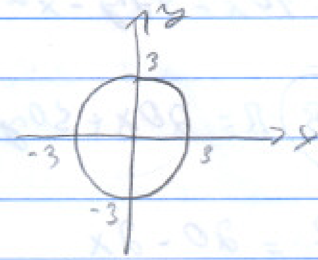
* PÁGINA 277 DO LIVRO MATEMÁTICA APLICADA À ECONOMIA (LÍLIA LADEIRA VERAS, 3ª EDIÇÃO, EDITORA ATLAS)

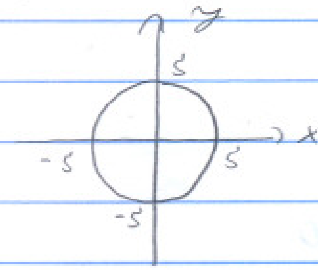
4.4 DERIVADAS PARCIAIS APLICADAS À ECONOMIA (LÍLIA VERAS)*

1.A

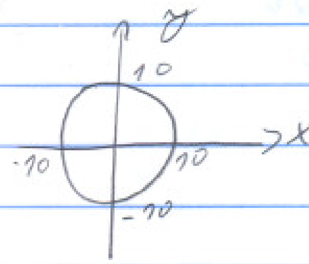
$$C = x^2 + y^2 + 20$$
$$29 = x^2 + y^2 + 20$$
$$29 - 20 = x^2 + y^2 \rightarrow 9 = x^2 + y^2$$
$$x^2 + y^2 = 9$$

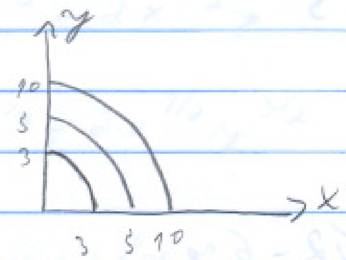


$$C = x^2 + y^2 + 20$$
$$45 = x^2 + y^2 + 20$$
$$45 - 20 = x^2 + y^2 \rightarrow 25 = x^2 + y^2$$
$$x^2 + y^2 = 25$$



$$C = x^2 + y^2 + 20$$
$$120 = x^2 + y^2 + 20$$
$$120 - 20 = x^2 + y^2 \rightarrow 100 = x^2 + y^2$$
$$x^2 + y^2 = 100$$





1.B

$$\frac{\partial C}{\partial x} = 2x \qquad \frac{\partial C}{\partial y} = 2y$$

2.A

$$R = x(27 - x - 2y) + y(22 - 3x - y)$$
$$= 27x - x^2 - 2xy + 22y - 3xy - y^2$$
$$= 27x - x^2 + 22y - y^2 - 5xy$$

$$\frac{\partial R}{\partial x} = 27 - 2x - 5y \qquad \frac{\partial R}{\partial y} = 22 - 2y - 5x$$

2B $L = R - C$

$= 27x - x^2 + 22y - y^2 - 3xy - (2x^2 + y^2)$

$= 27x - x^2 + 22y - y^2 - 3xy - 2x^2 - y^2$

$= 27x - 3x^2 + 22y - 2y^2 - 3xy$

$\frac{\partial L_m}{\partial x} = 27 - 6x - 3y$ $\frac{\partial L_m}{\partial y} = 22 - 4y - 3x$

3.A $\frac{\partial P}{\partial L} = 1{,}01\left[(L^{3/4})(K^{1/4})' + (K^{1/4})(L^{3/4})'\right]$

$= 1{,}01\left[(L^{3/4})(0) + (K^{1/4})\left(\frac{3}{4}L^{-1/4}\right)\right]$

$= 1{,}01\left[0 + (K^{1/4})\left(\frac{3L^{-1/4}}{4}\right)\right]$

$= 1{,}01\left(\frac{3}{4}K^{1/4}L^{-1/4}\right)$

$\frac{\partial P}{\partial L} = 1{,}01(0{,}75\,K^{1/4}L^{-1/4})$

$= 0{,}7575\,K^{1/4}L^{-1/4}$

$\frac{\partial P}{\partial K} = 1{,}01\left[(L^{3/4})(K^{1/4})' + (K^{1/4})(L^{3/4})'\right]$

