LIVRO MATEMÁTICA APLICADA À ECONOMIA $\Rightarrow$ LILIA LADEIRA VERAS

4.3 DERIVADAS PARCIAIS $\Rightarrow$ PÁGINA 170 DO LIVRO MATEMÁTICA APLICADA À ECONOMIA (LILIA LADEIRA VERAS, 3ª EDIÇÃO, EDITORA ATLAS)

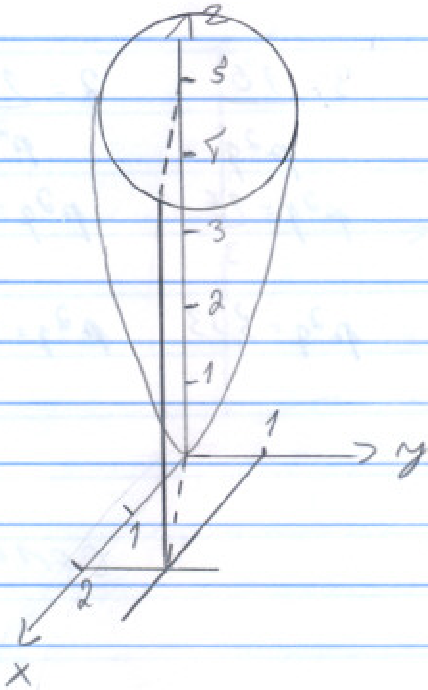
1.A $\frac{\partial z}{\partial x} = 2x$ $\qquad$ $\frac{\partial z}{\partial y} = 2y$

NO PONTO (2, 1)

$\frac{\partial z}{\partial x} = 2(2) = 4$ $\qquad$ $\frac{\partial z}{\partial y} = 2(1) = 2$

|178|

1.B



1.C $z = x^2 + y^2$

$z = x^2 + (1)^2$

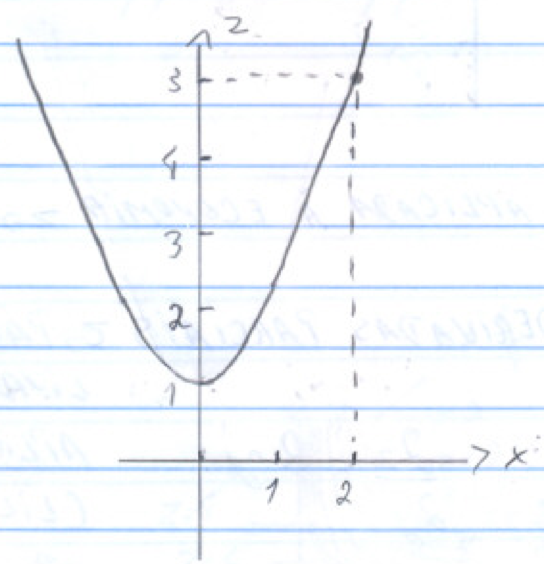
$z = x^2 + 1$

$x = 2 \Rightarrow z = x^2 + 1$

$z = (2)^2 + 1$

$z = 4 + 1$

$z = 5$



1.D $z = x^2 + y^2$

$z = (2)^2 + y^2$

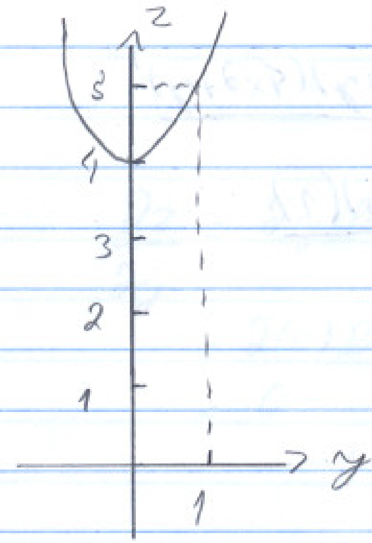
$z = 4 + y^2$

$y = 1 \Rightarrow z = 4 + y^2$

$z = 4 + (1)^2$

$z = 4 + 1$

$z = 5$



2.A $z = 4x^3y - 3x^2y + 5x - y$

$$\frac{\partial z}{\partial x} = 12x^2y - 6xy + 5 \qquad \frac{\partial z}{\partial y} = 4x^3 - 3x^2 - 1$$

2.B $z = x^{1/2}y$

$$\frac{\partial z}{\partial x} = \frac{1}{2}x^{-1/2}y \qquad \frac{\partial z}{\partial y} = x^{1/2}$$

2.C $z = \dfrac{x+y}{4x^2+y}$

$$\frac{\partial z}{\partial x} = \frac{(4x^2+y)(x+y)' - (x+y)(4x^2+y)'}{(4x^2+y)^2}$$

$$= \frac{(4x^2+y)(1) - (x+y)(8x)}{(4x^2+y)^2}$$

$$= \frac{4x^2+y-(8x^2+8xy)}{(4x^2+y)^2}$$

$$= \frac{4x^2+y-8x^2-8xy}{(4x^2+y)^2}$$

$$\to \frac{\partial z}{\partial x} = \frac{-4x^2-8xy+y}{(4x^2+y)^2}$$

|180|

$$\frac{\partial z}{\partial y} = \frac{(4x^2+y)(x+y)' - (x+y)(4x^2+y)'}{(4x^2+y)^2}$$

