr$

$u = r^{1/3}+2$

$\frac{du}{dr} = \frac{1}{3}r^{-2/3}$

$\frac{du}{dr} = \frac{1}{3r^{2/3}}$

$\frac{du}{dr} = \frac{1}{3\sqrt[3]{r^2}}$

$du = \frac{dr}{3\sqrt[3]{r^2}}$

$\frac{dr}{\sqrt[3]{r^2}} = 3\,du$

$\int u^4\, 3\,du$

$3\int u^4\,du$

$3\frac{u^{4+1}}{4+1}+C$

$3\frac{u^5}{5}+C$

$\frac{3(r^{1/3}+2)^5}{5}+C$

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44) $\int \left(t+\frac{1}{t}\right)^{3/2} \left(\frac{t^2-1}{t^2}\right) dt$

$u = t+\frac{1}{t}$ $\qquad \frac{du}{dt} = 1-\frac{1}{t^2}$

$\frac{du}{dt} = \frac{t^2-1}{t^2}$

$du = \left(\frac{t^2-1}{t^2}\right) dt$

$\int u^{3/2} du$

$\frac{u^{3/2+1}}{3/2+1} + C$

$\frac{u^{5/2}}{5/2} + C$

$\frac{2}{5} u^{5/2} + C \rightarrow \frac{2}{5}\left(t+\frac{1}{t}\right)^{5/2} + C$

45) $\int \frac{x^3}{(x^2+4)^{3/2}} dx$

$u = x^2+4$ $\qquad \frac{du}{dx} = 2x$

$x^2 = u-4$

$du = 2x\,dx$

$x\,dx = \frac{du}{2}$

$\int \frac{x x^2}{(x^2+4)^{3/2}} dx$

$\int \frac{(u-4)\frac{du}{2}}{u^{3/2}}$

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$$\int (u-4)u^{-3/2}\,\frac{du}{2} \qquad \to \frac{1}{2}\left(\frac{u^{-1/2+1}}{-1/2+1} - 4\frac{u^{-3/2+1}}{-3/2+1}\right)+C$$

$$\int \frac{1}{2}(u-4)u^{-3/2}\,du \qquad \frac{1}{2}\left(\frac{u^{1/2}}{1/2} - 4\frac{u^{-1/2}}{-1/2}\right)+C$$

$$\frac{1}{2}\int (u-4)u^{-3/2}\,du \qquad \frac{1}{2}\left(2u^{1/2} - 4(-2)u^{-1/2}\right)+C$$

$$\frac{1}{2}\int (u^{-1/2} - 4u^{-3/2})\,du \qquad \frac{1}{2}\left(2u^{1/2} + 8u^{-1/2}\right)+C$$

$$\frac{1}{2}\left(\int u^{-1/2}du - \int 4u^{-3/2}du\right) \qquad u^{1/2} + 4u^{-1/2} + C$$

$$\frac{1}{2}\left(\int u^{-1/2}du - 4\int u^{-3/2}du\right) \qquad u^{1/2} + \frac{4}{u^{1/2}} + C$$

$$(x^2+4)^{1/2} + \frac{4}{(x^2+4)^{1/2}} + C$$

46 $\displaystyle\int \frac{x^3}{\sqrt{1-2x^2}}\,dx$

$u = 1-2x^2 \qquad \frac{du}{dx} = -4x$

$2x^2 = 1-u \qquad du = -4x\,dx$

$x^2 = \frac{1-u}{2} \qquad x\,dx = \frac{du}{-4}$

$$\int \frac{x\,x^2}{\sqrt{1-2x^2}}\,dx$$

$$\int \frac{\left(\frac{1-u}{2}\right)\left(\frac{du}{-4}\right)}{\sqrt{u}}$$

$$\int \frac{\left(\frac{1-u}{2}\right)\left(\frac{du}{-4}\right)}{u^{1/2}}$$

$$\int \left(\frac{1-u}{2}\right)\left(\frac{du}{-4}\right)u^{-1/2}$$

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$$\int -\frac{1}{8}(1-u)u^{-1/2}\,du$$

$$-\frac{1}{8}\int (1-u)u^{-1/2}\,du$$

$$-\frac{1}{8}\int (u^{-1/2}-u^{1/2})\,du$$

$$-\frac{1}{8}\left(\int u^{-1/2}du-\int u^{1/2}du\right)$$

$$-\frac{1}{8}\left(\frac{u^{-1/2+1}}{-1/2+1}-\frac{u^{1/2+1}}{1/2+1}\right)+C$$

$$-\frac{1}{8}\left(\frac{u^{1/2}}{1/2}-\frac{u^{3/2}}{3/2}\right)+C$$

$$-\frac{1}{8}\left(2u^{1/2}-\frac{2}{3}u^{3/2}\right)+C$$

$$-\frac{1}{4}u^{1/2}+\frac{1}{12}u^{3/2}+C$$

$$-\frac{1}{4}(1-2x^2)^{1/2}+\frac{1}{12}(1-2x^2)^{3/2}+C$$