

10.2 FUNÇÕES DE MAIS DE UMA VARIÁVEL *

1.A) $f(x,y) = 3x + 2y - 5$ $f(3,-1) = 3(3) + 2(-1) - 5$
 $= 9 - 2 - 5$
 $= 9 - 7$
 $= 2$

1.B) $f(x,y) = 3x + 2y - 5$ $f(-4,2) = 3(-4) + 2(2) - 5$
 $= -12 + 4 - 5$
 $= -8 - 5$
 $= -13$

1.C) $f(x,y) = 3x + 2y - 5$ $f(a+1, b-2) = 3(a+1) + 2(b-2) - 5$
 $= 3a + 3 + 2b - 4 - 5$
 $= 3a + 2b - 6$

1.D) $f(x,y) = 3x + 2y - 5$ $f(x+1, y-2) = 3(x+1) + 2(y-2) - 5$
 $= 3x + 3 + 2y - 4 - 5$
 $= 3x + 2y - 6$

1.E) $f(x,y) = 3x + 2y - 5$ $f(2x, 3y) = 3(2x) + 2(3y) - 5$
 $= 6x + 6y - 5$

1.F) $f(x,y) = 3x + 2y - 5$ $f(x+a, y) - f(x,y) = 3(x+a) + 2y - 5 - (3x + 2y - 5)$
 $= 3x + 3a + 2y - 5 - 3x - 2y + 5$
 $= 3a$

1.G) $f(x,y) = 3x + 2y - 5$ $f(x, y+a) - f(x,y) = 3x + 2(y+a) - 5 - (3x + 2y - 5)$
 $= 3x + 2y + 2a - 5 - 3x - 2y + 5$
 $= 2a$

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$$\begin{aligned} 2.A \quad g(x,y) &= x^2 - 3y + 5 & g(-2,3) &= (-2)^2 - 3(3) + 5 \\ & & &= 4 - 9 + 5 \\ & & &= -5 + 5 \\ & & &= 0 \end{aligned}$$

$$\begin{aligned} 2.B \quad g(x,y) &= x^2 - 3y + 5 & g(4,7) &= (4)^2 - 3(7) + 5 \\ & & &= 16 - 21 + 5 \\ & & &= -5 + 5 \\ & & &= 0 \end{aligned}$$

$$\begin{aligned} 2.C \quad g(x,y) &= x^2 - 3y + 5 & g(4R, 25) &= (4R)^2 - 3(25) + 5 \\ & & &= 16R^2 - 65 + 5 \end{aligned}$$

$$\begin{aligned} 2.D \quad g(x,y) &= x^2 - 3y + 5 & g(4x, 2y) &= (4x)^2 - 3(2y) + 5 \\ & & &= 16x^2 - 6y + 5 \end{aligned}$$

$$\begin{aligned} 2.E \quad g(x,y) &= x^2 - 3y + 5 & g(x-2, 2-y) &= (x-2)^2 - 3(2-y) + 5 \\ & & &= x^2 - 4x + 4 - 6 + 3y + 5 \\ & & &= x^2 - 4x + 3y + 3 \end{aligned}$$

$$\begin{aligned} 2.F \quad g(x,y) &= x^2 - 3y + 5 & g(x+a, y) - g(x,y) &= (x+a)^2 - 3y + 5 - (x^2 - 3y + 5) \\ & & &= x^2 + 2xa + a^2 - 3y + 5 - x^2 + 3y - 5 \\ & & &= 2xa + a^2 \\ & & &= a(2x+a) \end{aligned}$$

$$\begin{aligned} 2.G \quad g(x,y) &= x^2 - 3y + 5 & g(x, y+a) - g(x,y) &= x^2 - 3(y+a) + 5 - (x^2 - 3y + 5) \\ & & &= x^2 - 3y - 3a + 5 - x^2 + 3y - 5 \\ & & &= -3a \end{aligned}$$

$$\begin{aligned} \text{3.A } f(x,y) &= \frac{x+y}{x-y} & f(-3,4) &= \frac{-3+4}{-3-4} \\ & & &= \frac{1}{-7} = -\frac{1}{7} \end{aligned}$$

$$\text{3.B } f(x,y) = \frac{x+y}{x-y} \quad f(x^2, y^2) = \frac{x^2+y^2}{x^2-y^2}$$

$$\begin{aligned} \text{3.C } f(x,y) &= \frac{x+y}{x-y} & [f(x,y)]^2 &= \frac{[x+y]^2}{[x-y]^2} \\ & & &= \frac{x^2+2xy+y^2}{x^2-2xy+y^2} \end{aligned}$$

$$\begin{aligned} \text{3.D } f(x,y) &= \frac{x+y}{x-y} & f(-x,y) - f(x,-y) &= \frac{-x+y}{-x-y} - \left(\frac{x-y}{x-(-y)} \right) \\ & & &= \frac{-x+y}{-x-y} - \left(\frac{x-y}{x+y} \right) \\ & & &= \frac{-x+y}{-x-y} - \frac{x-y}{x+y} \\ & & &= \frac{-x+y}{-x-y} - \frac{x+y}{-x-y} \\ & & &= 2 \left(\frac{-x+y}{-x-y} \right) = -2 \left(\frac{x-y}{x+y} \right) \end{aligned}$$

PERGUNTAR SOBRE MUDANÇA DE SINAL

$$\begin{aligned} \text{3.E } f(x,y) &= \frac{x+y}{x-y} & x-y &\neq 0 \\ & & x &\neq y \end{aligned}$$

