10.) LIMITES, CONTINUIDADE E EXTREMOS DE FUNÇÕES DE DUAS VARIÁVEIS. (LOUIS LEITHOLD) => PÁGINA 468 DO LIVRO MATEMÁTICA APLICADA À ECONOMIA E ADMINISTRAÇÃO (LOUIS LEITHOLD, EDITORA HARBRA)

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1) $\lim\limits_{(x,y)\to(3,2)} (3x-4y) = 3(3)-4(2) = 9-8 = 1$

2) $\lim\limits_{(x,y)\to(1,4)} (5x-3y) = 5(1)-3(4) = 5-12 = -7$

3) $\lim\limits_{(x,y)\to(1,1)} (x^2+y^2) = (1)^2+(1)^2 = 1+1 = 2$

4) $\lim\limits_{(x,y)\to(5,3)} (2x^2-y^2) = 2(5)^2-(3)^2 = 2(25)-9 = 50-9 = 41$

5) $\lim\limits_{(x,y)\to(-2,-4)} (x^2+2x-y) = (-2)^2+2(-2)-(-4) = 4-4+4 = 4$

6) $\lim\limits_{(x,y)\to(3,-1)} (x^2+y^2-4x+2y) = (3)^2+(-1)^2-4(3)+2(-1) = 9+1-12-2 = -4$

7) $\lim\limits_{(x,y)\to(-2,2)} y\sqrt[3]{x^3+4y}$ $\qquad x^3+4y \geq 0$

$\lim\limits_{(x,y)\to(-2,2)} y\sqrt[3]{x^3+4y} = 2\sqrt[3]{(-2)^3+4(2)} = 2\sqrt{-8+8} = 2\sqrt{0} = 2(0) = 0$

8) $\lim\limits_{(x,y)\to(5,2)} \sqrt{\dfrac{x^2+12y}{x-y^2}}$ $\qquad \dfrac{x^2+12y}{x-y^2} \geq 0 \qquad x-y^2 \neq 0 \qquad x-y^2 > 0$

$\lim\limits_{(x,y)\to(5,2)} \sqrt{\dfrac{x^2+12y}{x-y^2}} = \sqrt{\dfrac{(5)^2+12(2)}{5-(2)^2}} = \sqrt{\dfrac{25+24}{5-4}} = \sqrt{\dfrac{49}{1}} = 7$

9) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4-y^4}{x^2+y^2}$ $\qquad x^2+y^2 \neq 0$

$x^4 - y^4 = (x^2 + y^2)(x^2 - y^2)$

$$\lim_{(x,y)\to(0,0)} \frac{x^4 - y^4}{x^2 + y^2} = \frac{\cancel{(x^2+y^2)}(x^2-y^2)}{\cancel{(x^2+y^2)}} = x^2 - y^2 = (0)^2 - (0)^2 = 0$$

10) $\lim_{(x,y)\to(1,e)} \ln \frac{y}{x}$ $\qquad \frac{y}{x} > 0 \quad x \neq 0$

$$\lim_{(x,y)\to(1,e)} \ln \frac{y}{x} = \ln \frac{e}{1} = \ln e = 1$$

11) $\lim_{(x,y)\to(1,1)} \frac{e^x + e^y}{e^{-x} + e^{-y}}$ $\qquad e^{-x} + e^{-y} \neq 0$

$$\lim_{(x,y)\to(1,1)} \frac{e^x + e^y}{e^{-x} + e^{-y}} = \frac{e^1 + e^1}{e^{-1} + e^{-1}} = \frac{e + e}{2e^{-1}} = \frac{\cancel{2}e}{\cancel{2}e^{-1}} = \frac{e}{e^{-1}} = e \cdot e = e^2 = 7{,}3891$$

12) $\lim_{(x,y)\to(0,0)} \frac{e^x + e^y}{e^{-x} + e^{-y}}$ $\qquad e^{-x} + e^{-y} \neq 0$

$$\lim_{(x,y)\to(0,0)} \frac{e^x + e^y}{e^{-x} + e^{-y}} = \frac{e^0 + e^0}{e^0 + e^0} = \frac{1+1}{1+1} = \frac{2}{2} = 1$$