$$= \frac{(4x^2+y)(1) - (x+y)(1)}{(4x^2+y)^2}$$

$$= \frac{4x^2+y-(x+y)}{(4x^2+y)^2}$$

$$= \frac{4x^2+y-x-y}{(4x^2+y)^2}$$

$$= \frac{4x^2-x}{(4x^2+y)^2} = \frac{x(4x-1)}{(4x^2+y)^2}$$

2.D) $z = \ln x(y^2+3)$

$$\frac{\partial z}{\partial x} = (\ln x)(y^2+3)' + (y^2+3)(\ln x)'$$

$$= (\ln x)(0) + (y^2+3)\left(\frac{1}{x}\right)$$

$$= 0 + \frac{y^2+3}{x}$$

$$= \frac{y^2+3}{x}$$

$$\frac{\partial z}{\partial y} = (\ln x)(y^2+3)' + (y^2+3)(\ln x)'$$

$$= (\ln x)(2y) + (y^2+3)(0)$$

$$= 2y\ln x + 0$$

$$= 2y\ln x$$

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NO PONTO (1,0):

$$\frac{\partial z}{\partial x} = \frac{y^2+3}{x} = \frac{(0)^2+3}{1} = \frac{0+3}{1} = 3$$

$$\frac{\partial z}{\partial y} = 2y \ln x = 2(0)\ln 1 = 0$$

2.E $z = (x^2-y)^3$

$$\frac{\partial z}{\partial x} = 3(x^2-y)^2(x^2-y)' = 3(x^2-y)^2(2x) = 6x(x^2-y)^2$$

$$\frac{\partial z}{\partial y} = 3(x^2-y)^2(x^2-y)' = 3(x^2-y)^2(-1) = -3(x^2-y)^2$$

NO PONTO (2,2):

$$\frac{\partial z}{\partial x} = 6x(x^2-y)^2 = 6(2)[(2)^2-2]^2 = 12(4-2)^2 = 12(2)^2 = 12(4) = 48$$

$$\frac{\partial z}{\partial y} = -3(x^2-y)^2 = -3[(2)^2-2]^2 = -3(4-2)^2 = -3(2)^2 = -3(4) = -12$$

11/21

2.F $z = e^{2x+y^2}$

$$\frac{\partial z}{\partial x} = e^{2x+y^2}(2x+y^2)'$$
$$= e^{2x+y^2}(2)$$
$$= 2e^{2x+y^2}$$

$$\frac{\partial z}{\partial y} = e^{2x+y^2}(2x+y^2)'$$
$$= e^{2x+y^2}(2y)$$
$$= 2ye^{2x+y^2}$$

NO PONTO (0,1):

$$\frac{\partial z}{\partial x} = 2e^{2x+y^2}$$
$$= 2e^{2(0)+(1)^2}$$
$$= 2e^{0+1}$$
$$= 2e^1$$
$$= 2e$$

$$\frac{\partial z}{\partial y} = 2y\,e^{2x+y^2}$$
$$= 2(1)e^{2(0)+(1)^2}$$
$$= 2e^{0+1}$$
$$= 2e^1$$
$$= 2e$$

3.A $(x-2)^2 + (y-3)^2 = 0$

$$\frac{dy}{dx} = \frac{-\partial z/\partial x}{\partial z/\partial y}$$

$$\frac{\partial z}{\partial x} = 2(x-2)(x-2)' + 2(y-3)(y-3)'$$
$$= 2(x-2)(1) + 2(y-3)(0)$$
$$= 2(x-2) + 0$$
$$= 2(x-2)$$

$$\frac{\partial z}{\partial y} = 2(x-2)(x-2)' + 2(y-3)(y-3)'$$

$$= 2(x-2)(0) + 2(y-3)(1)$$

$$= 0 + 2(y-3)$$

$$= 2(y-3)$$

$$\frac{dy}{dx} = \frac{-\partial z/\partial x}{\partial z/\partial y}$$

$$= -\frac{\cancel{2}(x-2)}{\cancel{2}(y-3)}$$

$$= -\frac{(x-2)}{y-3} = \frac{-x+2}{y-3} = \frac{2-x}{y-3}$$

3.B $\frac{x^2+y}{y^2} - x + 1 = 0$

$$\frac{dy}{dx} = \frac{-\partial z/\partial x}{\partial z/\partial y}$$

$$\frac{\partial z}{\partial x} = \frac{(y^2)(x^2+y)' - (x^2+y)(y^2)'}{(y^2)^2} - 1$$

$$= \frac{(y^2)(2x) - (x^2+y)(0)}{y^4} - 1$$

$$= \frac{(y^2)(2x) - 0}{y^4} - 1$$

$$= \frac{\cancel{(y^2)}(2x)}{y^{\cancel{4}\,2}} - 1$$

$$= \frac{2x}{y^2} - 1$$

125

$$\frac{\partial z}{\partial y} = \frac{(y^2)(x^2+y)' - (x^2+y)(y^2)'}{(y^2)^2}$$