$D = \{(x,y) \in \mathbb{R}^2 / x \neq y\}$ PERGUNTAR SE O DOMÍNIO ESTÁ CERTO.

$$\begin{aligned} \text{4.A } g(x,y) &= \sqrt{x^2+y^2-25} & g(-4,3) &= \sqrt{(-4)^2+(3)^2-25} \\ & & &= \sqrt{16+9-25} \\ & & &= \sqrt{25-25} \\ & & &= \sqrt{0} \\ & & &= 0 \end{aligned}$$

$$\begin{aligned} \text{4.B } g(x, y) &= \sqrt{x^2 + y^2 - 25} & g(10, -3) &= \sqrt{(10)^2 + (-3)^2 - 25} \\ & & &= \sqrt{100 + 9 - 25} \\ & & &= \sqrt{84} \\ & & &= 10 \end{aligned}$$

$$\begin{aligned} \text{4.C } g(x, y) &= \sqrt{x^2 + y^2 - 25} & g(x^2, y^2) &= \sqrt{(x^2)^2 + (y^2)^2 - 25} \\ & & &= \sqrt{x^4 + y^4 - 25} \end{aligned}$$

$$\begin{aligned} \text{4.D } g(x, y) &= \sqrt{x^2 + y^2 - 25} & [g(x, y)]^2 &= (\sqrt{x^2 + y^2 - 25})^2 \\ & & &= x^2 + y^2 - 25 \end{aligned}$$

$$\begin{aligned} \text{4.E } g(x, y) &= \sqrt{x^2 + y^2 - 25} & g(-x, y) - g(x, -y) &= \sqrt{(-x)^2 + y^2 - 25} - (\sqrt{x^2 + (-y)^2 - 25}) \\ & & &= \sqrt{x^2 + y^2 - 25} - (\sqrt{x^2 + y^2 - 25}) \\ & & &= \sqrt{x^2 + y^2 - 25} - \sqrt{x^2 + y^2 - 25} \\ & & &= 0 \end{aligned}$$

$$\begin{aligned} \text{4.F } g(x, y) &= \sqrt{x^2 + y^2 - 25} & x^2 + y^2 - 25 &\geq 0 & x^2 + y^2 - 25 &= 0 \\ & & x^2 + y^2 &\geq 25 & x^2 + y^2 &= 25 \end{aligned}$$

$$D = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \geq 25\}$$

$$\text{4.G } \mathbb{R}_n = \mathbb{R}_+$$

$$\text{5.A } g(x, y, z) = \sqrt{4 - x^2 - y^2 - z^2}$$

$$\begin{aligned} g(1, -1, -1) &= \sqrt{4 - (1)^2 - (-1)^2 - (-1)^2} \\ &= \sqrt{4 - (1) - (1) - (1)} \\ &= \sqrt{4 - 1 - 1 - 1} \\ &= \sqrt{1} \\ &= 1 \end{aligned}$$

3.B) $g(x, y, z) = \sqrt{4 - x^2 - y^2 - z^2}$

$$\begin{aligned}
 g(-a, 2b, \frac{1}{2}c) &= \sqrt{4 - (-a)^2 - (2b)^2 - \left(\frac{1}{2}c\right)^2} \\
 &= \sqrt{4 - (a^2) - (4b^2) - \left(\frac{1}{4}c^2\right)} \\
 &= \sqrt{4 - a^2 - 4b^2 - \frac{1}{4}c^2} \\
 &= \sqrt{\frac{16 - 4a^2 - 16b^2 - c^2}{4}} \quad \text{POSSO TIRAR TUDO DA RAIZ?} \\
 &= \frac{\sqrt{16 - 4a^2 - 16b^2 - c^2}}{2}
 \end{aligned}$$

3.C) $g(x, y, z) = \sqrt{4 - x^2 - y^2 - z^2}$

$$\begin{aligned}
 g(y, -x, -y) &= \sqrt{4 - (y)^2 - (-x)^2 - (-y)^2} \\
 &= \sqrt{4 - (y^2) - (x^2) - (y^2)} \\
 &= \sqrt{4 - y^2 - x^2 - y^2} \\
 &= \sqrt{4 - x^2 - 2y^2}
 \end{aligned}$$

3.D) $g(x, y, z) = \sqrt{4 - x^2 - y^2 - z^2}$

$$\begin{aligned}
 [g(x, y, z)]^2 - [g(x+2, y+2, z)]^2 &= (\sqrt{4 - x^2 - y^2 - z^2})^2 - (\sqrt{4 - (x+2)^2 - (y+2)^2 - (z)^2})^2 \\
 &= 4 - x^2 - y^2 - z^2 - (4 - (x^2 + 4x + 4) - (y^2 + 4y + 4) - z^2) \\
 &= 4 - x^2 - y^2 - z^2 - (4 - x^2 - 4x - 4 - y^2 - 4y - 4 - z^2) \\
 &= 4 - x^2 - y^2 - z^2 - (-x^2 - 4x - y^2 - 4y - 4 - z^2) \\
 &= 4 - x^2 - y^2 - z^2 + x^2 + 4x + y^2 + 4y + 4 + z^2 \\
 &= 4 + 4x + 4y + 4 \\
 &= 4x + 4y + 8 \\
 &= 4(x + y + 2)
 \end{aligned}$$

