9.1 INTEGRAÇÃO POR PARTES ⇒ PÁGINA 387 DO LIVRO MATEMÁTICA APLICADA À ECONOMIA E ADMINISTRAÇÃO (LOUIS LEITHOLD, EDITORA HARBRA)

EXERCÍCIOS 9.1

1) $\int x e^{3x} dx$

$u = x \qquad \frac{du}{dx} = 1 \qquad du = dx$

$dv = e^{3x} dx$

$\int dv = \int e^{3x} dx$

$v = \int e^{3x} dx \Rightarrow v = \frac{1}{3} e^{3x}$

$u' = 3x \qquad \frac{du'}{dx} = 3 \qquad du' = 3dx \qquad dx = \frac{du'}{3}$

$\int e^{u'} \frac{du'}{3} \Rightarrow \frac{1}{3} \int e^{u'} du'$

$\int \frac{1}{3} e^{u'} du' \qquad \frac{1}{3} e^{u'} \Rightarrow \frac{1}{3} e^{3x}$

FORONI

$\int x e^{3x} dx = x\left(\frac{1}{3} e^{3x}\right) - \int \frac{1}{3} e^{3x} du$

$\int x e^{3x} dx = \frac{1}{3} x e^{3x} - \frac{1}{3} \int e^{3x} dx$

$u'' = 3x \quad \frac{du''}{dx} = 3 \qquad \int e^{u''} \frac{du''}{3}$

$du'' = 3dx \qquad \int \frac{1}{3} e^{u''} du''$

$dx = \frac{du''}{3} \qquad \frac{1}{3} \int e^{u''} du''$

$\frac{1}{3} e^{u''}$

$\frac{1}{3} e^{3x}$

$\int x e^{3x} dx = \frac{1}{3} x e^{3x} - \frac{1}{3}\left(\frac{1}{3} e^{3x}\right) + C$

$\int x e^{3x} dx = \frac{1}{3} x e^{3x} - \frac{1}{9} e^{3x} + C$

2) $\int x e^{-2x} dx$

$u = x \quad \frac{du}{dx} = 1 \qquad dv = e^{-2x} dx$

$\int dv = \int e^{-2x} dx$

$du = dx \qquad v = -\frac{1}{2} e^{-2x}$

$u' = -2x \quad \frac{du'}{dx} = -2 \qquad \int e^{u'} \frac{du'}{-2}$

$du' = -2dx \qquad \int -\frac{1}{2} e^{u'} du'$

$dx = \frac{du'}{-2} \qquad -\frac{1}{2} \int e^{u'} du'$

$-\frac{1}{2} e^{u'} \Rightarrow -\frac{1}{2} e^{-2x}$

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$$\int x e^{-2x} dx = x\left(\frac{-1}{2} e^{-2x}\right) - \int \frac{-1}{2} e^{-2x} du$$

$$\int x e^{-2x} dx = -\frac{1}{2} x e^{-2x} - \left(\frac{-1}{2}\right) \int e^{-2x} dx$$

$$\int x e^{-2x} dx = -\frac{1}{2} x e^{-2x} + \frac{1}{2} \int e^{-2x} dx$$

$u'' = -2x \qquad \frac{du''}{dx} = -2 \qquad \int e^{u''} \frac{du''}{-2}$

$du'' = -2dx \qquad \int \frac{-1}{2} e^{u''} du''$

$dx = \frac{du''}{-2} \qquad -\frac{1}{2} \int e^{u''} du''$

$\left.\right\} \to -\frac{1}{2} e^{u''}$

$-\frac{1}{2} e^{-2x}$

$$\int x e^{-2x} dx = -\frac{1}{2} x e^{-2x} + \frac{1}{2}\left(-\frac{1}{2} e^{-2x}\right) + C$$

$$\int x e^{-2x} dx = -\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x} + C$$

3) $\int x 3^x dx$

$u = x \qquad \frac{du}{dx} = 1 \qquad dv = 3^x dx$

$du = dx \qquad \int dv = \int 3^x dx$

$v = \frac{3^x}{\ln 3}$

$u' = x \qquad \frac{du'}{dx} = 1 \qquad \int 3^{u'} du'$

$du' = dx \qquad \frac{3^{u'}}{\ln 3}$

$\frac{3^x}{\ln 3}$

$\int x\, 3^x dx = x \dfrac{3^x}{\ln 3} - \int \dfrac{3^x}{\ln 3} du$

$\int x\, 3^x dx = x \dfrac{3^x}{\ln 3} - \int \dfrac{1}{\ln 3} 3^x dx$

$\int x\, 3^x dx = x \dfrac{3^x}{\ln 3} - \dfrac{1}{\ln 3} \int 3^x dx$

$u'' = x \quad \dfrac{du''}{dx} = 1 \quad \int 3^{u''} du''$

$du'' = dx \quad \dfrac{3^{u''}}{\ln 3} \rightarrow \dfrac{3^x}{\ln 3}$

$\int x\, 3^x dx = x \dfrac{3^x}{\ln 3} - \dfrac{1}{\ln 3} \left(\dfrac{3^x}{\ln 3} \right) + C$

$\int x\, 3^x dx = x \dfrac{3^x}{\ln 3} - \dfrac{3^x}{(\ln 3)^2} + C$

1) $\int x\, 10^{x/2} dx$

$u = x \quad \dfrac{du}{dx} = 1 \quad dv = 10^{x/2} dx$

$du = dx \quad \int dv = \int 10^{x/2} dx$

$v = \dfrac{2}{\ln 10} \cdot 10^{x/2}$

$u' = \dfrac{x}{2} \quad \dfrac{du'}{dx} = \dfrac{1}{2} \quad \int 10^{u'} 2\, du'$