$$= \frac{(y^2)(1) - (x^2+y)(2y)}{y^4}$$

$$= \frac{y^2 - (2x^2y + 2y^2)}{y^4}$$

$$= \frac{y^2 - 2x^2y - 2y^2}{y^4}$$

$$= \frac{-2x^2y - y^2}{y^4}$$

$$= \frac{\cancel{y}(-2x^2 - y)}{y^{\cancel{4}\,3}} = \frac{-2x^2 - y}{y^3}$$

$$\frac{dy}{dx} = \frac{-\partial z/\partial x}{\partial z/\partial y}$$

$$= \frac{-\left(\frac{2x}{y^2} - 1\right)}{\frac{-2x^2 - y}{y^3}}$$

$$= \left(-\frac{2x}{y^2} + 1\right)\left(\frac{y^3}{-2x^2 - y}\right)$$

$$= \left(\frac{-2x + y^2}{y^2}\right)\left(\frac{y^3}{-2x^2 - y}\right)$$

$$= \frac{y(-2x + y^2)}{-2x^2 - y}$$

$$= \frac{-2xy + y^3}{-2x^2 - y}$$

$$= \frac{-1(2xy - y^3)}{-1(2x^2 + y)}$$

$$= \frac{2xy - y^3}{2x^2 + y}$$

3.C $(2x-y)\ln(xy)=0$

$$\frac{dy}{dx}=-\frac{\partial z/\partial x}{\partial z/\partial y}$$

$$\frac{\partial z}{\partial x}=(2x-y)[\ln(xy)]'+\ln(xy)(2x-y)'$$

$$=(2x-y)\left[\frac{1}{xy}(xy)'\right]+\ln(xy)(2)$$

$$=(2x-y)\left[\frac{1}{x\cancel{y}}(\cancel{y})\right]+2\ln(xy)$$

$$=(2x-y)\left(\frac{1}{x}\right)+2\ln(xy)$$

$$=\frac{2x-y}{x}+2\ln(xy)$$

$$\frac{\partial z}{\partial y}=(2x-y)[\ln(xy)]'+\ln(xy)(2x-y)'$$

$$=(2x-y)\left[\frac{1}{xy}(xy)'\right]+\ln(xy)(-1)$$

$$=(2x-y)\left[\frac{1}{\cancel{x}y}(\cancel{x})\right]-\ln(xy)$$

$$=(2x-y)\left(\frac{1}{y}\right)-\ln(xy)$$

$$=\frac{2x-y}{y}-\ln(xy)$$

|18|

$$\frac{dy}{dx} = \frac{-\partial z/\partial x}{\partial z/\partial y}$$

$$= \frac{-\left[\frac{2x-y}{x} + 2\ln(xy)\right]}{\frac{2x-y}{y} - \ln(xy)}$$

$$= \frac{-\left[\frac{2x-y+2x\ln(xy)}{x}\right]}{\frac{2x-y-y\ln(xy)}{y}}$$

$$= -\left[\frac{2x-y+2x\ln(xy)}{x}\right]\left(\frac{y}{2x-y-y\ln(xy)}\right)$$

$$= -\left[\frac{2xy-y^2+2xy\ln(xy)}{2x^2-xy-xy\ln(xy)}\right]$$

$$= \frac{-2xy+y^2-2xy\ln(xy)}{2x^2-xy-xy\ln(xy)}$$

$$= \frac{-1\left[2xy-y^2+2xy\ln(xy)\right]}{-1\left[-2x^2+xy+xy\ln(xy)\right]}$$

$$= \frac{2xy\ln(xy)+2xy-y^2}{xy\ln(xy)-2x^2+xy}$$

3.D $e^{x+y} = xy+1$

$e^{x+y} - xy - 1 = 0$

$$\frac{dy}{dx} = \frac{-\partial z/\partial x}{\partial z/\partial y}$$

$$\frac{\partial z}{\partial x} = e^{x+y}(x+y)' - y$$

$$= e^{x+y}(1) - y$$

 $= e^{x+y} - y$

$$\frac{\partial z}{\partial y} = e^{x+y}(x+y)' - x$$
$$= e^{x+y}(1) - x$$
$$= e^{x+y} - x$$

$$\frac{dy}{dx} = \frac{-\partial z/\partial x}{\partial z/\partial y}$$
$$= \frac{-(e^{x+y} - y)}{e^{x+y} - x}$$
$$= \frac{-e^{x+y} + y}{e^{x+y} - x}$$

NO PONTO (0,0):

$$\frac{dy}{dx} = \frac{-e^{x+y} + y}{e^{x+y} - x}$$
$$= \frac{-e^{0+0} + 0}{e^{0+0} - 0}$$
$$= \frac{-e^{0}}{e^{0}}$$
$$= \frac{-1}{1} = -1$$

3.E $x^{1/2} y^{1/2} = 20$

$x^{1/2} y^{1/2} - 20 = 0$

$$\frac{dy}{dx} = \frac{-\partial z/\partial x}{\partial z/\partial y}$$

|1881|

$$\frac{\partial z}{\partial x} = (x^{1/2})(y^{1/2})' + (y^{1/2})(x^{1/2})'$$

$$= (x^{1/2})(0) + (y^{1/2})\left(\frac{1}{2}x^{-1/2}\right)$$

$$= 0 + (y^{1/2})\left(\frac{1}{2x^{1/2}}\right)$$

$$= \frac{y^{1/2}}{2x^{1/2}} = \frac{\sqrt{y}}{2\sqrt{x}}$$

$$\frac{\partial z}{\partial y} = (x^{1/2})(y^{1/2})' + (y^{1/2})(x^{1/2})'$$

$$= (x^{1/2})\left(\frac{1}{2}y^{-1/2}\right) + (y^{1/2})(0)$$

$$= (x^{1/2})\left(\frac{1}{2y^{1/2}}\right) + 0$$

$$= \frac{x^{1/2}}{2y^{1/2}} = \frac{\sqrt{x}}{2\sqrt{y}}$$

$$\frac{dy}{dx} = \frac{-\partial z/\partial x}{\partial z/\partial y}$$

$$= \frac{-\left(\frac{\sqrt{y}}{2\sqrt{x}}\right)}{\frac{\sqrt{x}}{2\sqrt{y}}}$$