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5.E $g(x, y, z) = \sqrt{4 - x^2 - y^2 - z^2}$

$$4 - x^2 - y^2 - z^2 \geq 0$$

$$x^2 + y^2 + z^2 \leq 4$$

$$4 - x^2 - y^2 - z^2 = 0$$

$$-x^2 - y^2 - z^2 = -4 \quad (-1)$$

$$x^2 + y^2 + z^2 = 4$$

$$D = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 \leq 4\}$$

5.F $\mathbb{Z}_m = \mathbb{R}_+$

6.A $f(x, y, z) = \frac{x+y+z}{x-y+z}$

6.C $f(x, y, z) = \frac{x+y+z}{x-y+z}$

$$f(1, -1, 1) = \frac{1 + (-1) + 1}{1 - (-1) + 1}$$

$$= \frac{1 - 1 + 1}{1 + 1 + 1}$$

$$= \frac{1}{3}$$

$$f(-y, z, -x) = \frac{-y+z-x}{-y-z-x} = \frac{-y+z-x}{-y-z-x} \cdot \frac{x+y+z}{x+y+z} = \frac{-y+z-x}{-y-z-x} \cdot \frac{x+y+z}{x+y+z}$$

6.D $f(x, y, z) = \frac{x+y+z}{x-y+z}$

6.B $f(x, y, z) = \frac{x+y+z}{x-y+z}$

$$f(R^2, 2RS, S^2) = \frac{R^2 + 2RS + S^2}{R^2 - 2RS + S^2} = \frac{(R+S)^2}{(R-S)^2}$$