13) $\lim_{(x,y)\to(0,0)} \frac{y^3 + x^2y + 3x^2 + 3y^2}{x^3 + xy^2 - 3x^2 - 3y^2}$ $\qquad x^3 + xy^2 - 3x^2 - 3y^2 \neq 0$

$$\frac{y^3 + x^2y + 3x^2 + 3y^2}{x^3 + xy^2 - 3x^2 - 3y^2} = \frac{y(y^2 + x^2) + 3(x^2 + y^2)}{x(x^2 + y^2) - 3(x^2 + y^2)} = \frac{\cancel{(x^2+y^2)}(y+3)}{\cancel{(x^2+y^2)}(x-3)} = \frac{y+3}{x-3}$$

$$\lim_{(x,y)\to(0,0)} \frac{y^3 + x^2y + 3x^2 + 3y^2}{x^3 + xy^2 - 3x^2 - 3y^2} = \frac{y+3}{x-3} = \frac{0+3}{0-3} = \frac{3}{-3} = -1$$

(14) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3+y^3+x^2y+xy^2}{x^2+y^2}$ $\qquad x^2+y^2 \neq 0$

$$\frac{x^3+y^3+x^2y+xy^2}{x^2+y^2} = \frac{x^3+xy^2+y^3+x^2y}{x^2+y^2} = \frac{x(x^2+y^2)+y(y^2+x^2)}{x^2+y^2} =$$

$$= \frac{\cancel{(x^2+y^2)}(x+y)}{\cancel{(x^2+y^2)}} = x+y$$

$$\lim_{(x,y)\to(0,0)} \frac{x^3+y^3+x^2y+xy^2}{x^2+y^2} = x+y = 0+0=0$$

(15) $f(x,y) = x^2y^2(x+y)^2$ $\quad$ f É CONTÍNUA EM TODO PONTO EM $\mathbb{R}^2$

(16) $f(x,y) = (2x-3)^2(y^2+1)^3$ $\quad$ f É CONTÍNUA EM TODO PONTO EM $\mathbb{R}^2$

(17) $f(x,y) = \dfrac{x^4-y^4}{x^2-y^2}$

$$\left.\begin{aligned} x^2-y^2 &\neq 0 \\ x^2 &\neq y^2 \\ x &\neq \sqrt{y^2} \end{aligned}\right\} \to x \neq y$$

$D = \{(x,y) \in \mathbb{R}^2 \mid x \neq y\}$ $\quad$ f É CONTÍNUA PARA TODO $(x,y) \in D$.

(18) $f(x,y) = \left(\dfrac{1}{x^2-9} - \dfrac{1}{y}\right)^{1/3} = \left(\dfrac{y-(x^2-9)}{y(x^2-9)}\right)^{1/3} = \left(\dfrac{y-x^2+9}{y(x^2-9)}\right)^{1/3} =$

$$= \sqrt[3]{\frac{y-x^2+9}{y(x^2-9)}}$$

$$\frac{y-x^2+9}{y(x^2-9)} \geq 0 \qquad \left.\begin{aligned} y(x^2-9) &\neq 0 \\ x^2-9 &\neq \frac{0}{y} \end{aligned}\right\} \begin{aligned} x^2-9 &\neq 0 \\ x^2 &\neq 9 \\ x &\neq \pm\sqrt{9} \\ x &\neq \pm 3 \end{aligned} \qquad \left.\begin{aligned} y(x^2-9) &> 0 \\ x^2-9 &> \frac{0}{y} \end{aligned}\right\} \begin{aligned} x^2-9 &> 0 \\ x^2 &> 9 \\ x &> \pm\sqrt{9} \\ x &> \pm 3 \end{aligned}$$

$D = \{(x,y) \in \mathbb{R}^2 \mid y-x^2+9 \geq 0 \text{ E } x > \pm 3\}$

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f É CONTÍNUA PARA TODO $(x,y) \in D$.

19) $f(x,y) = \ln x - \ln y$ $\quad x > 0 \quad y > 0$

$D = \{(x,y) \in \mathbb{R}^2 / x > 0 \text{ E } y > 0\}$

f É CONTÍNUA PARA TODO $(x,y) \in D$.