$du' = \dfrac{dx}{2} \quad 2 \int 10^{u'} du'$

$2 \dfrac{10^{u'}}{\ln 10}$

$dx = 2\, du' \quad \dfrac{2}{\ln 10} 10^{x/2}$

1961

$$\int x\,10^{x/2}dx = x\,\frac{2}{\ln 10}\,10^{x/2} - \int \frac{2}{\ln 10}\,10^{x/2}du$$

$$\int x\,10^{x/2}dx = \frac{2x}{\ln 10}\,10^{x/2} - \frac{2}{\ln 10}\int 10^{x/2}dx$$

$u'' = \frac{x}{2}$ $\quad \frac{du''}{dx} = \frac{1}{2}$ $\quad \int 10^{u''}2\,du''$

$du'' = \frac{dx}{2}$ $\quad 2\int 10^{u''}du''$

$dx = 2\,du''$ $\quad 2\,\frac{10^{u''}}{\ln 10}$

$\frac{2}{\ln 10}\,10^{x/2}$

$$\int x\,10^{x/2}dx = \frac{2x}{\ln 10}\,10^{x/2} - \frac{2}{\ln 10}\left(\frac{2}{\ln 10}\,10^{x/2}\right) + C$$

$$\int x\,10^{x/2}dx = \frac{2x}{\ln 10}\,10^{x/2} - \frac{4}{(\ln 10)^2}\,10^{x/2} + C$$

$$\int x\,10^{x/2}dx = \frac{2x\,10^{x/2}\ln 10 - 4\,10^{x/2}}{(\ln 10)^2} + C$$

$$\int x\,10^{x/2}dx = \frac{2(10^{x/2})(x\ln 10 - 2)}{(\ln 10)^2} + C$$

5) $\int \ln x\,dx$

$u = \ln x$ $\quad \frac{du}{dx} = \frac{1}{x}$ $\quad dv = dx$

$du = \frac{dx}{x}$ $\quad \int dv = \int dx$

$v = x$

$$\int \ln x\,dx = x\ln x - \int x\,du$$

$$\int \ln x\,dx = x\ln x - \int x\,\frac{dx}{x}$$

$\int \ln x \, dx = x \ln x - \int x(x^{-1}) \, dx$

$\int \ln x \, dx = x \ln x - \int x^0 \, dx$

$\int \ln x \, dx = x \ln x - \int 1 \, dx$

$\int \ln x \, dx = x \ln x - \int dx$

$\int \ln x \, dx = x \ln x - x + K$

6) $\int \log_{10} x \, dx$

$\int \frac{\ln x}{\ln 10} \, dx$

$\int \frac{1}{\ln 10} \ln x \, dx$

$\frac{1}{\ln 10} \int \ln x \, dx$

$\int \ln x \, dx$

$u = \ln x \qquad \frac{du}{dx} = \frac{1}{x} \qquad dv = dx$

$\int dv = \int dx$

$du = \frac{dx}{x} \qquad v = x$

$\int \ln x \, dx = x \ln x - \int x \, du$

$\int \ln x \, dx = x \ln x - \int x \frac{dx}{x}$

$\int \ln x \, dx = x \ln x - \int x(x^{-1}) \, dx \quad \int \ln x \, dx = x \ln x - x$

$\int \ln x \, dx = x \ln x - \int x^0 \, dx$

$\int \ln x \, dx = x \ln x - \int 1 \, dx$

$\int \ln x \, dx = x \ln x - \int dx$

1981

$$\int \log_{10} x \, dx = \frac{1}{\ln 10} \int \ln x \, dx$$

$$\int \log_{10} x \, dx = \frac{1}{\ln 10}(x \ln x - x) + C$$

(7) $\int x \log_{10} x \, dx$

$$\int x \frac{\ln x}{\ln 10} \, dx$$

$$\int \frac{1}{\ln 10} x \ln x \, dx$$

$$\frac{1}{\ln 10} \int x \ln x \, dx$$

$$\int x \ln x \, dx$$

$u = \ln x$ $\quad \frac{du}{dx} = \frac{1}{x}$ $\quad dv = x \, dx$

$\int dv = \int x \, dx$

$du = \frac{dx}{x}$ $\quad v = \frac{x^{1+1}}{1+1}$

$v = \frac{x^2}{2}$

$$\int x \ln x \, dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \, du$$

$$\int x \ln x \, dx = \frac{x^2}{2} \ln x - \int \frac{1}{2} x^2 \frac{dx}{x}$$

$$\int x \ln x \, dx = \frac{x^2}{2} \ln x - \frac{1}{2} \int x^2 (x^{-1}) \, dx$$

$$\int x \ln x \, dx = \frac{x^2}{2} \ln x - \frac{1}{2} \int x \, dx$$

$\int x \ln x \, dx = \frac{x^2}{2} \ln x - \frac{1}{2}\left(\frac{x^{1+1}}{1+1}\right)$

$\int x \ln x \, dx = \frac{x^2}{2} \ln x - \frac{1}{2}\left(\frac{x^2}{2}\right)$