6.E $x-y+z \neq 0$

$$f(-3, 2, -5) = \frac{-3+2+(-5)}{-3-2+(-5)}$$

$$= \frac{-3+2-5}{-3-2-5}$$

$$= \frac{-3+2-5}{-3-2-5}$$

$$= \frac{-3+2-5}{-3-2-5}$$

$$= \frac{-6}{-10}$$

$$= \frac{6}{10} = \frac{3}{5}$$

$$D = \{(x, y, z) \in \mathbb{R}^3 \mid x-y+z \neq 0\}$$

$$7) f(x, y) = \sqrt{16 - x^2 - y^2}$$

$$16 - x^2 - y^2 \geq 0$$

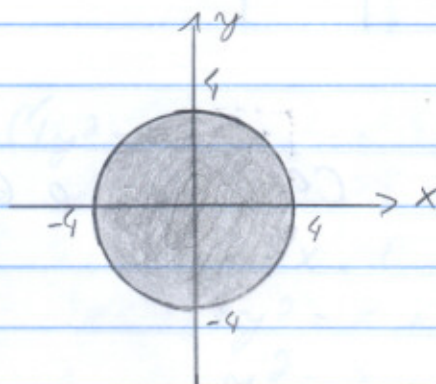
$$x^2 + y^2 \leq 16$$

$$D = \dots$$

$$16 - x^2 - y^2 = 0$$

$$-x^2 - y^2 = -16 \quad (-1)$$

$$x^2 + y^2 = 16$$



$$8) f(x, y) = \frac{\sqrt{16 - x^2 - y^2}}{x}$$

$$16 - x^2 - y^2 \geq 0$$

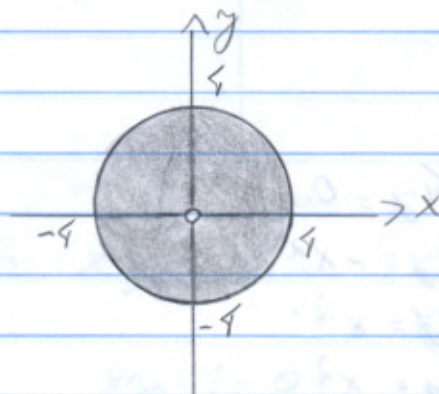
$$x^2 + y^2 \leq 16$$

$$16 - x^2 - y^2 = 0$$

$$-x^2 - y^2 = -16 \quad (-1)$$

$$x^2 + y^2 = 16$$

$$x \neq 0$$



O PUNTO $x=0$ ESTÁ
CERTO

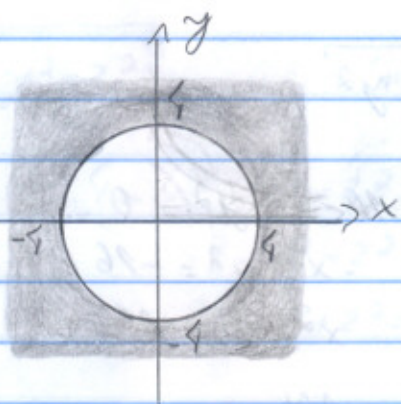
$$9) f(x, y) = \sqrt{x^2 + y^2 - 16}$$

$$x^2 + y^2 - 16 \geq 0$$

$$x^2 + y^2 \geq 16$$

$$x^2 + y^2 - 16 = 0$$

$$x^2 + y^2 = 16$$

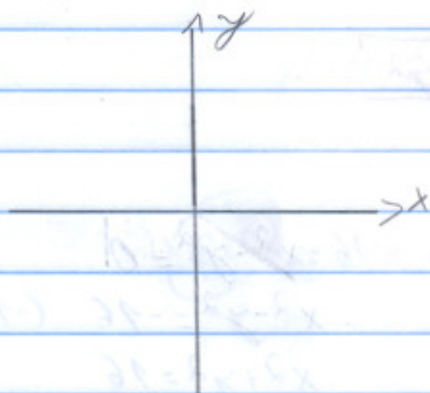


$$10) f(x, y) = \frac{x^2 - y^2}{x - y}$$

COMO FAZ O GRÁFICO

$$x - y \neq 0$$

$$x \neq y$$



$$11) f(x, y) = \ln(x^2 - 4y)$$

$$x^2 - 4y > 0$$

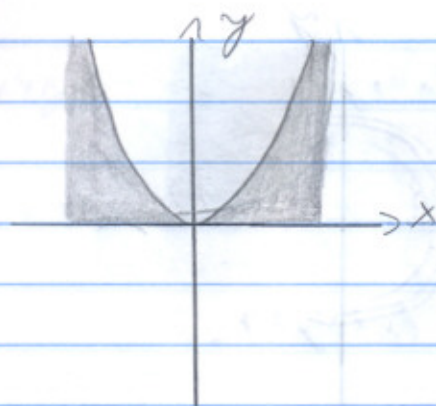
$$y < \frac{x^2}{4}$$

$$x^2 - 4y = 0$$

$$-4y = -x^2 \quad (-1)$$

$$4y = x^2$$

$$y = \frac{x^2}{4}$$



(12) $f(x, y) = \ln(4y^2 - x)$

$$4y^2 - x > 0$$

$$y > \frac{\sqrt{x}}{2}$$

$$x \geq 0$$

$$4y^2 - x = 0$$

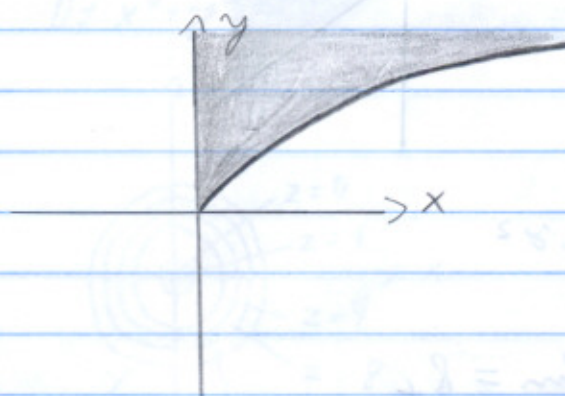
$$4y^2 = x$$

$$y^2 = \frac{x}{4}$$

$$y = \sqrt{\frac{x}{4}}$$

$$y = \frac{\sqrt{x}}{2}$$

$$y = \frac{\sqrt{x}}{2}$$



(13) $f(x, y) = \ln(xy - 1)$

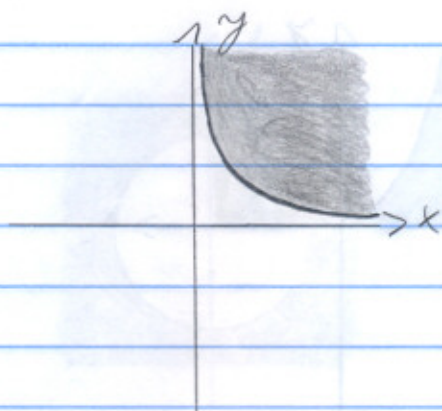
$$xy - 1 > 0$$

$$y > \frac{1}{x}$$

$$xy - 1 = 0$$

$$xy = 1$$

$$y = \frac{1}{x}$$



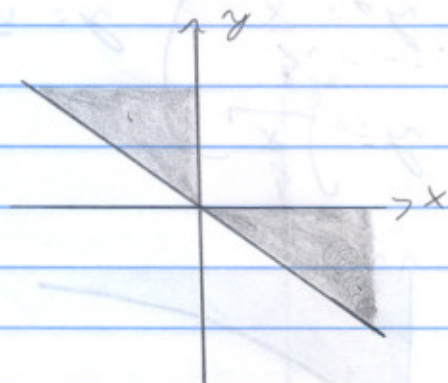
$$14) f(x, y) = \sqrt{x+y}$$

$$x+y \geq 0$$

$$x+y=0$$

$$y \geq -x$$

$$y = -x$$