$$= \frac{-\sqrt{y}}{2\sqrt{x}}\left(\frac{2\sqrt{y}}{\sqrt{x}}\right)$$

$$= \frac{-\cancel{2}y}{\cancel{2}x}$$

$$= \frac{-y}{x}$$

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NO PONTO (16,23):

$$\frac{dy}{dx} = -\frac{y}{x}$$

$$= -\frac{23}{16}$$

3.F $x^2y - xy^2 = 0$

$$\frac{dy}{dx} = -\frac{\partial z/\partial x}{\partial z/\partial y}$$

$$\frac{\partial z}{\partial x} = 2xy - y^2$$

$$\frac{\partial z}{\partial y} = x^2 - 2xy$$

$$\frac{dy}{dx} = -\frac{\partial z/\partial x}{\partial z/\partial y}$$

$$= -\frac{(2xy - y^2)}{x^2 - 2xy}$$

$$= \frac{-2xy + y^2}{x^2 - 2xy}$$

NO PONTO (3,3):

$$\frac{dy}{dx} = \frac{-2xy + y^2}{x^2 - 2xy}$$

$$= \frac{-2(3)(3) + (3)^2}{(3)^2 - 2(3)(3)}$$

$$= \frac{-18 + 9}{9 - 18}$$

$$= \frac{-9}{-9}$$

$$= 1$$

|1790|

4.A $\ln(x-y) = x^2 - 4y$

NO PONTO $(2,1)$:

O PONTO $(2,1)$ PERTENCE À CURVA

$\ln(2-1) = (2)^2 - 4(1)$

$\ln 1 = 4 - 4$

$0 = 0$

4.B

5.A $\dfrac{dR}{dy} = \dfrac{\partial R}{\partial q_1} \cdot \dfrac{dq_1}{dy} + \dfrac{\partial R}{\partial q_2} \cdot \dfrac{dq_2}{dy}$

5.B $R = 10q_1 + 30q_2$

$q_1 = 0{,}2y + 5$

$q_2 = 0{,}3y + 4$

$\dfrac{\partial R}{\partial q_1} = 10 \qquad \dfrac{\partial R}{\partial q_2} = 30 \qquad \dfrac{dq_1}{dy} = 0{,}2 \qquad \dfrac{dq_2}{dy} = 0{,}3$

5.C $\dfrac{dR}{dy} = \dfrac{\partial R}{\partial q_1} \cdot \dfrac{dq_1}{dy} + \dfrac{\partial R}{\partial q_2} \cdot \dfrac{dq_2}{dy}$

$= 10 \cdot 0{,}2 + 30 \cdot 0{,}3$

$= 2 + 9$

$= 11$

$R = 10q_1 + 30q_2$

$= 10(0{,}2y + 5) + 30(0{,}3y + 4)$

$= 2y + 50 + 9y + 120$

$= 11y + 170$

$\dfrac{dR}{dy} = 11$

6.A $\dfrac{\partial R}{\partial x} = \dfrac{dR}{dq} \cdot \dfrac{\partial q}{\partial x} \qquad \dfrac{\partial R}{\partial y} = \dfrac{dR}{dq} \cdot \dfrac{\partial q}{\partial y}$

6.B $R = -2q^2 + 4q$

$q = x^2 + y^2$

$$\frac{\partial R}{\partial x} = \frac{dR}{dq} \cdot \frac{\partial q}{\partial x}$$
$$= (-4q+4)(2x)$$
$$= [-4(x^2+y^2)+4](2x)$$
$$= (-4x^2-4y^2+4)(2x)$$
$$= -8x^3-8xy^2+8x$$

$$\frac{\partial R}{\partial y} = \frac{dR}{dq} \cdot \frac{\partial q}{\partial y}$$
$$= (-4q+4)(2y)$$
$$= [-4(x^2+y^2)+4](2y)$$
$$= (-4x^2-4y^2+4)(2y)$$
$$= -8x^2y-8y^3+8y$$

7.A $z = 3x^2-4xy+3y$

$$\frac{\partial z}{\partial x} = 6x-4y \qquad \frac{\partial z}{\partial y} = -4x+3$$

$$\frac{\partial^2 z}{\partial x^2} = 6 \qquad \frac{\partial^2 z}{\partial y^2} = 0$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial y}(6x-4y) = -4$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial x}(-4x+3) = -4$$

7.B $z = x^3-y^3$

$$\frac{\partial z}{\partial x} = 3x^2 \qquad \frac{\partial z}{\partial y} = -3y^2$$

$$\frac{\partial^2 z}{\partial x^2} = 6x \qquad \frac{\partial^2 z}{\partial y^2} = -6y$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial y}(3x^2) = 0$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial x}(-3y^2) = 0$$