$= 1{,}01\left[(L^{3/4})\left(\frac{1}{4}K^{-3/4}\right) + (K^{1/4})(0)\right]$

$= 1{,}01\left(\frac{1}{4}K^{-3/4}L^{3/4} + 0\right)$

$= 1{,}01(0{,}25\,K^{-3/4}L^{3/4})$

$= 0{,}2525\,K^{-3/4}L^{3/4}$

3.B $P = f(L, K)$

$f(L, K) = 1{,}01\,L^{3/4}K^{1/4}$

$f(\kappa L, \kappa K) = 1{,}01(\kappa L)^{3/4}(\kappa K)^{1/4}$

$f(\kappa L, \kappa K) = 1{,}01\,\kappa^{3/4}L^{3/4}\kappa^{1/4}K^{1/4}$

$= 1{,}01\,\kappa L^{3/4}K^{1/4}$

$= \kappa(1{,}01\,L^{3/4}K^{1/4})$

$= \kappa P$ → $m = 1$ (HOMOGENEIDADE DE GRAU 1)

$$\frac{\partial P}{\partial L} = f(K,L)$$

$$f(K,L) = 0{,}7575\,K^{1/4}L^{-1/4}$$

$$f(kK,kL) = 0{,}7575(kK)^{1/4}(kL)^{-1/4}$$

$$= 0{,}7575\,k^{1/4}K^{1/4}k^{-1/4}L^{-1/4}$$

$$= 0{,}7575\,k^{0}K^{1/4}L^{-1/4}$$

$$= k^{0}\left(0{,}7575\,K^{1/4}L^{-1/4}\right)$$

$$= k^{0}\frac{\partial P}{\partial L}$$

$\to m = 0$ (HOMOGENEIDADE DE GRAU ZERO)

$$\frac{\partial P}{\partial K} = f(K,L)$$

$$f(K,L) = 0{,}2525\,K^{-3/4}L^{3/4}$$

$$f(kK,kL) = 0{,}2525(kK)^{-3/4}(kL)^{3/4}$$

$$= 0{,}2525\,k^{-3/4}K^{-3/4}k^{3/4}L^{3/4}$$

$$= 0{,}2525\,k^{0}K^{-3/4}L^{3/4}$$

$$= k^{0}\left(0{,}2525\,K^{-3/4}L^{3/4}\right)$$

$$= k^{0}\frac{\partial P}{\partial K}$$

$\to m = 0$ (HOMOGENEIDADE DE GRAU ZERO)

5.A $z = f(x,y)$

$$f(x,y) = x^{0{,}25}y^{0{,}75}$$

$$f(kx,ky) = (kx)^{0{,}25}(ky)^{0{,}75}$$

$$= k^{0{,}25}x^{0{,}25}k^{0{,}75}y^{0{,}75}$$

$$= k\,x^{0{,}25}y^{0{,}75}$$

$$= kz$$

$\to m = 1$ (HOMOGENEIDADE DE GRAU 1)

4.B $\frac{\partial z}{\partial x} = 0{,}25x^{-0{,}75}y^{0{,}75}$ $\frac{\partial z}{\partial y} = 0{,}75x^{0{,}25}y^{-0{,}25}$

4.C $z = \frac{\partial z}{\partial x}x + \frac{\partial z}{\partial y}y$

$= (0{,}25x^{-0{,}75}y^{0{,}75})x + (0{,}75x^{0{,}25}y^{-0{,}25})y$

$= 0{,}25x^{0{,}25}y^{0{,}75} + 0{,}75x^{0{,}25}y^{0{,}75}$

$= 1x^{0{,}25}y^{0{,}75}$

$= x^{0{,}25}y^{0{,}75}$

5.A $\frac{\partial P}{\partial x} = 2x + 2y$ $\frac{\partial P}{\partial y} = 2x + 2y$

AO NÍVEL DE $x=5$ E $y=4$

$\frac{\partial P}{\partial x} = 2(5)+2(4)$ $\frac{\partial P}{\partial y} = 2(5)+2(4)$

$= 10+8$ $= 10+8$

$= 18$ $= 18$

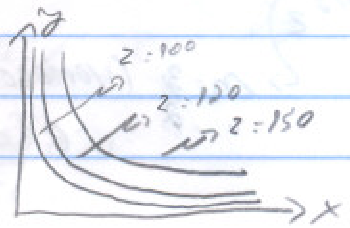
5.B 9 5.C 3,6 5.D 12,6

6.A $z = 10x^{1/2}y$ $z = 10x^{1/2}y$ $z = 10x^{1/2}y$

$100 = 10x^{1/2}y$ $120 = 10x^{1/2}y$ $130 = 10x^{1/2}y$

$\frac{100}{10} = x^{1/2}y$ $\frac{120}{10} = x^{1/2}y$ $\frac{130}{10} = x^{1/2}y$