$$15) g(x, y, z) = |x|e^{y^2}$$

$$D = \mathbb{R}$$

$$Z_m = \mathbb{R}_+$$

$$16) h(x, y, z) = \ln(x^2 + y^2 + z^2 - 1)$$

$$x^2 + y^2 + z^2 - 1 > 0$$

$$x^2 + y^2 + z^2 - 1 = 0$$

$$x^2 + y^2 + z^2 > 1$$

$$x^2 + y^2 + z^2 = 1$$

$$D = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 > 1\}$$

$$Z_m = (-\infty, +\infty)$$

17 $f(x, y) = \sqrt{x^2 + y^2} = z$

$$f(x, y) = k$$

$$k = \sqrt{x^2 + y^2}$$

$$k = \sqrt{(x^2 + y^2)}$$

$$16 = \sqrt{(x^2 + y^2)}$$

$$\sqrt{16} = \sqrt{x^2 + y^2}$$

$$\sqrt{16} = \sqrt{x^2 + y^2}$$

$$4 = \sqrt{x^2 + y^2}$$

$$1 = \sqrt{(x^2 + y^2)}$$

$$\sqrt{1} = \sqrt{x^2 + y^2}$$

$$\sqrt{1} = \sqrt{x^2 + y^2}$$

$$12 = \sqrt{(x^2 + y^2)}$$

$$\sqrt{12} = \sqrt{x^2 + y^2}$$

$$\sqrt{12} = \sqrt{x^2 + y^2}$$

$$3 = \sqrt{x^2 + y^2}$$

$$0 = \sqrt{(x^2 + y^2)}$$

$$\sqrt{0} = \sqrt{x^2 + y^2}$$

$$\sqrt{0} = \sqrt{x^2 + y^2}$$

$$0 = x^2 + y^2$$

$$8 = \sqrt{(x^2 + y^2)}$$

$$\sqrt{8} = \sqrt{x^2 + y^2}$$

$$\sqrt{8} = \sqrt{x^2 + y^2}$$

$$2 = \sqrt{x^2 + y^2}$$

$$1 = \sqrt{(x^2 + y^2)}$$

$$\sqrt{1} = \sqrt{x^2 + y^2}$$

$$\sqrt{1} = \sqrt{x^2 + y^2}$$

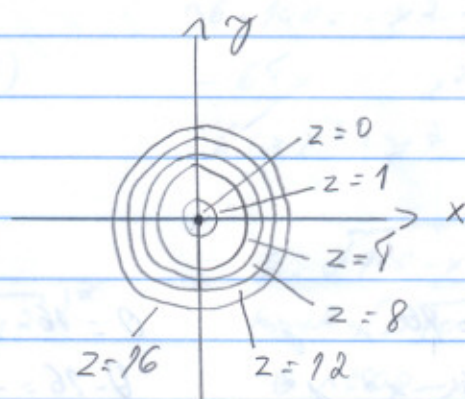
$$1 = x^2 + y^2$$

$$\sqrt{1} = \sqrt{(x^2 + y^2)}$$

$$\sqrt{1} = \sqrt{x^2 + y^2}$$

$$\sqrt{1} = \sqrt{x^2 + y^2}$$

$$1 = x^2 + y^2$$



$z=0$ ESTÁ CERTO?

18 $f(x, y) = \frac{1}{4}(x^2 + y^2)$

$$f(x, y) = k$$

$$k = \frac{1}{4}(x^2 + y^2)$$

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$$5 = \frac{1}{4}(x^2 + y^2)$$

$$\sqrt{5} = \frac{1}{4}(x^2 + y^2)$$

$$3 = \frac{1}{4}(x^2 + y^2)$$

$$2 = \frac{1}{4}(x^2 + y^2)$$

$$20 = x^2 + y^2$$

$$16 = x^2 + y^2$$

$$12 = x^2 + y^2$$

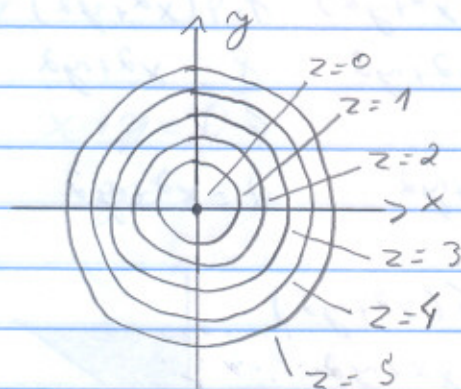
$$8 = x^2 + y^2$$

$$1 = \frac{1}{4}(x^2 + y^2)$$

$$0 = \frac{1}{4}(x^2 + y^2)$$

$$\sqrt{5} = x^2 + y^2$$

$$0 = x^2 + y^2$$



z=0 ESTÁ CERCA?