7.c $z = \ln(x^2+y^2)$

$$\frac{\partial z}{\partial x} = \frac{1}{(x^2+y^2)}(x^2+y^2)' = \frac{1}{x^2+y^2}(2x) = \frac{2x}{x^2+y^2}$$

$$\frac{\partial z}{\partial y} = \frac{1}{(x^2+y^2)}(x^2+y^2)' = \frac{1}{x^2+y^2}(2y) = \frac{2y}{x^2+y^2}$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{(x^2+y^2)(2x)' - (2x)(x^2+y^2)'}{(x^2+y^2)^2} = \frac{(x^2+y^2)(2) - (2x)(2x)}{(x^2+y^2)^2} = \frac{2x^2+2y^2-4x^2}{(x^2+y^2)^2} = \frac{2y^2-2x^2}{(x^2+y^2)^2}$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{(x^2+y^2)(2y)' - (2y)(x^2+y^2)'}{(x^2+y^2)^2} = \frac{(x^2+y^2)(2) - (2y)(2y)}{(x^2+y^2)^2} = \frac{2x^2+2y^2-4y^2}{(x^2+y^2)^2} = \frac{2x^2-2y^2}{(x^2+y^2)^2}$$

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$$\frac{\partial z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$= \frac{\partial}{\partial y}\left(\frac{2x}{x^2+y^2}\right)$$

$$= \frac{\partial}{\partial y}\left[\frac{(x^2+y^2)(2x)' - (2x)(x^2+y^2)'}{(x^2+y^2)^2}\right]$$

$$= \frac{(x^2+y^2)(0) - 2x(2y)}{(x^2+y^2)^2}$$

$$= \frac{0 - 4xy}{(x^2+y^2)^2}$$

$$= \frac{-4xy}{(x^2+y^2)^2}$$

$$\frac{\partial z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$

$$= \frac{\partial}{\partial x}\left(\frac{2y}{x^2+y^2}\right)$$

$$= \frac{\partial}{\partial x}\left[\frac{(x^2+y^2)(2y)' - (2y)(x^2+y^2)'}{(x^2+y^2)^2}\right]$$

$$= \frac{(x^2+y^2)(0) - 2y(2x)}{(x^2+y^2)^2}$$

$$= \frac{0 - 4xy}{(x^2+y^2)^2}$$

$$= \frac{-4xy}{(x^2+y^2)^2}$$

7.D $z = \dfrac{x}{x+2y}$

$$\frac{\partial z}{\partial x} = \frac{(x+2y)(x)' - (x)(x+2y)'}{(x+2y)^2}$$

$$= \frac{(x+2y)(1) - x(1)}{(x+2y)^2}$$

$$= \frac{\cancel{x}+2y-\cancel{x}}{(x+2y)^2}$$

$$= \frac{2y}{(x+2y)^2}$$

$$\frac{\partial z}{\partial y} = \frac{(x+2y)(x)' - (x)(x+2y)'}{(x+2y)^2}$$

$$= \frac{(x+2y)(0) - x(2)}{(x+2y)^2}$$

$$= \frac{0-2x}{(x+2y)^2}$$

$$= \frac{-2x}{(x+2y)^2}$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{(x+2y)^2(2y)' - (2y)[(x+2y)^2]'}{[(x+2y)^2]^2}$$

$$= \frac{(x+2y)^2(0) - 2y[2(x+2y)(x+2y)']}{(x+2y)^4}$$

$$= \frac{0-2y[2(x+2y)(1)]}{(x+2y)^4}$$

$$= \frac{-2y(2x+4y)}{(x+2y)^4}$$

$$= \frac{-4xy-8y^2}{(x+2y)^4}$$

$$= \frac{-4y\cancel{(x+2y)}}{(x+2y)^{\cancel{4}\,3}} = \frac{-4y}{(x+2y)^3}$$

1061

$$\frac{\partial^2 z}{\partial y^2} = \frac{(x+2y)^2(-2x)' - (-2x)[(x+2y)^2]'}{[(x+2y)^2]^2}$$

$$= \frac{(x+2y)^2(0) + 2x[2(x+2y)(x+2y)']}{(x+2y)^4}$$

$$= \frac{0 + 2x[2(x+2y)(2)]}{(x+2y)^4}$$

$$= \frac{2x[4(x+2y)]}{(x+2y)^4}$$

$$= \frac{2x(4x+8y)}{(x+2y)^4}$$

$$= \frac{8x^2+16xy}{(x+2y)^4}$$

$$= \frac{8x\cancel{(x+2y)}}{(x+2y)^{\cancel{4}\,3}}$$

$$= \frac{8x}{(x+2y)^3}$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$= \frac{\partial}{\partial y}\left(\frac{2y}{(x+2y)^2}\right)$$

$$= \frac{\partial}{\partial y}\left\{\frac{(x+2y)^2(2y)' - (2y)[(x+2y)^2]'}{[(x+2y)^2]^2}\right\}$$

$$= \frac{\partial}{\partial y}\left\{\frac{(x+2y)^2(2) - 2y[2(x+2y)(x+2y)']}{(x+2y)^4}\right\}$$

$$= \frac{\partial}{\partial y}\left\{\frac{2(x+2y)^2 - 2y[2(x+2y)(2)]}{(x+2y)^4}\right\}$$