$10 = x^{1/2}y$ $12 = x^{1/2}y$ $13 = x^{1/2}y$



6-B $\frac{\partial z}{\partial x} = 10\left[(x^{1/2})(y)' + (y)(x^{1/2})'\right]$

$= 10\left[(x^{1/2})(0) + (y)\left(\frac{1}{2}x^{-1/2}\right)\right]$

$= 10\left(0 + \frac{1}{2}x^{-1/2}y\right)$

$\frac{\partial z}{\partial x} = 10\left(\frac{1}{2}x^{-1/2}y\right)$

$= 5x^{-1/2}y$

$\frac{\partial z}{\partial y} = 10\left[(x^{1/2})(y)' + (y)(x^{1/2})'\right]$

$= 10\left[(x^{1/2})(1) + (y)(0)\right]$

$= 10(x^{1/2} + 0)$

$\frac{\partial z}{\partial y} = 10(x^{1/2})$

$= 10x^{1/2}$

AO NÍVEL $x = 4$ E $y = 6$

$\frac{\partial z}{\partial x} = 5x^{-1/2}y$

$= 5(4)^{-1/2}6$

$= \frac{30}{4^{1/2}}$

$= \frac{30}{\sqrt{4}}$

$= \frac{30}{2} = 15$

$\frac{\partial z}{\partial y} = 10x^{1/2}$

$= 10(4)^{1/2}$

$= 10\sqrt{4}$

$= 10(2)$

$= 20$

6-C $z = f(x,y)$

$f(x,y) = 10x^{1/2}y$

$f(kx,ky) = 10(kx)^{1/2}ky)$

$= 10k^{1/2}x^{1/2}ky$

$= 10k^{3/2}x^{1/2}y$

$f(kx,ky) = k^{3/2}(10x^{1/2}y)$

$= k^{3/2}z$

$\to m = \frac{3}{2}$ (HOMOGENEIDADE DE GRAU $\frac{3}{2}$)

$\frac{\partial z}{\partial x} = f(x,y)$

$f(x,y) = 3x^{-1/2}y$

$f(Kx,Ky) = 3(Kx)^{-1/2}Ky$

$= 3K^{-1/2}x^{-1/2}Ky$

$= 3K^{1/2}x^{-1/2}y$

$= K^{1/2}(3x^{-1/2}y)$

$f(Kx,Ky) = K^{1/2}\frac{\partial z}{\partial x} \rightarrow m = \frac{1}{2}$ (HOMOGENEIDADE DE GRAU $\frac{1}{2}$)

$\frac{\partial z}{\partial y} = f(x)$

$f(x) = 10x^{1/2}$

$f(Kx) = 10(Kx)^{1/2}$

$= 10K^{1/2}x^{1/2}$

$= K^{1/2}(10x^{1/2})$

$= K^{1/2}\frac{\partial z}{\partial y} \rightarrow m = \frac{1}{2}$ (HOMOGENEIDADE DE GRAU $\frac{1}{2}$)

6.D

$z = 10x^{1/2}y$

$= 10(4)^{1/2}6$

$= 60.2$

$= 120$

$z = 10x^{1/2}y$

$= 10(16)^{1/2}24$

$= 240.4$

$= 960$

A PRODUÇÃO FICA MULTIPLICADA POR 8.

$\frac{\partial z}{\partial x} = 3x^{-1/2}y$

$= 3(16)^{-1/2}24$

$= \frac{120}{16^{1/2}} = \frac{120}{4} = 30$

$\frac{\partial z}{\partial y} = 10x^{1/2}$

$= 10(16)^{1/2}$

$= 10(4) = 40$

AS PRODUÇÕES MARGINAIS FICAM MULTIPLICADAS POR 2.

12/1

6.E RETORNOS CRESCENTES NA PRODUÇÃO, PORQUE A FUNÇÃO É HOMOGÊNEA DE GRAU MAIOR QUE UM.