19) $f(x, y) = 16 - x^2 - y^2$

$$f(x, y) = K$$

$$K = 16 - x^2 - y^2$$

$$8 = 16 - x^2 - y^2$$

$$\sqrt{5} = 16 - x^2 - y^2$$

$$0 = 16 - x^2 - y^2$$

$$8 - 16 = -x^2 - y^2$$

$$\sqrt{5} - 16 = -x^2 - y^2$$

$$0 - 16 = -x^2 - y^2$$

$$-8 = -x^2 - y^2 \quad (-1)$$

$$-12 = -x^2 - y^2 \quad (-1)$$

$$-16 = -x^2 - y^2 \quad (-1)$$

$$8 = x^2 + y^2$$

$$12 = x^2 + y^2$$

$$16 = x^2 + y^2$$

$$-4 = 16 - x^2 - y^2$$

$$-8 = 16 - x^2 - y^2$$

$$-4 - 16 = -x^2 - y^2$$

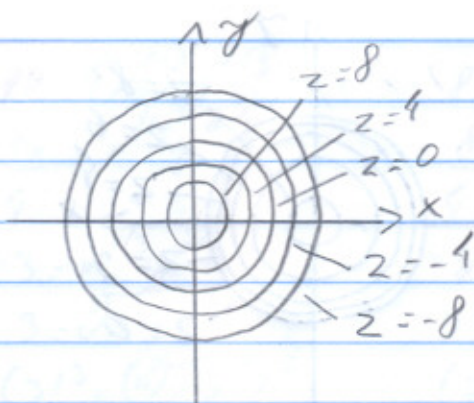
$$-8 - 16 = -x^2 - y^2$$

$$-20 = -x^2 - y^2 \quad (-1)$$

$$-24 = -x^2 - y^2 \quad (-1)$$

$$20 = x^2 + y^2$$

$$24 = x^2 + y^2$$



$$(20) f(x, y) = \sqrt{100 - x^2 - y^2}$$

$$f(x, y) = K$$

$$K = \sqrt{100 - x^2 - y^2}$$

$$8 = \sqrt{100 - x^2 - y^2}$$

$$(8)^2 = (\sqrt{100 - x^2 - y^2})^2$$

$$64 = 100 - x^2 - y^2$$

$$64 - 100 = -x^2 - y^2$$

$$-36 = -x^2 - y^2 (-1)$$

$$36 = x^2 + y^2$$

$$6 = \sqrt{100 - x^2 - y^2}$$

$$(6)^2 = (\sqrt{100 - x^2 - y^2})^2$$

$$36 = 100 - x^2 - y^2$$

$$36 - 100 = -x^2 - y^2$$

$$-64 = -x^2 - y^2 (-1)$$

$$64 = x^2 + y^2$$

$$4 = \sqrt{100 - x^2 - y^2}$$

$$(4)^2 = (\sqrt{100 - x^2 - y^2})^2$$

$$16 = 100 - x^2 - y^2$$

$$16 - 100 = -x^2 - y^2$$

$$-84 = -x^2 - y^2 (-1)$$

$$84 = x^2 + y^2$$

$$2 = \sqrt{100 - x^2 - y^2}$$

$$(2)^2 = (\sqrt{100 - x^2 - y^2})^2$$

$$4 = 100 - x^2 - y^2$$

$$4 - 100 = -x^2 - y^2$$

$$-96 = -x^2 - y^2 (-1)$$

$$96 = x^2 + y^2$$

$$0 = \sqrt{100 - x^2 - y^2}$$

$$(0)^2 = (\sqrt{100 - x^2 - y^2})^2$$

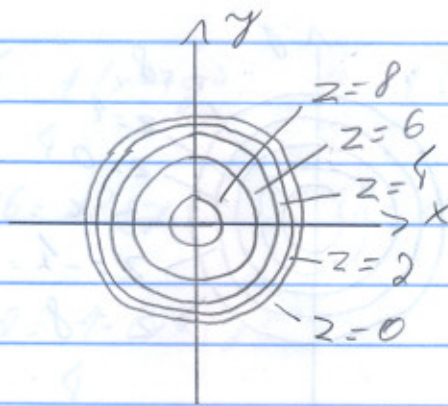
$$0 = 100 - x^2 - y^2$$

$$0 - 100 = -x^2 - y^2$$

$$-100 = -x^2 - y^2 (-1)$$

$$100 = x^2 + y^2$$

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21) $f(x, y) = 6xy$

$$f(x, y) = K$$

$$K = 6xy$$

$$30 = 6xy$$

$$24 = 6xy$$

$$18 = 6xy$$

$$12 = 6xy$$

$$6 = 6xy$$

$$\frac{30}{6} = xy$$

$$\frac{24}{6} = xy$$

$$\frac{18}{6} = xy$$

$$\frac{12}{6} = xy$$

$$\frac{6}{6} = xy$$

$$5 = xy$$

$$4 = xy$$

$$3 = xy$$

$$2 = xy$$

$$1 = xy$$

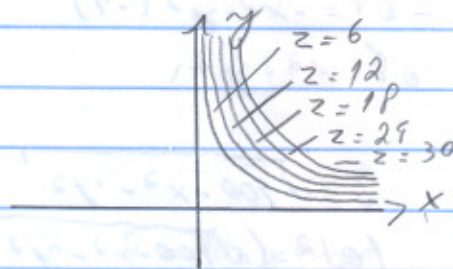
$$5 = xy$$

$$4 = xy$$

$$3 = xy$$

$$2 = xy$$

$$1 = xy$$



22) $f(x, y) = 4x^{1/3}y^{2/3}$

$$f(x, y) = K$$

$$K = 4x^{1/3}y^{2/3}$$

$$16 = 4x^{1/3}y^{2/3}$$

$$\frac{16}{4} = x^{1/3}y^{2/3}$$

$$4$$

$$4 = x^{1/3}y^{2/3}$$

$$4 = \sqrt[3]{xy^2}$$

$$(4)^3 = (\sqrt[3]{xy^2})^3$$

$$64 = xy^2$$

$$12 = 4x^{1/3}y^{2/3}$$

$$\frac{12}{4} = x^{1/3}y^{2/3}$$

$$3$$

$$3 = x^{1/3}y^{2/3}$$

$$3 = \sqrt[3]{xy^2}$$

$$(3)^3 = (\sqrt[3]{xy^2})^3$$

$$27 = xy^2$$

$$8 = 4x^{1/3}y^{2/3}$$

$$\frac{8}{4} = x^{1/3}y^{2/3}$$

$$2$$

$$2 = x^{1/3}y^{2/3}$$

$$2 = \sqrt[3]{xy^2}$$

$$(2)^3 = (\sqrt[3]{xy^2})^3$$

$$8 = xy^2$$

$$4 = 4x^{1/3}y^{2/3}$$

$$\frac{4}{4} = x^{1/3}y^{2/3}$$

$$1$$

$$1 = x^{1/3}y^{2/3}$$

$$1 = \sqrt[3]{xy^2}$$

$$(1)^3 = (\sqrt[3]{xy^2})^3$$

$$1 = xy^2$$

$$2 = 4x^{1/3}y^{2/3}$$

$$\frac{2}{4} = x^{1/3}y^{2/3}$$

$$\frac{1}{2}$$

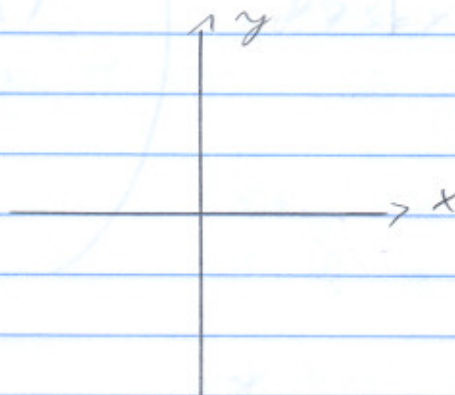
$$\frac{1}{2} = x^{1/3}y^{2/3}$$

$$\frac{1}{2} = \sqrt[3]{xy^2}$$

$$\frac{1}{2}$$

$$\left(\frac{1}{2}\right)^3 = (\sqrt[3]{xy^2})^3$$

$$\frac{1}{8} = xy^2$$



COMO FAZ O GRÁFICO?