$$= \frac{2(x+2y)^2 - 2y[4(x+2y)]}{(x+2y)^4}$$

$$= \frac{2(x+2y)^2 - 8y(x+2y)}{(x+2y)^4}$$

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$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\cancel{(x+2y)}[2(x+2y)-8y]}{(x+2y)^{\cancel{4}\,3}}$$

$$= \frac{2x+4y-8y}{(x+2y)^3}$$

$$= \frac{2x-4y}{(x+2y)^3}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$

$$= \frac{\partial}{\partial x}\left(\frac{-2x}{(x+2y)^2}\right)$$

$$= \frac{\partial}{\partial x}\left\{\frac{(x+2y)^2(-2x)' - (-2x)[(x+2y)^2]'}{[(x+2y)^2]^2}\right\}$$

$$= \frac{\partial}{\partial x}\left\{\frac{(x+2y)^2(-2) + 2x[2(x+2y)(x+2y)']}{(x+2y)^4}\right\}$$

$$= \frac{\partial}{\partial x}\left\{\frac{-2(x+2y)^2 + 2x[2(x+2y)(1)]}{(x+2y)^4}\right\}$$

$$= \frac{-2(x+2y)^2 + 2x[2(x+2y)]}{(x+2y)^4}$$

$$= \frac{-2(x+2y)^2 + 4x(x+2y)}{(x+2y)^4}$$

$$= \frac{\cancel{(x+2y)}[-2(x+2y)+4x]}{(x+2y)^{\cancel{4}\,3}}$$

$$= \frac{-2x-4y+4x}{(x+2y)^3}$$

$$= \frac{2x-4y}{(x+2y)^3}$$

7.E $z = (2x-3y^2+10)^3$

1081

$$\frac{\partial z}{\partial x} = 3(2x-3y^2+10)^2(2x-3y^2+10)'$$
$$= 3(2x-3y^2+10)^2(2)$$
$$= 6(2x-3y^2+10)^2$$

$$\frac{\partial z}{\partial y} = 3(2x-3y^2+10)^2(2x-3y^2+10)'$$
$$= 3(2x-3y^2+10)^2(-6y)$$
$$= -18y(2x-3y^2+10)^2$$

$$\frac{\partial^2 z}{\partial x^2} = 12(2x-3y^2+10)(2x-3y^2+10)'$$
$$= 12(2x-3y^2+10)(2)$$
$$= 24(2x-3y^2+10)$$

$$\frac{\partial^2 z}{\partial y^2} = (-18y)[(2x-3y^2+10)^2]' + (2x-3y^2+10)^2(-18y)'$$
$$= -18y[2(2x-3y^2+10)(2x-3y^2+10)'] + (2x-3y^2+10)^2(-18)$$
$$= -18y[2(2x-3y^2+10)(-6y)] - 18(2x-3y^2+10)^2$$
$$= -18y[-12y(2x-3y^2+10)] - 18(2x-3y^2+10)^2$$
$$= 216y^2(2x-3y^2+10) - 18(2x-3y^2+10)^2$$

$$\frac{\partial^2 z}{\partial y\partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$
$$= \frac{\partial}{\partial y}[6(2x-3y^2+10)^2]$$
$$= \frac{\partial}{\partial y}[12(2x-3y^2+10)(2x-3y^2+10)']$$
$$= \frac{\partial}{\partial y}[12(2x-3y^2+10)(-6y)]$$

$$\frac{\partial^2 z}{\partial y \partial x} = -72y(2x-3y^2+10)$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$

$$= \frac{\partial}{\partial x}[-18y(2x-3y^2+10)^2]$$

$$= \frac{\partial}{\partial x}\{(-18y)[(2x-3y^2+10)^2]' + (2x-3y^2+10)^2(-18y)'\}$$

$$= \frac{\partial}{\partial x}\{-18y[2(2x-3y^2+10)(2x-3y^2+10)'] + (2x-3y^2+10)^2(0)\}$$

$$= \frac{\partial}{\partial x}\{-18y[2(2x-3y^2+10)(2)] + 0\}$$

$$= -18y[4(2x-3y^2+10)]$$

$$= -72y(2x-3y^2+10)$$

7.F $z = xe^y$

$$\frac{\partial z}{\partial x} = e^y$$

$$\frac{\partial z}{\partial y} = (x)(e^y)' + (e^y)(x)'$$

$$= x[e^y(y)' + e^y(0)]$$

$$= x[e^y(1) + 0]$$

$$= x(e^y)$$

$$= xe^y$$

$$\frac{\partial^2 z}{\partial x^2} = e^y(y)'$$

$$= e^y(0)$$

$$= 0$$

$$\frac{\partial^2 z}{\partial y^2} = (x)(e^y)' + (e^y)(x)'$$

$$= x[e^y(y)'] + e^y(0)$$

$$= x[e^y(1)] + 0$$

$$= x(e^y) = xe^y$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$
$$= \frac{\partial}{\partial y}(e^y)$$
$$= \frac{\partial}{\partial y}\left[e^y (y)'\right]$$
$$= \frac{\partial}{\partial y}\left[e^y (1)\right]$$
$$= e^y$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$
$$= \frac{\partial}{\partial x}(xe^y)$$
$$= e^y$$