20) $f(x,y) = \ln(9 - x^2 - y^2) + \ln(x^2 + y^2 - 1)$

$9 - x^2 - y^2 > 0$
$-x^2 - y^2 > -9 \quad (-1)$
$x^2 + y^2 < 9$

$x^2 + y^2 - 1 > 0$
$x^2 + y^2 > 1$

$D = \{(x,y) \in \mathbb{R}^2 / 1 < x^2 + y^2 < 9\}$ $\quad$ f É CONTÍNUA PARA TODO $(x,y) \in D$.

21) $f(x,y) = 2x^2 + y^2 - 8x - 2y + 14$

$\frac{\partial z}{\partial x} = 4x - 8$ $\qquad$ $\frac{\partial z}{\partial y} = 2y - 2$

$4x - 8 = 0$
$4x = 8$
$x = \frac{8}{4} = 2$

$2y - 2 = 0$
$2y = 2$
$y = \frac{2}{2} = 1$

PONTO CRÍTICO: $(2,1)$

$\frac{\partial^2 z}{\partial x^2} = 4$ $\qquad$ $\frac{\partial^2 z}{\partial y^2} = 2$

$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial x}(2y - 2) = 0$

$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial y}(4x - 8) = 0$

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$$H(x,y) = \frac{\partial^2 z}{\partial x^2} \cdot \frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x \partial y} \cdot \frac{\partial^2 z}{\partial y \partial x}$$
$$= 4 \cdot 2 - 0 \cdot 0$$
$$= 8 - 0 = 8$$

COMO O $H > 0$ E A $\frac{\partial^2 z}{\partial x^2} > 0$, O PONTO É MÍNIMO RELATIVO.

22) $f(x,y) = x^2 + y^2 - 2x - y + 1$

$$\frac{\partial z}{\partial x} = 2x - 2 \qquad \frac{\partial z}{\partial y} = 2y - 1$$

$$2x - 2 = 0 \qquad 2y - 1 = 0$$
$$2x = 2 \qquad 2y = 1$$
$$x = \frac{2}{2} = 1 \qquad y = \frac{1}{2}$$

PONTO CRÍTICO: $\left(1, \frac{1}{2}\right)$

$$\frac{\partial^2 z}{\partial x^2} = 2 \qquad \frac{\partial^2 z}{\partial y^2} = 2$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial x}(2y - 1) = 0$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial y}(2x - 2) = 0$$

$$H(x,y) = \frac{\partial^2 z}{\partial x^2} \cdot \frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x \partial y} \cdot \frac{\partial^2 z}{\partial y \partial x}$$
$$= 2 \cdot 2 - 0 \cdot 0$$
$$= 4 - 0 = 4$$

COMO O $H > 0$ E A $\frac{\partial^2 z}{\partial x^2} > 0$, O PONTO É MÍNIMO RELATIVO

23) $f(x,y) = x^3 + y^2 - 6x^2 + y - 1$

$\frac{\partial z}{\partial x} = 3x^2 - 12x$ $\qquad$ $\frac{\partial z}{\partial y} = 2y + 1$

$3x^2 - 12x = 0$
$3x(x-4) = 0$
$x = 0 \quad x = 4$

$2y + 1 = 0$
$2y = -1$
$y = -\frac{1}{2}$

PONTOS CRÍTICOS $\left(4, -\frac{1}{2}\right)$ $\left(0, -\frac{1}{2}\right)$

$\frac{\partial^2 z}{\partial x^2} = 6x - 12$ $\qquad$ $\frac{\partial^2 z}{\partial y^2} = 2$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial x}(2y+1) = 0$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial y}(3x^2 - 12x) = 0$$

$$H(x,y) = \frac{\partial^2 z}{\partial x^2} \cdot \frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x \partial y} \cdot \frac{\partial^2 z}{\partial y \partial x}$$
$$= (6x-12)2 - 0.0$$
$$= 12x - 24 - 0$$
$$= 12x - 24$$

NO PONTO $\left(4, -\frac{1}{2}\right)$:

$H(x,y) = 12x - 24$
$= 12(4) - 24$
$= 48 - 24$
$= 24$

$\frac{\partial^2 z}{\partial x^2} = 6x - 12$
$= 6(4) - 12$
$= 24 - 12$
$= 12$

COMO $H > 0$ E $\frac{\partial^2 z}{\partial x^2} > 0$, O PONTO É MÍNIMO RELATIVO

NO PONTO $\left(0, -\frac{1}{2}\right)$:

$H(x,y) = 12x - 24$
$= 12(0) - 24$
$= 0 - 24$
$= -24$

COMO O $H < 0$, ESSE É UM PONTO DE SELA.