23 $f(x, y) = 2x^2 + 2y^2$

$$f(x, y) = K$$

$$K = 2x^2 + 2y^2$$

$$K = 2(x^2 + y^2)$$

172

$$12 = 2(x^2 + y^2) \quad f = 2(x^2 + y^2) \quad f = 2(x^2 + y^2) \quad 2 = 2(x^2 + y^2)$$

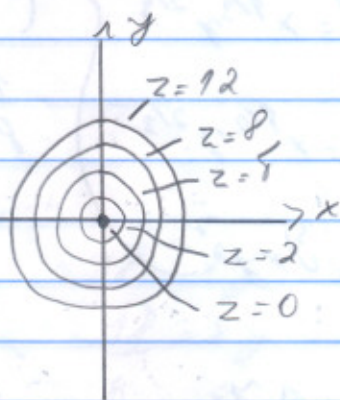
$$\frac{12}{2} = x^2 + y^2 \quad \frac{f}{2} = x^2 + y^2 \quad \frac{f}{2} = x^2 + y^2 \quad \frac{2}{2} = x^2 + y^2$$

$$6 = x^2 + y^2 \quad f = x^2 + y^2 \quad 2 = x^2 + y^2 \quad 1 = x^2 + y^2$$

$$0 = 2(x^2 + y^2)$$

$$\frac{0}{2} = x^2 + y^2$$

$$0 = x^2 + y^2$$



29 $f(x, y) = \sqrt{9 - x^2 - y^2}$

$$f(x, y) = k$$

$$k = \sqrt{9 - x^2 - y^2}$$

$$16 = \sqrt{9 - x^2 - y^2}$$

$$16 = \sqrt{9 - x^2 - y^2}$$

$$\sqrt{}$$

$$\sqrt{} = \sqrt{9 - x^2 - y^2}$$

$$(\sqrt{})^2 = (\sqrt{9 - x^2 - y^2})^2$$

$$16 = 9 - x^2 - y^2$$

$$16 - 9 = -x^2 - y^2$$

$$7 = -x^2 - y^2 \quad (-1)$$

$$-7 = x^2 + y^2$$

$$12 = 4\sqrt{9-x^2-y^2}$$

$$12 = \sqrt{9-x^2-y^2}$$

4

$$3 = \sqrt{9-x^2-y^2}$$

$$(3)^2 = (\sqrt{9-x^2-y^2})^2$$

$$9 = 9-x^2-y^2$$

$$9-9 = -x^2-y^2$$

$$0 = -x^2-y^2 \quad (-1)$$

$$0 = x^2+y^2$$

$$8 = 4\sqrt{9-x^2-y^2}$$

$$8 = \sqrt{9-x^2-y^2}$$

4

$$2 = \sqrt{9-x^2-y^2}$$

$$(2)^2 = (\sqrt{9-x^2-y^2})^2$$

$$4 = 9-x^2-y^2$$

$$4-9 = -x^2-y^2$$

$$-5 = -x^2-y^2 \quad (-1)$$

$$5 = x^2+y^2$$

$$4 = 4\sqrt{9-x^2-y^2}$$

$$4 = \sqrt{9-x^2-y^2}$$

4

$$1 = \sqrt{9-x^2-y^2}$$

$$(1)^2 = (\sqrt{9-x^2-y^2})^2$$

$$1 = 9-x^2-y^2$$

$$1-9 = -x^2-y^2$$

$$-8 = -x^2-y^2 \quad (-1)$$

$$8 = x^2+y^2$$

$$2 = 4\sqrt{9-x^2-y^2}$$

$$2 = \sqrt{9-x^2-y^2}$$

4

$$1 = \sqrt{9-x^2-y^2}$$

2

$$\left(\frac{1}{2}\right)^2 = (\sqrt{9-x^2-y^2})^2$$

$$\frac{1}{4} = 9-x^2-y^2$$

4

$$\frac{1}{4} - 9 = -x^2-y^2$$

4

$$\frac{1-36}{4} = -x^2-y^2$$

4

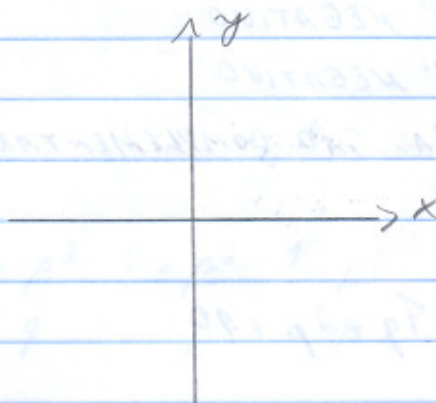
$$-\frac{35}{4} = -x^2-y^2$$

4

$$-\frac{35}{4} = -x^2-y^2 \quad (-1)$$

$$\frac{35}{4} = x^2+y^2$$

y



COMO FAZ O GRÁFICO

1741

$$(25) \quad x = -p - 3q + 6$$

$$y = -2q - p + 8$$

COEFICIENTE DE "q" NEGATIVO

COEFICIENTE DE "p" NEGATIVO

ENTÃO AS MERCADORIAS SÃO COMPLEMENTARES

$$(26) \quad x = -4p + 2q + 6$$

$$y = 3p - q + 10 \Rightarrow y = -q + 3p + 10$$

COEFICIENTE DE "q" POSITIVO

COEFICIENTE DE "p" POSITIVO

ENTÃO AS MERCADORIAS SÃO CONCORRENTES

$$(27) \quad x = 6 - 3p - 2q \Rightarrow x = -3p - 2q + 6$$