$\int x \ln x \, dx = \frac{x^2}{2} \ln x - \frac{x^2}{4}$

$\frac{1}{\ln 10}\left(\frac{x^2 \ln x}{2} - \frac{x^2}{4}\right) + C$

8) $\int \ln 2x^2 \, dx$

$u = \ln 2x^2 \qquad \frac{du}{dx} = \frac{2}{x} \qquad dv = dx$

$\qquad du = \frac{2dx}{x} \qquad \int dv = \int dx$

$\qquad v = x$

$\int \ln 2x^2 dx = x \ln 2x^2 - \int x \, du$

$\int \ln 2x^2 dx = x \ln 2x^2 - \int x \frac{2dx}{x}$

$\int \ln 2x^2 dx = x \ln 2x^2 - 2\int x(x^{-1}) dx$

$\int \ln 2x^2 dx = x \ln 2x^2 - 2\int x^0 dx$

$\int \ln 2x^2 dx = x \ln 2x^2 - 2\int 1 dx$

$\int \ln 2x^2 dx = x \ln 2x^2 - 2\int dx$

$\int \ln 2x^2 dx = x \ln 2x^2 - 2x + C$

9) $\int x^2 \ln x \, dx$

$u = \ln x \qquad \frac{du}{dx} = \frac{1}{x} \qquad dv = x^2 dx$

$\qquad du = \frac{dx}{x} \qquad \int dv = \int x^2 dx$

$\qquad v = \frac{x^{2+1}}{2+1} \Rightarrow v = \frac{x^3}{3}$

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$\int x^2 \ln x\, dx = \frac{x^3}{3} \ln x - \int \frac{x^3}{3} du$

$\int x^2 \ln x\, dx = \frac{x^3}{3} \ln x - \int \frac{1}{3} x^3 \frac{dx}{x}$

$\int x^2 \ln x\, dx = \frac{x^3}{3} \ln x - \frac{1}{3} \int x^3 (x^{-1})\, dx$

$\int x^2 \ln x\, dx = \frac{x^3}{3} \ln x - \frac{1}{3} \int x^2\, dx$

$\int x^2 \ln x\, dx = \frac{x^3}{3} \ln x - \frac{1}{3}\left(\frac{x^{2+1}}{2+1}\right) + C$

$\int x^2 \ln x\, dx = \frac{x^3}{3} \ln x - \frac{1}{3}\left(\frac{x^3}{3}\right) + C$

$\int x^2 \ln x\, dx = \frac{x^3}{3} \ln x - \frac{x^3}{9} + C = \frac{3x^3 \ln x - x^3}{9} + C = \frac{x^3(3 \ln x - 1)}{9} + C$

40) $\int x^2 \log_2 x\, dx$

$\int x^2 \frac{\ln x}{\ln 2}\, dx$

$\int \frac{1}{\ln 2} x^2 \ln x\, dx$

$\frac{1}{\ln 2} \int x^2 \ln x\, dx$

$\int x^2 \ln x\, dx$

$u = \ln x \quad \frac{du}{dx} = \frac{1}{x} \quad du = \frac{dx}{x}$

$dv = x^2 dx$

$\int dv = \int x^2 dx$

$v = \frac{x^{2+1}}{2+1}$

$v = \frac{x^3}{3}$

$$\int x^2 \ln x \, dx = \frac{x^3}{3} \ln x - \int \frac{x^3}{3} \, dx$$

$$\int x^2 \ln x \, dx = \frac{x^3}{3} \ln x - \int \frac{1}{3} x^3 \frac{dx}{x}$$

$$\int x^2 \ln x \, dx = \frac{x^3}{3} \ln x - \frac{1}{3} \int x^3 (x^{-1}) dx$$

$$\int x^2 \ln x \, dx = \frac{x^3}{3} \ln x - \frac{1}{3} \int x^2 dx$$

$$\int x^2 \ln x \, dx = \frac{x^3}{3} \ln x - \frac{1}{3} \left(\frac{x^{2+1}}{2+1} \right)$$

$$\int x^2 \ln x \, dx = \frac{x^3}{3} \ln x - \frac{1}{3} \left(\frac{x^3}{3} \right)$$

$$\int x^2 \ln x \, dx = \frac{x^3}{3} \ln x - \frac{x^3}{9}$$

$$\int x^2 \log_2 x \, dx = \frac{1}{\ln 2} \int x^2 \ln x \, dx$$

$$\int x^2 \log_2 x \, dx = \frac{1}{\ln 2} \left(\frac{x^3}{3} \ln x - \frac{x^3}{9} \right) + C$$

$$\int x^2 \log_2 x \, dx = \frac{1}{\ln 2} \left(\frac{3x^3 \ln x - x^3}{9} \right) + C$$