24) $f(x,y) = x^2 - 4xy + y^3 + 4y$

$\frac{\partial z}{\partial x} = 2x - 4y$

$\frac{\partial z}{\partial y} = -4x + 3y^2 + 4$

$2x - 4y = 0$
$2x = 4y$
$x = \frac{4y}{2}$
$x = 2y$

$-4x + 3y^2 + 4 = 0$
$-4(2y) + 3y^2 + 4 = 0$
$-8y + 3y^2 + 4 = 0$
$3y^2 - 8y + 4 = 0 < \begin{matrix} y = 2 \\ y = \frac{2}{3} \end{matrix}$

$x = 2(2)$
$x = 4$

$x = 2\left(\frac{2}{3}\right)$
$x = \frac{4}{3}$

PONTOS CRÍTICOS: $(4,2)\left(\frac{4}{3},\frac{2}{3}\right)$

$$\frac{\partial^2 z}{\partial x^2} = 2 \qquad \frac{\partial^2 z}{\partial y^2} = 6y$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial x}(-4x+3y^2+4) = -4$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial y}(2x-4y) = -4$$

$$H(x,y) = \frac{\partial^2 z}{\partial x^2}\cdot\frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x \partial y}\cdot\frac{\partial^2 z}{\partial y \partial x}$$
$$= (2)(6y)-(-4)(-4)$$
$$= 12y-(16)$$
$$= 12y-16$$

NO PONTO $(4,2)$:

$$H(x,y) = 12y-16 \qquad \frac{\partial^2 z}{\partial x^2} = 2$$
$$= 12(2)-16$$
$$= 24-16$$
$$= 8$$

COMO $H>0$ E $\frac{\partial^2 z}{\partial x^2} > 0$, O PONTO É MÍNIMO RELATIVO.

NO PONTO $\left(\frac{4}{3},\frac{2}{3}\right)$:

COMO O $H<0$, ESSE É UM PONTO DE SELA.

$$H(x,y) = 12y-16 = \overset{4}{\cancel{12}}\left(\frac{2}{\cancel{3}_1}\right)-16 = 8-16 = -8$$

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25) $f(x,y) = \frac{1}{x} - \frac{64}{y} + xy$

$\frac{\partial z}{\partial x} = -\frac{1}{x^2} + y$

$\frac{\partial z}{\partial y} = \frac{64}{y^2} + x$

$-\frac{1}{x^2} + y = 0$

$\frac{-1 + x^2 y = 0(x^2)}{x^2}$

$x^2 y - 1 = 0$

$x^2 y = 1$

$x(xy) = 1$

$xy = \frac{1}{x}$

$\frac{64}{y^2} + x = 0$

$\frac{64 + xy^2 = 0(y^2)}{y^2}$

$xy^2 + 64 = 0$

$xy^2 = -64$

$y(xy) = -64$

COMO $xy = \frac{1}{x}$, ENTÃO:

$y\left(\frac{1}{x}\right) = -64$

$\frac{y}{x} = -64$

$y = -64x$

$y = -\overset{16}{\cancel{64}}\left(-\frac{1}{\cancel{4}_1}\right)$

$y = 16$

$x^2 y = 1$

$x^2(-64x) = 1$

$-64x^3 = 1$

$x^3 = -\frac{1}{64}$

$x = \sqrt[3]{-\frac{1}{64}}$

$x = -\frac{1}{4}$

PONTO CRÍTICO: $\left(-\frac{1}{4}, 16\right)$

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$$\frac{\partial^2 z}{\partial x^2} = \frac{2}{x^3} \qquad \frac{\partial^2 z}{\partial y^2} = -\frac{128}{y^3}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial x}\left(\frac{64}{y^2} + x\right) = 1$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial y}\left(\frac{-1}{x^2} + y\right) = 1$$