$$y = 4 + 2p - q \Rightarrow y = -q + 2p + 4$$

COEFICIENTE DE "q" NEGATIVO

COEFICIENTE DE "p" POSITIVO

AS MERCADORIAS NÃO SÃO CONCORRENTES NEM COMPLEMENTARES

$$(28) \quad x = -7q - p + 7 \Rightarrow x = -p - 7q + 7$$

$$y = 18 - 3q - 9p \Rightarrow y = -3q - 9p + 18$$

COEFICIENTE DE "q" NEGATIVO

COEFICIENTE DE "p" NEGATIVO

ENTÃO AS MERCADORIAS SÃO COMPLEMENTARES

$$(29) \quad x = -3p + 3q + 13$$

$$y = 2p - 4q + 10 \Rightarrow y = -4q + 2p + 10$$

COEFICIENTE DE "q" POSITIVO

COEFICIENTE DE "p" POSITIVO

ENTÃO AS MERCADORIAS SÃO CONCORRENTES

30 $x = 9 - 3p + q \Rightarrow x = -3p + q + 9$

$y = 10 - 2p - 5q \Rightarrow y = -5q - 2p + 10$

COEFICIENTE DE "q" POSITIVO

COEFICIENTE DE "p" NEGATIVO

AS MERCADORIAS NÃO SÃO CONCORRENTES NEM COMPLEMENTARES

31 $p \times = q$

$x = q$

p

$5 = q$

p

$4 = q$

p

$3 = q$

p

$2 = q$

p

$1 = q$

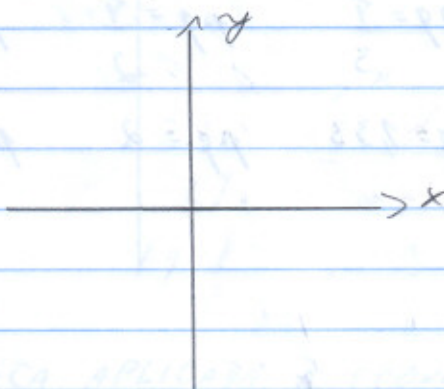
p

$\frac{1}{2} = q$

p

$\frac{1}{5} = q$

p



$q \cdot y = p^2$

$y = \frac{p^2}{q}$

$5 = \frac{p^2}{q}$

q

$4 = \frac{p^2}{q}$

q

$3 = \frac{p^2}{q}$

q

$2 = \frac{p^2}{q}$

q

$1 = \frac{p^2}{q}$

q

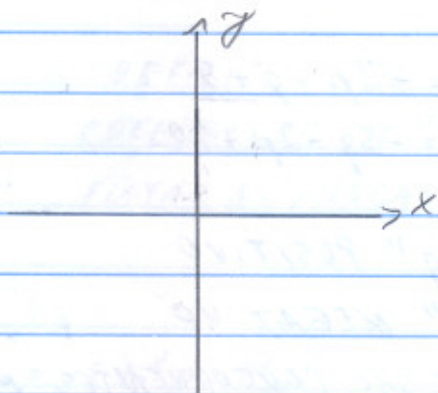
$\frac{1}{2} = \frac{p^2}{q}$

q

$\frac{1}{5} = \frac{p^2}{q}$

q

176



32 $pq \cdot x = 4$
 $x = \frac{4}{pq}$

pq

$5 = \frac{4}{pq}$

$4 = \frac{4}{pq}$

$3 = \frac{4}{pq}$

$2 = \frac{4}{pq}$

$1 = \frac{4}{pq}$

$\frac{1}{2} = \frac{4}{pq}$

pq

pq

pq

pq

pq

$2 \cdot pq$

$pq = \frac{4}{5}$

$pq = \frac{4}{4}$

$pq = \frac{4}{3}$

$pq = \frac{4}{2}$

$pq = \frac{4}{1}$

$pq = \frac{4}{\frac{1}{2}}$

$pq = 0,8$

$pq = 1$

$pq = 1,33$

$pq = 2$

$pq = 4$

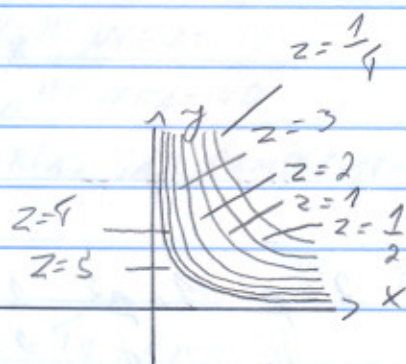
$pq = 8$

$\frac{1}{4} = \frac{4}{pq}$

pq

$pq = \frac{16}{1}$

$pq = 16$



$p^2 q y = 16$

$y = \frac{16}{p^2 q}$

$p^2 q$

177

$$5 = \frac{16}{p^2 q}$$

$$p^2 q = \frac{16}{5}$$

$$p^2 q = 3,20$$

$$4 = \frac{16}{p^2 q}$$

$$p^2 q = \frac{16}{4}$$

$$p^2 q = 4$$

$$3 = \frac{16}{p^2 q}$$

$$p^2 q = \frac{16}{3}$$

$$p^2 q = 5,33$$

$$2 = \frac{16}{p^2 q}$$

$$p^2 q = \frac{16}{2}$$

$$p^2 q = 8$$

$$1 = \frac{16}{p^2 q}$$

$$p^2 q = \frac{16}{1}$$

$$p^2 q = 16$$

$$1 = \frac{16}{p^2 q}$$

$$p^2 q = \frac{32}{1}$$

$$p^2 q = 32$$

$$1 = \frac{16}{p^2 q}$$

$$p^2 q = \frac{64}{1}$$

$$p^2 q = 64$$

COMO FAZ O GRÁFICO?

p^2