8.A $z = (x-1)^4 + (y-3)^4 + 10$

$$\frac{\partial z}{\partial x} = 4(x-1)^3 (x-1)' + 4(y-3)^3 (y-3)'$$
$$= 4(x-1)^3 (1) + 4(y-3)^3 (0)$$
$$= 4(x-1)^3 + 0$$
$$= 4(x-1)^3$$

$$\frac{\partial z}{\partial y} = 4(x-1)^3 (x-1)' + 4(y-3)^3 (y-3)'$$
$$= 4(x-1)^3 (0) + 4(y-3)^3 (1)$$
$$= 0 + 4(y-3)^3$$
$$= 4(y-3)^3$$

$\frac{\partial z}{\partial x} = 0$

$4(x-1)^3 = 0$

$(x-1)^3 = \frac{0}{4}$

$(x-1)^3 = 0$

$x-1 = \sqrt[3]{0}$

$x-1 = 0$

$x = 1$

$\frac{\partial z}{\partial y} = 0$

$4(y-3)^3 = 0$

$(y-3)^3 = \frac{0}{4}$

$(y-3)^3 = 0$

$y-3 = \sqrt[3]{0}$

$y-3 = 0$

$y = 3$

8.B $z = \ln(2x - x^2 + y^2)$

$$\frac{\partial z}{\partial x} = \frac{1}{2x-x^2+y^2}(2x-x^2+y^2)'$$
$$= \frac{1}{2x-x^2+y^2}(2-2x)$$
$$= \frac{2-2x}{2x-x^2+y^2}$$

$$\frac{\partial z}{\partial y} = \frac{1}{2x-x^2+y^2}(2x-x^2+y^2)'$$
$$= \frac{1}{2x-x^2+y^2}(2y)$$
$$= \frac{2y}{2x-x^2+y^2}$$

$\frac{\partial z}{\partial x} = 0$

$\frac{2-2x}{2x-x^2+y^2} = 0$

$\to 2-2x = 0(2x-x^2+y^2)$

$2-2x = 0$

$-2x = -2$

$x = \frac{-2}{-2} = 1$

1121

$$\frac{\partial z}{\partial y} = 0$$

$$\frac{2y}{2x - x^2 + y^2} = 0$$

$$2y = 0(2x - x^2 + y^2)$$

$$2y = 0$$

$$y = \frac{0}{2}$$

$$y = 0$$

P.C $z = \dfrac{2xy}{x+y}$

$$\frac{\partial z}{\partial x} = \frac{(x+y)(2xy)' - (2xy)(x+y)'}{(x+y)^2}$$

$$= \frac{(x+y)(2y) - 2xy(1)}{(x+y)^2}$$

$$= \frac{\cancel{2xy} + 2y^2 - \cancel{2xy}}{(x+y)^2}$$

$$= \frac{2y^2}{(x+y)^2}$$

$$\frac{\partial z}{\partial y} = \frac{(x+y)(2xy)' - (2xy)(x+y)'}{(x+y)^2}$$

$$= \frac{(x+y)(2x) - 2xy(1)}{(x+y)^2}$$

$$= \frac{2x^2 + \cancel{2xy} - \cancel{2xy}}{(x+y)^2}$$

$$= \frac{2x^2}{(x+y)^2}$$

$$\frac{\partial z}{\partial x} = 0$$

$$\frac{2y^2}{(x+y)^2} = 0$$

$$2y^2 = 0(x+y)^2$$

$$2y^2 = 0$$

$$y^2 = \frac{0}{2}$$

$$y^2 = 0$$

$$y = \sqrt{0}$$

$$y = 0$$

$$\frac{\partial z}{\partial y} = 0$$

$$\frac{2x^2}{(x+y)^2} = 0$$

$$2x^2 = 0(x+y)^2$$

$$2x^2 = 0$$

$$x^2 = \frac{0}{2}$$

$$x^2 = 0$$

$$x = \sqrt{0}$$

$$x = 0$$

8.D $z = \sqrt{x+y}$

$$z = (x+y)^{1/2}$$

$$\frac{\partial z}{\partial x} = \frac{1}{2}(x+y)^{-1/2}(x+y)'$$

$$= \frac{1}{2(x+y)^{1/2}}(1)$$

$$= \frac{1}{2\sqrt{x+y}}$$

$$\frac{\partial z}{\partial y} = \frac{1}{2}(x+y)^{-1/2}(x+y)'$$

$$= \frac{1}{2(x+y)^{1/2}}(1)$$

$$= \frac{1}{2\sqrt{x+y}}$$

$$\frac{\partial z}{\partial x} = 0$$

$$\frac{1}{2\sqrt{x+y}} = 0$$

$$1 = 0(2\sqrt{x+y})$$

"O SISTEMA NÃO TEM SOLUÇÃO".

$$\frac{\partial z}{\partial y} = 0$$

$$\frac{1}{2\sqrt{x+y}} = 0$$

$$1 = 0(2\sqrt{x+y})$$

"O SISTEMA NÃO TEM SOLUÇÃO".