$$H(x,y) = \frac{\partial^2 z}{\partial x^2} \cdot \frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x \partial y} \cdot \frac{\partial^2 z}{\partial y \partial x}$$

$$= \frac{2}{x^3} \cdot \left(\frac{-128}{y^3}\right) - (1)(1)$$

$$= -\frac{256}{x^3 y^3} - 1$$

NO PONTO $\left(-\frac{1}{4}, 16\right)$:

$$H(x,y) = -\frac{256}{x^3 y^3} - 1 = \frac{-256}{\left(\frac{-1}{4}\right)^3 (16)^3} - 1 = \frac{-256}{\frac{-1}{64}(4.096)} - 1 = \frac{-256}{-64} - 1 = 4 - 1 = 3$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{2}{x^3} = \frac{2}{\left(\frac{-1}{4}\right)^3} = \frac{2}{\frac{-1}{64}} = 2(-64) = -128$$

COMO O $H > 0$ E $\frac{\partial^2 z}{\partial x^2} < 0$, O PONTO É MÁXIMO RELATIVO.

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26 $f(x,y) = 18x^2 - 32y^2 - 36x - 128y - 110$

$\frac{\partial z}{\partial x} = 36x - 36$ $\frac{\partial z}{\partial y} = 64y - 128$

$36x - 36 = 0$
$36x = 36$
$x = \frac{36}{36}$
$x = 1$

$64y - 128 = 0$
$64y = 128$
$y = \frac{128}{64}$
$y = 2$

PONTO CRÍTICO: $(1, 2)$

$\frac{\partial^2 z}{\partial x^2} = 36$ $\frac{\partial^2 z}{\partial y^2} = 64$

$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$
$= \frac{\partial}{\partial x}(64y - 128)$
$= 0$

$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$
$= \frac{\partial}{\partial y}(36x - 36)$
$= 0$

$H(x,y) = \frac{\partial^2 z}{\partial x^2} \cdot \frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x \partial y} \cdot \frac{\partial^2 z}{\partial y \partial x}$
$= 36 \cdot 64 - 0 \cdot 0$
$= 2.304$

O PONTO NÃO É EXTREMO RELATIVO.

27 $f(x,y) = 4xy^2 - 2x^2y - x$

$\frac{\partial z}{\partial x} = 4y^2 - 4xy - 1$ $\frac{\partial z}{\partial y} = 8xy - 2x^2$

$$\begin{cases} 4y^2 - 4xy - 1 = 0 \\ 8xy - 2x^2 = 0 \end{cases} \quad (2) \quad \begin{cases} 8y^2 - \cancel{8xy} - 2 = 0 \\ \cancel{8xy} - 2x^2 = 0 \end{cases}$$

$$\begin{cases} 8y^2 - 2 = 0 \\ -2x^2 = 0 \end{cases}$$

$8y^2 - 2 = 0$

$8y^2 = 2$

$y^2 = \frac{2}{8}$

$y^2 = \frac{1}{4}$

$y = \pm\sqrt{\frac{1}{4}}$

$y = \pm\frac{1}{2}$

$-2x^2 = 0$

$x^2 = \frac{0}{-2}$

$x^2 = 0$

$x = 0$

PONTOS CRÍTICOS: $\left(0, \frac{1}{2}\right)\left(0, -\frac{1}{2}\right)$

$$\frac{\partial^2 z}{\partial x^2} = -4y \qquad \frac{\partial^2 z}{\partial y^2} = 8x$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial x}(8xy - 2x^2) = 8y - 4x$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial y}(4y^2 - 4xy - 1) = 8y - 4x$$

$$H(x,y) = \frac{\partial^2 z}{\partial x^2} \cdot \frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x \partial y} \cdot \frac{\partial^2 z}{\partial y \partial x}$$

$$= -4y(8x) - (8y - 4x)(8y - 4x)$$

$$= -32xy - (8y - 4x)^2$$

NO PONTO $\left(0, \frac{1}{2}\right)$:

$$H(x,y) = -32xy - (8y-4x)^2$$
$$= -32(0)\left(\frac{1}{2}\right) - \left[\cancel{8}^{4}\left(\frac{1}{\cancel{2}_1}\right) - 4(0)\right]^2$$
$$= 0 - [4(1) - 0]^2$$
$$= -(4)^2$$
$$= -16$$

COMO O $H<0$, ESSE É UM PONTO DE SELA.