12 A
$$\frac{\partial q_1}{\partial p_1} = \frac{1}{2}(20-p_1)^{-1/2}(20-p_1)'$$
$$= \frac{1}{2}(20-p_1)^{-1/2}(-1)$$
$$= -\frac{1}{2}(20-p_1)^{-1/2}$$

$$\frac{\partial q_2}{\partial p_2} = -2$$

$$\frac{Eq_1}{Ep_1} = \frac{p_1}{q_1} \cdot \frac{\partial q_1}{\partial p_1}$$
$$= \frac{p_1}{(20-p_1)^{1/2}} \cdot -\frac{1}{2}(20-p_1)^{-1/2}$$
$$= -\frac{1}{2}\frac{p_1}{(20-p_1)} = \frac{-p_1}{40-2p_1}$$

$$\frac{Eq_2}{Ep_2} = \frac{p_2}{q_2} \cdot \frac{\partial q_2}{\partial p_2}$$
$$= \frac{p_2}{10-2p_2} \cdot -2$$
$$= \frac{-2p_2}{10-2p_2} = \frac{-\cancel{2}p_2}{\cancel{2}(5-p_2)} = \frac{-p_2}{5-p_2}$$

12 B $R = p \cdot q$

$q_1 = (20 - p_1)^{1/2}$

$q_1 = \sqrt{20 - p_1}$

$(q_1)^2 = (\sqrt{20 - p_1})^2$

$q_1^2 = 20 - p_1$

$p_1 = 20 - q_1^2$

$q_2 = 10 - 2p_2$

$2p_2 = 10 - q_2$

$p_2 = \frac{10 - q_2}{2}$

$p_2 = 5 - \frac{q_2}{2}$

$R = (20 - q_1^2)q_1 + \left(5 - \frac{q_2}{2}\right)q_2$

$R = 20q_1 - q_1^3 + 5q_2 - \frac{q_2^2}{2}$

$\frac{\partial R}{\partial q_1} = 20 - 3q_1^2$

$\frac{\partial R}{\partial q_2} = 5 - \frac{1}{2}(2q_2)$

$= 5 - q_2$

13.A

$q_1 = -2p_1 + 4p_2 + 10$

$20 = -2p_1 + 4p_2 + 10$

$20 - 10 = -2(p_1 - 2p_2)$

$10 = -2(p_1 - 2p_2)$

$\frac{10}{-2} = p_1 - 2p_2$

$p_1 - 2p_2 = -5$

$\Downarrow$

$0 - 2p_2 = -5$

$p_2 = \frac{-5}{-2}$

$p_2 = 2,5$

$q_1 = -2p_1 + 4p_2 + 10$

$30 = -2p_1 + 4p_2 + 10$

$30 - 10 = -2(p_1 - 2p_2)$

$20 = -2(p_1 - 2p_2)$

$\frac{20}{-2} = p_1 - 2p_2$

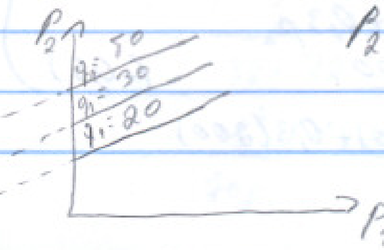
$p_1 - 2p_2 = -10$

$\Downarrow$

$0 - 2p_2 = -10$

$p_2 = \frac{-10}{-2}$

$p_2 = 5$



$q_1 = -2p_1 + 4p_2 + 10$

$40 = -2p_1 + 4p_2 + 10$

$40 - 10 = -2(p_1 - 2p_2)$

$30 = -2(p_1 - 2p_2)$

$\frac{30}{-2} = p_1 - 2p_2$

$p_1 - 2p_2 = -15$

$\Downarrow$

$0 - 2p_2 = -15$

$p_2 = \frac{-15}{-2}$

$p_2 = 7,5$

126.1

13.B $\frac{\partial q_1}{\partial p_1} = -2 \qquad \frac{\partial q_1}{\partial p_2} = 4$

$$\frac{Eq_1}{Ep_1} = \frac{p_1}{q_1} \cdot \frac{\partial q_1}{\partial p_1}$$
$$= \frac{p_1}{-2p_1+4p_2+10}(-2)$$
$$= \frac{2}{-2(2)+4(3)+10}(-2)$$
$$= \frac{2}{-4+12+10}(-2)$$

$$\frac{Eq_1}{Ep_1} = \frac{\cancel{2}^{1}}{\cancel{18}_{9}}(-2)$$
$$= \frac{1}{9}(-2)$$
$$= -\frac{2}{9}$$

$$\frac{Eq_1}{Ep_2} = \frac{p_2}{q_1} \cdot \frac{\partial q_1}{\partial p_2}$$
$$= \frac{p_2}{-2p_1+4p_2+10}(4)$$
$$= \frac{3}{-2(2)+4(3)+10}(4)$$
$$= \frac{3}{-4+12+10}(4)$$

$$\frac{Eq_1}{Ep_2} = \frac{\cancel{3}^{1}}{\cancel{18}_{6}}(4)$$
$$= \frac{1(\cancel{4}^{2})}{\cancel{6}_{3}}$$
$$= \frac{2}{3}$$