$$\int x^2 \log_2 x \, dx = \frac{3x^3 \ln x - x^3}{9 \ln 2} + C$$

$$\int x^2 \log_2 x \, dx = \frac{x^3 (3 \ln x - 1)}{9 \ln 2} + C$$

11) $\int (\ln x)^2 dx$

$\int \ln x \ln x \, dx$

$u = \ln x \quad \dfrac{du}{dx} = \dfrac{1}{x} \qquad dv = \ln x \, dx$

$\int dv = \int \ln x \, dx$

$du = \dfrac{dx}{x} \qquad v = x \ln x - x$

$\int \ln x \, dx$

$u' = \ln x \quad \dfrac{du'}{dx} = \dfrac{1}{x} \qquad dv' = dx$

$\int dv' = \int dx$

$du' = \dfrac{dx}{x} \qquad v' = x$

$\int \ln x \, dx = x \ln x - \int x \, du'$

$\int \ln x \, dx = x \ln x - \int x \dfrac{dx}{x}$

$\int \ln x \, dx = x \ln x - \int x (x^{-1}) dx$

$\int \ln x \, dx = x \ln x - \int x^0 dx$

$\int \ln x \, dx = x \ln x - \int 1 dx$

$\int \ln x \, dx = x \ln x - \int dx$

$\int \ln x \, dx = x \ln x - x$

$\int (\ln x)^2 dx = \ln x (x \ln x - x) - \int x \ln x - x \, du$

$\int (\ln x)^2 dx = \ln x (x \ln x - x) - \int x \ln x - x \dfrac{dx}{x}$

$\int x \ln x - x \dfrac{dx}{x}$

$\int x (\ln x - 1) x^{-1} dx \rightarrow \int (\ln x - 1) dx$

$\int x^0 (\ln x - 1) dx$

$\int 1 (\ln x - 1) dx$

$u'' = \ln x - 1$ $\quad \frac{du''}{dx} = \frac{1}{x}$ $\quad dv'' = dx$

$\int dv'' = \int dx$

$du'' = \frac{dx}{x}$ $\quad v'' = x$

$\int(\ln x - 1)dx = x(\ln x - 1) - \int x \, du''$

$\int(\ln x - 1)dx = x(\ln x - 1) - \int x \frac{dx}{x}$

$\int(\ln x - 1)dx = x(\ln x - 1) - \int x(x^{-1})dx$

$\int(\ln x - 1)dx = x(\ln x - 1) - \int x^0 dx$

$\int(\ln x - 1)dx = x(\ln x - 1) - \int 1 dx$

$\int(\ln x - 1)dx = x(\ln x - 1) - \int dx$

$\int(\ln x - 1)dx = x(\ln x - 1) - x$

$\int(\ln x)^2 dx = \ln x(x \ln x - x) - [x(\ln x - 1) - x] + C$

$\int(\ln x)^2 dx = x(\ln x)^2 - x\ln x - [x\ln x - x - x] + C$

$\int(\ln x)^2 dx = x(\ln x)^2 - x\ln x - [x\ln x - 2x] + C$

$\int(\ln x)^2 dx = x(\ln x)^2 - x\ln x - x\ln x + 2x + C$

$\int(\ln x)^2 dx = x(\ln x)^2 - 2x\ln x + 2x + C$

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$\int \ln x (\ln x)^2 dx$

$u = \ln x$ $\quad \frac{du}{dx} = \frac{1}{x}$ $\quad dv = (\ln x)^2 dx$

$\int dv = \int(\ln x)^2 dx$

$du = \frac{dx}{x}$ $\quad v = x(\ln x)^2 - 2x\ln x + 2x$

$\int(\ln x)^2 dx$

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$\int \ln x \ln x \, dx$

$u' = \ln x \quad \frac{du'}{dx} = \frac{1}{x} \quad du' = \frac{dx}{x}$

$dv' = \ln x \, dx$
$\int dv' = \int \ln x \, dx$
$v' = x \ln x - x$

$u'' = \ln x \quad \frac{du''}{dx} = \frac{1}{x} \quad du'' = \frac{dx}{x}$

$dv'' = dx$
$\int dv'' = \int dx$
$v'' = x$

$\int \ln x \, dx = x \ln x - \int x \, du''$
$\int \ln x \, dx = x \ln x - \int x \frac{dx}{x}$
$\int \ln x \, dx = x \ln x - \int x (x^{-1}) dx$
$\int \ln x \, dx = x \ln x - \int x^0 dx$

$\int \ln x \, dx = x \ln x - \int 1 dx$
$\int \ln x \, dx = x \ln x - \int dx$
$\int \ln x \, dx = x \ln x - x$

$\int (\ln x)^2 dx = \ln x (x \ln x - x) - \int x \ln x - x \, du'$
$\int (\ln x)^2 dx = \ln x (x \ln x - x) - \int x \ln x - x \frac{dx}{x}$
$\int (\ln x)^2 dx = \ln x (x \ln x - x) - \int (x \ln x - x) x^{-1} dx$
$\int (\ln x)^2 dx = \ln x (x \ln x - x) - \int (x^0 \ln x - x^0) dx$
$\int (\ln x)^2 dx = \ln x (x \ln x - x) - \int (1 \ln x - 1) dx$
$\int (\ln x)^2 dx = \ln x (x \ln x - x) - \int (\ln x - 1) dx$

$\int (\ln x - 1) dx$

$u''' = \ln x - 1 \quad \frac{du'''}{dx} = \frac{1}{x} \quad du''' = \frac{dx}{x}$

$dv''' = dx$
$\int dv''' = \int dx$
$v''' = x$

$\int(\ln x-1)\,dx = x(\ln x-1) - \int x\, du'''$

$\int(\ln x-1)\,dx = x(\ln x-1) - \int x \frac{dx}{x}$

$\int(\ln x-1)\,dx = x(\ln x-1) - \int x(x^{-1})\,dx \rightarrow \int(\ln x-1)\,dx = x(\ln x-1) - \int dx$

$\int(\ln x-1)\,dx = x(\ln x-1) - \int x^{0}\,dx \quad \int(\ln x-1)\,dx = x(\ln x-1) - x$

$\int(\ln x-1)\,dx = x(\ln x-1) - \int 1\,dx$

$\int(\ln x)^2 dx = \ln x(x\ln x - x) - [x(\ln x-1) - x]$

$\int(\ln x)^2 dx = x(\ln x)^2 - x\ln x - [x\ln x - x - x]$

$\int(\ln x)^2 dx = x(\ln x)^2 - x\ln x - [x\ln x - 2x]$

$\int(\ln x)^2 dx = x(\ln x)^2 - 2x\ln x + 2x$

$\int(\ln x)^3 dx = \ln x(x(\ln x)^2 - 2x\ln x + 2x) - \int(x(\ln x)^2 - 2x\ln x + 2x)\,du$

$\int(\ln x)^3 dx = \ln x(x(\ln x)^2 - 2x\ln x + 2x) - \int(x(\ln x)^2 - 2x\ln x + 2x)\frac{dx}{x}$

$\int(\ln x)^3 dx = x(\ln x)^3 - 2x(\ln x)^2 + 2x\ln x - \int(x(\ln x)^2 - 2x\ln x + 2x)\,x^{-1}dx$

$\int(\ln x)^3 dx = x(\ln x)^3 - 2x(\ln x)^2 + 2x\ln x - \int(x^0(\ln x)^2 - 2x^0\ln x + 2x^0)\,dx$

$\int(\ln x)^3 dx = x(\ln x)^3 - 2x(\ln x)^2 + 2x\ln x - \int[1(\ln x)^2 - 2(1)\ln x + 2(1)]\,dx$