NO PONTO $\left(0, -\frac{1}{2}\right)$:

$$H(x,y) = -32xy - (8y-4x)^2$$
$$= -32(0)\left(-\frac{1}{2}\right) - \left[\cancel{8}^{4}\left(\frac{-1}{\cancel{2}_1}\right) - 4(0)\right]^2$$
$$= 0 - [4(-1) - 0]^2$$
$$= -(-4)^2$$
$$= -(16)$$
$$= -16$$

COMO O $H<0$, ESSE É UM PONTO DE SELA.

28) $f(x,y) = x^3 + y^3 - 18xy$

$$\frac{\partial z}{\partial x} = 3x^2 - 18y \qquad \frac{\partial z}{\partial y} = 3y^2 - 18x$$

$$3x^2 - 18y = 0$$
$$3x^2 = 18y$$
$$y = \frac{\cancel{3}^1 x^2}{\cancel{18}_6} = \frac{1}{6}x^2$$

$$3y^2 - 18x = 0$$
$$3\left(\frac{1}{6}x^2\right)^2 - 18x = 0$$
$$\cancel{3}^1\left(\frac{1}{\cancel{36}_{12}}x^4\right) - 18x = 0$$

$$\frac{1}{12}x^4 - 18x = 0$$
$$\frac{x^4 - 216x = 0(12)}{12}$$
$$x^4 - 216x = 0$$
$$x(x^3 - 216) = 0$$
$$x = 0 \quad x = 6$$

$y = \frac{1}{6} x^2$ $\qquad$ $y = \frac{1}{6} x^2$

$y = \frac{1}{6}(0)^2$ $\qquad$ $y = \frac{1}{6}(6)^2$

$y = 0$ $\qquad$ $y = \frac{1}{\cancel{6}_1}(\cancel{36}^{6}) \Rightarrow y = 6$

PONTOS CRÍTICOS: $(0,0)$ $(6,6)$

$\frac{\partial^2 z}{\partial x^2} = 6x$ $\qquad$ $\frac{\partial^2 z}{\partial y^2} = 6y$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial x}(3y^2 - 18x) = -18$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial y}(3x^2 - 18y) = -18$$

$$H(x,y) = \frac{\partial^2 z}{\partial x^2} \cdot \frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x \partial y} \cdot \frac{\partial^2 z}{\partial y \partial x}$$
$$= 6x \cdot 6y - (-18)(-18)$$
$$= 36xy - 324$$

NO PONTO $(0,0)$:

$$H(x,y) = 36xy - 324$$
$$= 36(0)(0) - 324$$
$$= 0 - 324$$
$$= -324$$

COMO O $H<0$, ESSE É UM PONTO DE SELA.

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NO PONTO (6,6):

$H(x,y) = 36xy - 324$
$= 36(6)(6) - 324$
$= 1.296 - 324$
$= 972$

$\frac{\partial^2 z}{\partial x^2} = 6x$
$= 6(6)$
$= 36$

COMO O $H > 0$ E $\frac{\partial^2 z}{\partial x^2} > 0$, O PONTO É MÍNIMO RELATIVO.