FORONI

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9.A $z = \dfrac{x^2+y^2}{x}$

$$z(kx,ky) = \frac{(kx)^2+(ky)^2}{kx}$$

$$= \frac{k^2x^2+k^2y^2}{kx}$$

$$= \frac{k^{\cancel{2}}(x^2+y^2)}{\cancel{k}x}$$

$$= k\cdot\frac{x^2+y^2}{x}$$

$$= kz$$

O GRAU DE HOMOGENEIDADE É $m=1$

9.B $$\frac{\partial z}{\partial x} = \frac{(x)(x^2+y^2)' - (x^2+y^2)(x)'}{x^2}$$

$$= \frac{x(2x)-(x^2+y^2)(1)}{x^2}$$

$$= \frac{2x^2-(x^2+y^2)}{x^2}$$

$$= \frac{2x^2-x^2-y^2}{x^2}$$

$$= \frac{x^2-y^2}{x^2}$$

$$\frac{\partial z}{\partial y} = \frac{(x)(x^2+y^2)' - (x^2+y^2)(x)'}{x^2}$$

$$= \frac{x(2y)-(x^2+y^2)(0)}{x^2}$$

$$= \frac{2xy-0}{x^2} = \frac{2\cancel{x}y}{x^{\cancel{2}1}} = \frac{2y}{x}$$

$$\frac{\partial z}{\partial x}x + \frac{\partial z}{\partial y}y = mz$$

$$\left(\frac{x^2-y^2}{x^2}\right)x + \left(\frac{2y}{x}\right)y = 1\left(\frac{x^2+y^2}{x}\right)$$

$$\frac{x^2-y^2}{x} + \frac{2y^2}{x} = \frac{x^2+y^2}{x}$$

$$\frac{x^2-y^2+2y^2}{x} = \frac{x^2+y^2}{x}$$

$$\frac{x^2+y^2}{x} = \frac{x^2+y^2}{x}$$

10.A $z = x^{1/2}y^{3/2}$

O GRAU DE HOMOGENEIDADE É $m=2$

$$z(kx,ky) = (kx)^{1/2}(ky)^{3/2}$$
$$= k^{1/2}x^{1/2}k^{3/2}y^{3/2}$$
$$= k^{4/2}x^{1/2}y^{3/2}$$
$$= k^2x^{1/2}y^{3/2}$$

10.B POR "2" SEMPRE É O VALOR DA HOMOGENEIDADE?

10.C $\frac{\partial z}{\partial x} = (x^{1/2})(y^{3/2})' + (y^{3/2})(x^{1/2})'$

$$= (x^{1/2})(0) + y^{3/2}\left(\frac{1}{2}x^{-1/2}\right)$$

$$= 0 + y^{3/2}\left(\frac{1}{2x^{1/2}}\right)$$

$$= \frac{y^{3/2}}{2x^{1/2}}$$

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1/16 1

$$\frac{\partial z}{\partial y} = (x^{1/2})(y^{3/2})' + (y^{3/2})(x^{1/2})'$$

$$= x^{1/2}\left(\frac{3}{2}y^{1/2}\right) + y^{3/2}(0)$$

$$= x^{1/2}\left(\frac{3}{2}y^{1/2}\right) + 0$$

$$= \frac{3}{2}x^{1/2}y^{1/2}$$

$$\frac{\partial z}{\partial x}x + \frac{\partial z}{\partial y}y = mz$$

$$\frac{y^{3/2}}{2x^{1/2}}(x) + \frac{3}{2}x^{1/2}y^{1/2}(y) = 2(x^{1/2}y^{3/2})$$

$$\frac{y^{3/2}x^{1/2}}{2} + \frac{3}{2}x^{1/2}y^{3/2} = 2x^{1/2}y^{3/2}$$

$$\frac{\cancel{4}\,x^{1/2}y^{3/2}}{\cancel{2}\,1} = 2x^{1/2}y^{3/2}$$

$$2x^{1/2}y^{3/2} = 2x^{1/2}y^{3/2}$$

11.A $C = 10x + 8y$

$$R = 2x(10) + 5y(10) - \frac{x^2(10) + 3y2(10)}{10}$$

$$= 20x + 50y - \frac{\cancel{10}(x^2 + 3y^2)}{\cancel{10}\,1}$$

$$= 20x + 50y - (x2 + 3y^2)$$

$$= 20x + 50y - x^2 - 3y2$$

$$L = R - C$$

$$= 20x + 50y - x^2 - 3y2 - (10x + 8y)$$

FORONI

$L = 20x + 50y - x^2 - 3y^2 - 10x - 8y$

$= 10x + 42y - x^2 - 3y^2$

11.B $R = 20x + 50y - x^2 - 3y^2$

$\frac{\partial R}{\partial x} = 20 - 2x$

$\frac{\partial R}{\partial y} = 50 - 6y$

$\frac{\partial R}{\partial x} = 0$

$20 - 2x = 0$

$-2x = -20$

$x = \frac{-20}{-2}$

$x = 10$

$\frac{\partial R}{\partial y} = 0$

$50 - 6y = 0$

$-6y = -50$

$y = \frac{-\cancel{50}^{25}}{-\cancel{6}_{3}}$

$y = \frac{25}{3}$

$L = 10x + 42y - x^2 - 3y^2$

$\frac{\partial L}{\partial x} = 10 - 2x$

$\frac{\partial L}{\partial y} = 42 - 6y$

$\frac{\partial L}{\partial x} = 0$

$10 - 2x = 0$

$-2x = -10$

$x = \frac{-10}{-2}$

$x = 5$

$\frac{\partial L}{\partial y} = 0$

$42 - 6y = 0$

$-6y = -42$

$y = \frac{-42}{-6}$

$y = 7$