$\int(\ln x)^3 dx = x(\ln x)^3 - 2x(\ln x)^2 + 2x\ln x - \int[(\ln x)^2 - 2\ln x + 2]\,dx$

$\int[(\ln x)^2 - 2\ln x + 2]\,dx = \int(\ln x)^2 dx - \int 2\ln x\,dx + \int 2\,dx$

$\int[(\ln x)^2 - 2\ln x + 2]\,dx = \int(\ln x)^2 dx - 2\int\ln x\,dx + 2\int dx$

$\int[(\ln x)^2 - 2\ln x + 2]\,dx = x(\ln x)^2 - 2x\ln x + 2x - 2[x\ln x - x] + 2x + C$

$\int[(\ln x)^2 - 2\ln x + 2]\,dx = x(\ln x)^2 - 2x\ln x + 2x - 2x\ln x + 2x + 2x + C$

$\int[(\ln x)^2 - 2\ln x + 2]\,dx = x(\ln x)^2 - 4x\ln x + 6x + C$

$\int(\ln x)^3 dx = x(\ln x)^3 - 2x(\ln x)^2 + 2x\ln x - [x(\ln x)^2 - 4x\ln x + 6x] + C$

$\int(\ln x)^3 dx = x(\ln x)^3 - 2x(\ln x)^2 + 2x\ln x - x(\ln x)^2 + 4x\ln x - 6x + C$

$\int(\ln x)^3 dx = x(\ln x)^3 - 3x(\ln x)^2 + 6x\ln x - 6x + C$

(13) $\int x\, a^x\, dx$

$b = x \qquad \frac{db}{dx} = 1$

$db = dx$

$\int b\, a^b\, db$

$u = b \qquad \frac{du}{db} = 1 \qquad dv = a^b\, db$

$du = db \qquad \int dv = \int a^b\, db$

$v = \frac{a^b}{\ln a}$

$\int x\, a^x\, dx = b\, \frac{a^b}{\ln a} - \int \frac{a^b}{\ln a}\, du$

$\int x\, a^x\, dx = b\, \frac{a^b}{\ln a} - \int \frac{1}{\ln a}\, a^b\, db$

$\int x\, a^x\, dx = b\, \frac{a^b}{\ln a} - \frac{1}{\ln a} \int a^b\, db$

$\int a^b\, db = \frac{a^b}{\ln a} + C$

$\int x\, a^x\, dx = b\, \frac{a^b}{\ln a} - \frac{1}{\ln a} \left(\frac{a^b}{\ln a} \right) + C$

$\int x\, a^x\, dx = b\, \frac{a^b}{\ln a} - \frac{a^b}{(\ln a)^2} + C$

$\int x \cdot a^x\, dx = \frac{b\, a^b (\ln a) - a^b}{(\ln a)^2} + C$

$\int x\, a^x\, dx = \frac{a^b (b \ln a - 1)}{(\ln a)^2} + C = \frac{a^x (x \ln a - 1)}{(\ln a)^2} + C$

14) $\int x \log_a x \, dx$

$\int x \frac{\ln x}{\ln a} dx$

$\int \frac{1}{\ln a} (x \ln x) dx$

$\frac{1}{\ln a} \int (x \ln x) dx$

$\int x \ln x \, dx$

$u = \ln x \quad \frac{du}{dx} = \frac{1}{x} \quad du = \frac{dx}{x}$

$dv = x \, dx$

$\int dv = \int x \, dx$

$v = \frac{x^{1+1}}{1+1}$

$v = \frac{x^2}{2}$

$\int x \ln x \, dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} du$

$\int x \ln x \, dx = \frac{x^2}{2} \ln x - \int \frac{1}{2} x^2 \frac{dx}{x}$

$\int x \ln x \, dx = \frac{x^2}{2} \ln x - \frac{1}{2} \int x^2 (x^{-1}) dx$

$\int x \ln x \, dx = \frac{x^2 \ln x}{2} - \frac{1}{2} \int x^1 dx$

$\int x \ln x \, dx = \frac{x^2 \ln x}{2} - \frac{1}{2} \left(\frac{x^{1+1}}{1+1} \right) + C$

$\int x \ln x \, dx = \frac{x^2 \ln x}{2} - \frac{1}{2} \left(\frac{x^2}{2} \right) + C$

$\int x \ln x \, dx = \frac{x^2 \ln x}{2} - \frac{x^2}{4} + C$

$\int x \ln x \, dx = \frac{2x^2 \ln x - x^2}{4} + C$

$\int x \ln x \, dx = \frac{x^2 (2 \ln x - 1)}{4} + C$

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$\int x \log_a x \, dx = \frac{1}{\ln a}\left(\frac{x^2(2\ln x - 1)}{4}\right) + C$

$\int x \log_a x \, dx = \frac{x^2(2\ln x - 1)}{4\ln a} + C$

(13) $\int x^2 e^{-2x} dx$

$u = -2x$ $\quad \frac{du}{dx} = -2$

$x = \frac{u}{-2}$ $\quad du = -2dx$

$dx = \frac{du}{-2}$

$\int \left(\frac{u}{-2}\right)^2 e^u \frac{du}{-2}$

$\int \frac{u^2}{4} e^u \frac{du}{-2}$

$\int -\frac{1}{8} u^2 e^u du$

$-\frac{1}{8} \int u^2 e^u du$

$a = u^2$ $\quad \frac{da}{du} = 2u$ $\quad db = e^u du$

$da = 2u\,du$ $\quad \int db = \int e^u du$

$b = e^u$

$\int u^2 e^u du = u^2 e^u - \int e^u da$

$\int u^2 e^u du = u^2 e^u - \int e^u 2u\,du$

$\int u^2 e^u du = u^2 e^u - 2\int e^u u\,du$

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$a' = u \qquad \frac{da'}{du} = 1 \qquad db' = e^u du$