29) $f(x,y) = \frac{2}{xy} + x^2 + y^2$

$\frac{\partial z}{\partial x} = -\frac{2}{x^2 y} + 2x$

$\frac{\partial z}{\partial y} = -\frac{2}{xy^2} + 2y$

$-\frac{2}{x^2y} + 2x = 0$

$\frac{-2 + 2x^3y = 0(x^2y)}{x^2y}$

$-2 + 2x^3y = 0$

$2x^3y = 2$

$x^3y = \frac{2^1}{2_1}$

$x^3y = 1$

$y = \frac{1}{x^3}$

$-\frac{2}{xy^2} + 2y = 0$

$\frac{-2 + 2xy^3 = 0(xy^2)}{xy^2}$

$-2 + 2xy^3 = 0$

$2xy^3 = 2$

$xy^3 = \frac{2^1}{2_1}$

$xy^3 = 1$

$x\left(\frac{1}{x^3}\right)^3 = 1$

$x\left(\frac{1}{x^9}\right) = 1$

$\frac{1}{x^8} = 1$

$x^8 = 1$

$x = \pm 1$

$y = \frac{1}{(1)^3} = 1 \quad y = \frac{1}{(-1)^3} = -1$

PONTOS CRÍTICOS: (1,1)(-1,-1)

$$\frac{\partial^2 z}{\partial x^2} = \frac{4}{x^3 y} + 2 \qquad \frac{\partial^2 z}{\partial x^2} = \frac{4}{x y^3} + 2$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \qquad \frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$= \frac{\partial}{\partial x}\left(-\frac{2}{x y^2} + 2y\right) \qquad = \frac{\partial}{\partial y}\left(\frac{-2}{x^2 y} + 2x\right)$$

$$= \frac{2}{x^2 y^2} \qquad = \frac{2}{x^2 y^2}$$

$$H(x,y) = \frac{\partial^2 z}{\partial x^2} \cdot \frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x \partial y} \cdot \frac{\partial^2 z}{\partial y \partial x}$$

$$= \left(\frac{4}{x^3 y} + 2\right)\left(\frac{4}{x y^3} + 2\right) - \left(\frac{2}{x^2 y^2}\right)\left(\frac{2}{x^2 y^2}\right)$$

NO PONTO (1,1):

$$H(x,y) = \left(\frac{4}{x^3 y} + 2\right)\left(\frac{4}{x y^3} + 2\right) - \left(\frac{2}{x^2 y^2}\right)\left(\frac{2}{x^2 y^2}\right)$$

$$= \left(\frac{4}{(1)^3 1} + 2\right)\left(\frac{4}{1(1)^3} + 2\right) - \left(\frac{2}{(1)^2(1)^2}\right)\left(\frac{2}{(1)^2(1)^2}\right)$$

$$= (4+2)(4+2) - (2)(2)$$

$$= 6.6 - 4$$

$$= 36 - 4$$

$$= 32$$

COMO O $H > 0$ E A $\frac{\partial^2 z}{\partial x^2} > 0$, O PONTO É MÍNIMO RELATIVO.

$$\frac{\partial^2 z}{\partial x^2} = \frac{4}{x^3 y} + 2 = \frac{4}{(1)^3 1} + 2 = 4 + 2 = 6$$

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30) $f(x,y) = \dfrac{2x+2y+1}{x^2+y^2+1}$

$$\frac{\partial z}{\partial x} = \frac{-2x^2+2y^2+2-4xy-2x}{(x^2+y^2+1)^2} \qquad \frac{\partial z}{\partial y} = \frac{2x^2-2y^2+2-4xy-2y}{(x^2+y^2+1)^2}$$

$$\frac{-2x^2+2y^2+2-4xy-2x}{(x^2+y^2+1)^2} = 0 \qquad \frac{2x^2-2y^2+2-4xy-2y}{(x^2+y^2+1)^2} = 0$$

$$-2x^2+2y^2+2-4xy-2x = 0(x^2+y^2+1)^2 \qquad 2x^2-2y^2+2-4xy-2y = 0(x^2+y^2+1)^2$$

$$-2x^2+2y^2+2-4xy-2x = 0 \qquad 2x^2-2y^2+2-4xy-2y = 0$$

$$\begin{cases} -2x^2+2y^2+2-4xy-2x = 0 \\ 2x^2-2y^2+2-4xy-2y = 0 \end{cases}$$

$$4-8xy-2x-2y = 0$$

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31) $f(x,y) = e^{xy}$

$\frac{\partial z}{\partial x} = e^{xy}(xy)'$

$= e^{xy} y$

$= y e^{xy}$

$\frac{\partial z}{\partial y} = e^{xy}(xy)'$

$= e^{xy} x$

$= x e^{xy}$

$y e^{xy} = 0$

$y = \frac{0}{e^{xy}}$

$y = 0$

$x e^{xy} = 0$

$x = \frac{0}{e^{xy}}$

$x = 0$

PONTO CRÍTICO: (0,0)