14.A $\frac{\partial q_1}{\partial p_1} = -0{,}6$

$$\frac{Eq_1}{Ep_1} = \frac{p_1}{q_1} \cdot \frac{\partial q_1}{\partial p_1}$$
$$= \frac{p_1}{200-0{,}6p_1+0{,}3p_2}(-0{,}6)$$
$$= \frac{300}{200-0{,}6(300)+0{,}3(200)}(-0{,}6)$$

$$\frac{Eq_1}{Ep_1} = \frac{300}{200-180+60}(-0{,}6)$$
$$= \frac{30\cancel{0}}{8\cancel{0}}(-0{,}6)$$
$$= 3{,}75(-0{,}6)$$
$$= -2{,}25$$

14 B) $\dfrac{\partial q_1}{\partial p_2} = 0{,}3$

$\dfrac{Eq_1}{Ep_2} = \dfrac{p_2}{q_1}\dfrac{\partial q_1}{\partial p_2}$

$= \dfrac{p_2}{200-0{,}6p_1+0{,}3p_2}(0{,}3)$

$= \dfrac{200}{200-0{,}6(300)+0{,}3(200)}(0{,}3)$

$= \dfrac{200}{200-180+60}(0{,}3)$

$\dfrac{Eq_1}{Ep_2} = \dfrac{20\cancel{0}}{8\cancel{0}}(0{,}3)$

$= 2{,}5(0{,}3)$

$= 0{,}75$

14 C) SUBSTITUTOS, PORQUE A ELASTICIDADE CRUZADA É POSITIVA.

15. A) $q_1 = -2p_1^2 - p_2 + 77$

$\dfrac{\partial q_1}{\partial p_1} = -4p_1$ $\qquad$ $\dfrac{\partial q_1}{\partial p_2} = -1$

$\dfrac{Eq_1}{Ep_1} = \dfrac{p_1}{q_1}\dfrac{\partial q_1}{\partial p_1}$

$= \dfrac{p_1}{-2p_1^2-p_2+77}(-4p_1)$

$= \dfrac{4}{-2(4)^2-3+77}[-4(4)]$

$= \dfrac{4}{-2(16)+72}(-16)$

$= \dfrac{4}{-32+72}(-16)$

$= \dfrac{\cancel{4}^{1}}{\cancel{40}_{\cancel{10}_5}}(-\cancel{16}^{8}) = -\dfrac{8}{5} = -1{,}6$

$\dfrac{Eq_2}{Ep_2} = \dfrac{p_2}{q_1}\dfrac{\partial q_1}{\partial q_2}$

$= \dfrac{p_2}{-2p_1^2-p_2+77}(-1)$

$= \dfrac{3}{-2(4)^2-3+77}(-1)$

$= \dfrac{3}{-2(16)+72}(-1)$

$= \dfrac{3}{-32+72}(-1)$

$= \dfrac{\cancel{5}^{1}}{\cancel{40}_8}(-1) = -\dfrac{1}{8} = -0{,}125$

12B)

$q_2 = -p_1 - 3p_2 + 39$

$\frac{\partial q_2}{\partial p_1} = -1$

$\frac{\partial q_2}{\partial p_2} = -3$

$\frac{Eq_2}{Ep_1} = \frac{p_1}{q_2}\frac{\partial q_2}{\partial p_1}$

$= \frac{p_1}{-p_1-3p_2+39}(-1)$

$= \frac{4}{-4-3(5)+39}(-1)$

$= \frac{4}{-19+39}(-1)$

$= \frac{4^1}{20_5}(-1)$

$= -\frac{1}{5} = -0,2$

$\frac{Eq_2}{Ep_2} = \frac{p_2}{q_2}\frac{\partial q_2}{\partial p_2}$

$= \frac{p_2}{-p_1-3p_2+39}(-3)$

$= \frac{5}{-4-3(5)+39}(-3)$

$= \frac{5}{-19+39}(-3)$

$= \frac{5^1}{20_4}(-3)$

$= \frac{-3}{4} = -0,75$

13B) q_1 SOFRERÁ UM DECRÉSCIMO DE 16%, E q_2 SOFRERÁ UM DECRÉSCIMO DE 2%.