$\int db' = \int e^u du$

$da' = du \qquad b' = e^u$

$\int e^u u\,du = u e^u - \int e^u da'$

$\int e^u u\,du = u e^u - \int e^u du$

$\int e^u u\,du = u e^u - e^u + C$

$\int u^2 e^u du = u^2 e^u - 2\int e^u u\,du$

$\int u^2 e^u du = u^2 e^u - 2(u e^u - e^u) + C$

$\int u^2 e^u du = u^2 e^u - 2u e^u + 2e^u + C$

$\int x^2 e^{-2x} dx = -\frac{1}{8} \int u^2 e^u du$

$\int x^2 e^{-2x} dx = -\frac{1}{8} (u^2 e^u - 2u e^u + 2e^u) + C$

$\int x^2 e^{-2x} dx = -\frac{1}{8} [(-2x)^2 e^{-2x} - 2(-2x)e^{-2x} + 2e^{-2x}] + C$

$\int x^2 e^{-2x} dx = -\frac{1}{8} [4x^2 e^{-2x} + 4x e^{-2x} + 2e^{-2x}] + C$

$\int x^2 e^{-2x} dx = \frac{-4x^2 e^{-2x} - 4x e^{-2x} - 2e^{-2x}}{8} + C$

$\int x^2 e^{-2x} dx = \frac{-\cancel{2} e^{-2x}(2x^2 + 2x + 1)}{84} + C$

$\int x^2 e^{-2x} dx = \frac{-e^{-2x}(2x^2 + 2x + 1)}{4} + C$

(16) $\int x^3 e^x dx$

$u = x \qquad \frac{du}{dx} = 1 \Rightarrow du = dx$

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$\int u^3 e^u du$

$a = u^3$ $\quad \frac{da}{du} = 3u^2$ $\quad db = e^u du$

$da = 3u^2 du$ $\quad \int db = \int e^u du$

$b = e^u$

$\int u^3 e^u du = u^3 e^u - \int e^u da$

$\int u^3 e^u du = u^3 e^u - \int e^u 3u^2 du$

$\int u^3 e^u du = u^3 e^u - 3\int e^u u^2 du$

$\int e^u u^2 du$

$a' = u^2$ $\quad \frac{da'}{du} = 2u$ $\quad db' = e^u du$

$da' = 2u du$ $\quad \int db' = \int e^u du$

$b' = e^u$

$\int e^u u^2 du = u^2 e^u - \int e^u da'$

$\int e^u u^2 du = u^2 e^u - \int e^u 2u du$

$\int e^u u^2 du = u^2 e^u - 2\int e^u u du$

$\int e^u u du$

$a'' = u$ $\quad \frac{da''}{du} = 1$ $\quad db'' = e^u du$

$da'' = du$ $\quad \int db'' = \int e^u du$

$b'' = e^u$

$\int e^u u du = u e^u - \int e^u da''$

$\int e^u u du = u e^u - \int e^u du$

$\int e^u u du = u e^u - e^u + C$

$$\int e^u u^2 du = u^2 e^u - 2(ue^u - e^u) + C$$
$$\int e^u u^2 du = u^2 e^u - 2ue^u + 2e^u + C$$

$$\int u^3 e^u du = u^3 e^u - 3(u^2 e^u - 2ue^u + 2e^u) + C$$
$$\int u^3 e^u du = u^3 e^u - 3u^2 e^u + 6ue^u - 6e^u + C$$

$$\int x^3 e^x dx = x^3 e^x - 3x^2 e^x + 6xe^x - 6e^x + C$$
$$\int x^3 e^x dx = e^x(x^3 - 3x^2 + 6x - 6) + C$$

17) $\int \frac{xe^x}{(x+1)^2} dx$

(18) $\int \frac{\ln(x+1)}{\sqrt{x+1}}\,dx$

$u = \ln(x+1)$ $\quad \frac{du}{dx} = \frac{(x+1)'}{x+1}$ $\quad dv = \frac{1}{\sqrt{x+1}}\,dx$

$\frac{du}{dx} = \frac{1}{x+1}$ $\quad \int dv = \int \frac{1}{\sqrt{x+1}}\,dx$

$du = \frac{dx}{x+1}$ $\quad \int dv = \int \frac{1}{(x+1)^{1/2}}\,dx$

$\int dv = \int (x+1)^{-1/2}\,dx$

$v = 2\sqrt{x+1}$

$\int (x+1)^{-1/2}\,dx$

$\int u'^{-1/2}\,du$ $\quad u' = x+1$ $\quad \frac{du'}{dx} = 1$

$\frac{u'^{-1/2+1}}{-1/2+1}$

$du' = dx$

$\frac{u'^{1/2}}{1/2}$

$2u'^{1/2} = 2(x+1)^{1/2} = 2\sqrt{x+1}$

$\int \frac{\ln(x+1)}{\sqrt{x+1}}\,dx = \ln(x+1)(2\sqrt{x+1}) - \int 2(x+1)^{1/2}\,du$

$\int \frac{\ln(x+1)}{\sqrt{x+1}}\,dx = \ln(x+1)(2\sqrt{x+1}) - 2\int (x+1)^{1/2}\,\frac{dx}{x+1}$

$\int \frac{\ln(x+1)}{\sqrt{x+1}}\,dx = \ln(x+1)(2\sqrt{x+1}) - 2\int (x+1)^{1/2}(x+1)^{-1}\,dx$

$\int \frac{\ln(x+1)}{\sqrt{x+1}}\,dx = \ln(x+1)(2\sqrt{x+1}) - 2\int (x+1)^{-1/2}\,dx$