$\frac{\partial^2 z}{\partial x^2} = y^2 e^{xy}$

$\frac{\partial^2 z}{\partial y^2} = x^2 e^{xy}$

$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$

$= \frac{\partial}{\partial x}(x e^{xy})$

$= e^{xy}(xy+1)$

$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$

$= \frac{\partial}{\partial y}(y e^{xy})$

$= e^{xy}(xy+1)$

$H(x,y) = \frac{\partial^2 z}{\partial x^2} \frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x \partial y} \cdot \frac{\partial^2 z}{\partial y \partial x}$

$= (y^2 e^{xy})(x^2 e^{xy}) - [e^{xy}(xy+1)][e^{xy}(xy+1)]$

NO PONTO (0,0):

FORONI

$H(x,y) = (y^2 e^{xy})(x^2 e^{xy}) - [e^{xy}(xy+1)][e^{xy}(xy+1)]$

$= [(0)^2 e^{(0)(0)}][(0)^2 e^{(0)(0)}] - \{e^{(0)(0)}[(0)(0)+1]\}\{e^{(0)(0)}[(0)(0)+1]\}$

$= (0.e^0)(0.e^0) - (e^0.1)(e^0.1)$

$= (0.1)(0.1) - (1.1)(1.1)$

$= 0.0 - (1)(1)$

$= 0 - 1$

$= -1$

COMO O $H < 0$, ESSE É UM PONTO DE SELA.

32 $f(x,y) = x^3 + x^2 + y^2 - xy + 8$

$\frac{\partial z}{\partial x} = 3x^2 + 2x - y$

$\frac{\partial z}{\partial y} = 2y - x$

$3x^2 + 2x - y = 0$

COMO $x = 2y$, ENTÃO:

$3(2y)^2 + 2(2y) - y = 0$

$3(4y^2) + 4y - y = 0$

$12y^2 + 3y = 0$

$3y(4y + 1) = 0$

$y = 0$ E $y = -\frac{1}{4}$

$2y - x = 0$

$x = 2y$

COMO $y = 0$ E $y = -\frac{1}{4}$, ENTÃO:

$x = 2(0)$

$x = 0$

$x = \cancel{2}\left(-\frac{1}{\cancel{4}_2}\right)$

$x = -\frac{1}{2}$

PONTOS CRÍTICOS: $(0,0)$ $\left(-\frac{1}{2}, -\frac{1}{4}\right)$

$\frac{\partial^2 z}{\partial x^2} = 6x + 2$

$\frac{\partial^2 z}{\partial y^2} = 2$

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$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$
$$= \frac{\partial}{\partial x}(2y - x)$$
$$= -1$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$
$$= \frac{\partial}{\partial y}(3x^2 + 2x - y)$$
$$= -1$$

$$H(x,y) = \frac{\partial^2 z}{\partial x^2} \cdot \frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x \partial y} \cdot \frac{\partial^2 z}{\partial y \partial x}$$
$$= (6x+2)2 - (-1)(-1)$$
$$= 12x + 4 - (1)$$
$$= 12x + 4 - 1$$
$$= 12x + 3$$

NO PONTO $(0,0)$:

$$H(x,y) = 12x + 3$$
$$= 12(0) + 3$$
$$= 0 + 3$$
$$= 3$$

$$\frac{\partial^2 z}{\partial x^2} = 6x + 2$$
$$= 6(0) + 2$$
$$= 0 + 2$$
$$= 2$$

COMO O $H > 3$ E A $\frac{\partial^2 z}{\partial x^2} > 0$ O PONTO É DE MÍNIMO RELATIVO.

NO PONTO $\left(-\frac{1}{2}, -\frac{1}{4}\right)$:

$$H(x,y) = 12x + 3$$
$$= \cancel{12}^{6}\left(\frac{-1}{\cancel{2}_{1}}\right) + 3$$
$$= -6 + 3 = -3$$

COMO O $H < 0$, ESSE É UM PONTO DE SELA.