$\int (x+1)^{-1/2}\,dx$

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$u'' = x+1 \qquad \frac{du''}{dx} = 1$

$du'' = dx$

$\int u''^{-1/2} \, du''$

$\frac{u''^{-1/2+1}}{-1/2+1} + C$

$\frac{u''^{1/2}}{1/2} + C$

$2u''^{1/2} + C \Rightarrow 2(x+1)^{1/2} + C$

$\int \frac{\ln(x+1)}{\sqrt{x+1}} = \ln(x+1)(2\sqrt{x+1}) - 2\left[2(x+1)^{1/2}\right] + C$

$\int \frac{\ln(x+1)}{\sqrt{x+1}} = \ln(x+1)(2\sqrt{x+1}) - 4(x+1)^{1/2} + C$

$\int \frac{\ln(x+1)}{\sqrt{x+1}} = \ln(x+1)(2\sqrt{x+1}) - 4\sqrt{x+1} + C$

$\int \frac{\ln(x+1)}{\sqrt{x+1}} = 2\sqrt{x+1}(\ln(x+1) - 2) + C$

19 $\int \frac{\ln x}{x^2} \, dx$

$u = \ln x \qquad \frac{du}{dx} = \frac{1}{x} \qquad dv = \frac{1}{x^2} dx \qquad v = \frac{x^{-1}}{-1}$

$du = \frac{dx}{x} \qquad \int dv = \int \frac{1}{x^2} dx \qquad v = -\frac{1}{x}$

$\int dv = \int x^{-2} dx$

$v = \frac{x^{-2+1}}{-2+1}$

$\int \frac{\ln x}{x^2}\,dx = -\frac{1}{x}\ln x - \int -\frac{1}{x}\,du$

$\int \frac{\ln x}{x^2}\,dx = -\frac{\ln x}{x} - \int -1x^{-1}\frac{dx}{x}$

$\int \frac{\ln x}{x^2}\,dx = -\frac{\ln x}{x} - (-1)\int x^{-1}(x^{-1})\,dx$

$\int \frac{\ln x}{x^2}\,dx = -\frac{\ln x}{x} + \int x^{-2}\,dx$

$\int \frac{\ln x}{x^2}\,dx = -\frac{\ln x}{x} + \frac{x^{-2+1}}{-2+1} + C$

$\int \frac{\ln x}{x^2}\,dx = -\frac{\ln x}{x} + \frac{x^{-1}}{-1} + C$

$\int \frac{\ln x}{x^2}\,dx = -\frac{\ln x}{x} - \frac{1}{x} + C = \frac{-\ln x - 1}{x} + C$

20) $\int \frac{\ln x}{x^3}\,dx$

$u = \ln x$ $\quad \frac{du}{dx} = \frac{1}{x}$ $\quad dv = \frac{1}{x^3}\,dx$

$du = \frac{dx}{x}$ $\quad \int dv = \int \frac{1}{x^3}\,dx$

$\int dv = \int x^{-3}\,dx$

$v = \frac{x^{-3+1}}{-3+1}$

$v = \frac{x^{-2}}{-2}$

$v = -\frac{1}{2x^2}$

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$$\int \frac{\ln x}{x^3}\,dx = \ln x\left(-\frac{1}{2x^2}\right) - \int -\frac{1}{2x^2}\,du$$

$$\int \frac{\ln x}{x^3}\,dx = -\frac{\ln x}{2x^2} - \int -\frac{1}{2}\,x^{-2}\,\frac{dx}{x}$$

$$\int \frac{\ln x}{x^3}\,dx = -\frac{\ln x}{2x^2} - \left(-\frac{1}{2}\right)\int x^{-2}(x^{-1})\,dx$$

$$\int \frac{\ln x}{x^3}\,dx = -\frac{\ln x}{2x^2} + \frac{1}{2}\int x^{-3}\,dx$$

$$\int \frac{\ln x}{x^3}\,dx = -\frac{\ln x}{2x^2} + \frac{1}{2}\left(\frac{x^{-3+1}}{-3+1}\right) + C$$

$$\int \frac{\ln x}{x^3}\,dx = -\frac{\ln x}{2x^2} + \frac{1}{2}\left(\frac{x^{-2}}{-2}\right) + C$$

$$\int \frac{\ln x}{x^3}\,dx = -\frac{\ln x}{2x^2} - \frac{1}{4x^2} + C = \frac{-2\ln x - 1}{4x^2} + C$$

21) $\int \frac{x^3\,dx}{\sqrt{1-x^2}}$

$u = x^2$ $\quad \frac{du}{dx} = 2x$ $\quad du = 2x\,dx$

$dv = \frac{x\,dx}{\sqrt{1-x^2}}$

$\int dv = \int \frac{x\,dx}{\sqrt{1-x^2}}$

$\int dv = \int \frac{x\,dx}{(1-x^2)^{1/2}}$

$\int dv = \int x(1-x^2)^{-1/2}\,dx \rightarrow v = -(1-x^2)^{1/2}$

$u' = 1-x^2$ $\quad \frac{du'}{dx} = -2x$ $\quad du' = -2x\,dx$ $\quad x\,dx = \frac{du'}{-2}$

$\int u'^{-1/2}\,\frac{du'}{-2} \rightarrow -\frac{1}{2}\int u'^{-1/2}\,du'$

$\int -\frac{1}{2}\,u'^{-1/2}\,du' \rightarrow -\frac{1}{2}\,\frac{u'^{-1/2+1}}{-1/2+1}$

$\Rightarrow$ NA OUTRA PÁGINA

$$\int \frac{x^3 dx}{\sqrt{1-x^2}} = x^2\left(-(1-x^2)^{1/2}\right) - \int -(1-x^2)^{1/2} du$$

$$\int \frac{x^3 dx}{\sqrt{1-x^2}} = -x^2(1-x^2)^{1/2} - (-1)\int (1-x^2)^{1/2}\, 2x\, dx$$

$$\int \frac{x^3 dx}{\sqrt{1-x^2}} = -x^2(1-x^2)^{1/2} + 1(2)\int (1-x^2)^{1/2} x\, dx$$

$$\int \frac{x^3 dx}{\sqrt{1-x^2}} = -x^2(1-x^2)^{1/2} + 2\int (1-x^2)^{1/2} x\, dx$$

$$\int (1-x^2)^{1/2} x\, dx$$

$u'' = 1-x^2 \qquad \frac{du''}{dx} = -2x$

$du'' = -2x\,dx$

$x\,dx = \frac{du''}{-2}$

$$\int u''^{1/2} \frac{du''}{-2}$$

$$\int -\frac{1}{2} u''^{1/2} du''$$

$$-\frac{1}{2}\int u''^{1/2} du$$

$$-\frac{1}{2}\frac{u''^{1/2+1}}{1/2+1} + C$$

$$-\frac{1}{2}\frac{u''^{3/2}}{3/2} + C$$

$$-\frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{3}\right) u''^{3/2} + C$$

$$-\frac{1}{3} u''^{3/2} + C$$

$$-\frac{1}{3}(1-x^2)^{3/2} + C$$

$$\int \frac{x^3 dx}{\sqrt{1-x^2}} = -x^2(1-x^2)^{1/2} + 2\left(-\frac{1}{3}(1-x^2)^{3/2}\right) + C$$

$$\int \frac{x^3 dx}{\sqrt{1-x^2}} = -x^2(1-x^2)^{1/2} - \frac{2}{3}(1-x^2)^{3/2} + C$$

(*) CONTINUAÇÃO $-\frac{1}{2}\frac{u'^{1/2}}{1/2}$

$-\frac{1}{\cancel{2}_1}(\cancel{2}^1)u'^{1/2}$

$-u'^{1/2}$

$-(1-x^2)^{1/2}$

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(22) $\int x^3\sqrt{1-x^2}\,dx$

$u = x^2 \qquad \frac{du}{dx} = 2x \qquad dv = x\sqrt{1-x^2}\,dx$

$\int dv = \int x\sqrt{1-x^2}\,dx$

$du = 2x\,dx \qquad v = -\frac{1}{3}(1-x^2)^{3/2}$

$u' = 1-x^2 \qquad \frac{du'}{dx} = -2x \qquad \int \sqrt{u'}\,\frac{du'}{-2} \qquad -\frac{1}{2}\left(\frac{u'^{3/2}}{3/2}\right)$

$du' = -2x\,dx \qquad \int -\frac{1}{2}u'^{1/2}\,du' \qquad -\frac{1}{2}\cdot\frac{2}{3}u'^{3/2}$

$x\,dx = \frac{du'}{-2} \qquad -\frac{1}{2}\int u'^{1/2}\,du' \qquad -\frac{1}{3}u'^{3/2}$

$-\frac{1}{2}\left(\frac{u'^{1/2+1}}{1/2+1}\right) \qquad -\frac{1}{3}(1-x^2)^{3/2}$

$\int x^3\sqrt{1-x^2}\,dx = x^2\left(-\frac{1}{3}(1-x^2)^{3/2}\right) - \int -\frac{1}{3}(1-x^2)^{3/2}\,du$

$\int x^3\sqrt{1-x^2}\,dx = -\frac{1}{3}x^2(1-x^2)^{3/2} - \int -\frac{1}{3}(1-x^2)^{3/2}\,2x\,dx$

$\int x^3\sqrt{1-x^2}\,dx = -\frac{1}{3}x^2(1-x^2)^{3/2} - \int -\frac{2}{3}x(1-x^2)^{3/2}\,dx$

$\int x^3\sqrt{1-x^2}\,dx = -\frac{1}{3}x^2(1-x^2)^{3/2} - \left(-\frac{2}{3}\right)\int x(1-x^2)^{3/2}\,dx$

$\int x^3\sqrt{1-x^2}\,dx = -\frac{1}{3}x^2(1-x^2)^{3/2} + \frac{2}{3}\int x(1-x^2)^{3/2}\,dx$

$\int x(1-x^2)^{3/2}\,dx$

$u'' = 1-x^2 \qquad \frac{du''}{dx} = -2x \Rightarrow du'' = -2x\,dx$

$x\,dx = \frac{du''}{-2}$

$\int x(1-x^2)^{3/2}\,dx$

$\int u^{3/2}\,\dfrac{du}{-2}$

$\int -\dfrac{1}{2}u^{3/2}\,du$

$-\dfrac{1}{2}\int u^{3/2}\,du$

$-\dfrac{1}{2}\,\dfrac{u^{3/2+1}}{3/2+1}+C$

$-\dfrac{1}{2}\,\dfrac{u^{5/2}}{5/2}+C$

$-\dfrac{1}{2}\cdot\dfrac{2}{5}u^{5/2}+C$

$-\dfrac{\cancel{2}^{\,1}}{\cancel{10}_{\,5}}u^{5/2}+C$

$-\dfrac{1}{5}u^{5/2}+C$

$-\dfrac{1}{5}(1-x^2)^{5/2}+C$

$\int x^3\sqrt{1-x^2}\,dx=-\dfrac{1}{3}x^2(1-x^2)^{3/2}+\dfrac{2}{3}\left(-\dfrac{1}{5}(1-x^2)^{5/2}\right)+C$

$\int x^3\sqrt{1-x^2}\,dx=-\dfrac{1}{3}x^2(1-x^2)^{3/2}-\dfrac{2}{15}(1-x^2)^{5/2}+C$

28) $\int(2^x+x)^2\,dx$