LISTA III MATEMÁTICA I

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EXERCÍCIO 1:

1.A $\begin{pmatrix} 3 & 0 \\ 4 & 5 \end{pmatrix}$

CALCULANDO OS AUTOVALORES

$$\det(A - mI) = \begin{vmatrix} 3-m & 0 \\ 4 & 5-m \end{vmatrix}$$

$$= (3-m)(5-m) - (0)(4)$$

$$= 15 - 3m - 5m + m^2 - 0$$

$$= m^2 - 8m + 15$$

FÓRMULA DE BHASKARA

$m^2 - 8m + 15 = 0$

$\Delta = b^2 - 4ac$

$= (-8)^2 - 4(1)(15)$

$= 64 - 60$

$= 4$

$$m = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$= \frac{-(-8) \pm \sqrt{4}}{2(1)}$$

$$= \frac{8 \pm 2}{2} \begin{matrix} \nearrow 5 \\ \searrow 3 \end{matrix}$$

$m_1 = 5$ E $m_2 = 3$

CALCULANDO OS AUTOVETORES

PARA $m_1 = 5$

$$(A - 5I)V = \begin{bmatrix} -2 & 0 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$v_1 = 0 \text{ E } v_2 = 1$$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

PARA $m_2 = 3$

$$(A - 3I)V = \begin{bmatrix} 0 & 0 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$v_1 = 1 \text{ E } v_2 = -2$$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

1.B $\begin{pmatrix} -1 & 3 \\ -2 & 4 \end{pmatrix}$

CALCULANDO OS AUTOVALORES:

$$\det(A - mI) = \begin{vmatrix} -1-m & 3 \\ -2 & 4-m \end{vmatrix}$$

$$= (-1-m)(4-m) - 3(-2)$$

$$= -4 + m - 4m + m^2 + 6$$

$$= m^2 - 3m + 2$$

FÓRMULA DE BHASKARA:

$$m^2 - 3m + 2 = 0$$

$$\Delta = b^2 - 4ac$$

$$= (-3)^2 - 4(1)(2) \rightarrow \Delta = 9 - 8 = 1$$

$$m = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-(-3) \pm \sqrt{1}}{2(1)} \rightarrow m = \frac{3 \pm 1}{2}$$

$$m_1 = 2 \text{ E } m_2 =$$

CALCULANDO OS AUTOVETORES

PARA $r_1 = 2$

$$(A-2I)V = \begin{bmatrix} -3 & 3 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$v_1 = 1$ E $v_2 = 1$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

PARA $r_2 = 1$

$$(A-1I)V = \begin{bmatrix} -2 & 3 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$v_1 = 3$ E $v_2 = 2$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

1.C $\begin{pmatrix} 0 & -2 \\ 1 & -3 \end{pmatrix}$

CALCULANDO OS AUTOVALORES

$$\det(A - rI) = \begin{vmatrix} -r & -2 \\ 1 & -3-r \end{vmatrix}$$

$$= (-r)(-3-r) - (-2)(1)$$

$$= 3r + r^2 + 2$$

$$= r^2 + 3r + 2$$

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FÓRMULA DE BHASKARA:

$r^2 + 3r + 2 = 0$

$r = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$\Delta = b^2 - 4ac$

$= (3)^2 - 4(1)(2)$

$= 9 - 8$

$= 1$

$= \dfrac{-3 \pm \sqrt{1}}{2(1)}$

$= \dfrac{-3 \pm 1}{2}$ $\nearrow -1$ $\searrow -2$

$r_1 = -1$ e $r_2 = -2$

CALCULANDO OS AUTOVETORES

PARA $r_1 = -1$

$(A - (-1)I)V = \begin{bmatrix} 1 & -2 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$v_1 = 2$ e $v_2 = 1$.

$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

PARA $r_2 = -2$

$(A - (-2)I)V = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$v_1 = 1$ e $v_2 = 1$

$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

1-I) $\begin{bmatrix} 0 & 0 & -2 \\ 0 & 0 & 0 \\ 1 & 0 & -3 \end{bmatrix}$

CALCULANDO OS AUTOVALORES

$$\det(A-\lambda I) = \begin{vmatrix} -\lambda & 0 & -2 \\ 0 & -\lambda & 0 \\ 1 & 0 & -3-\lambda \end{vmatrix}$$

$$= -(0)\cancel{\begin{vmatrix} 0 & -2 \\ 0 & -3-\lambda \end{vmatrix}} + (-\lambda)\begin{vmatrix} -\lambda & -2 \\ 1 & -3-\lambda \end{vmatrix} - (0)\cancel{\begin{vmatrix} -1 & 0 \\ 1 & 0 \end{vmatrix}}$$

$$= -\lambda[(-\lambda)(-3-\lambda)-(-2)(1)]$$

$$= -\lambda(3\lambda+\lambda^2+2)$$

$$= -\lambda(\lambda^2+3\lambda+2)$$

$-\lambda = 0$

$\lambda_1 = 0$

FÓRMULA DE BHASKARA:

$\lambda^2+3\lambda+2=0$

$\Delta = b^2-4ac$

$= (3)^2-4(1)(2)$

$= 9-8$

$= 1$

$\lambda = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$= \dfrac{-3 \pm \sqrt{1}}{2(1)}$

$= \dfrac{-3 \pm 1}{2}$ $\nearrow -1$, $\searrow -2$

$\lambda_2 = -1$ E $\lambda_3 = -2$

CALCULANDO OS AUTOVETORES

PARA $r_1 = 0$

$$(A - 0I)V = \begin{bmatrix} 0 & 0 & -2 \\ 0 & 0 & 0 \\ 1 & 0 & -3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$v_1 = 0$, $v_2 = 1$ E $v_3 = 0$

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

PARA $r_2 = -1$

$$(A - (-1)I)V = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 1 & 0 & -2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$v_1 = 2$, $v_2 = 0$ E $v_3 = 1$

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

PARA $r_3 = -2$

$$(A - (-2)I)V = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 2 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$x_1 = 1, x_2 = 0$ E $x_3 = 1$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

1.E $\begin{pmatrix} 3 & 5 \\ -2 & 1 \end{pmatrix}$

CALCULANDO OS AUTOVALORES

$$\det(A - \lambda I) = \begin{vmatrix} 3-\lambda & 5 \\ -2 & 1-\lambda \end{vmatrix}$$

$$= (3-\lambda)(1-\lambda) - 5(-2)$$

$$= 3 - 3\lambda - \lambda + \lambda^2 + 10$$

$$= \lambda^2 - 4\lambda + 13$$

FÓRMULA DE BHASKARA:

$\lambda^2 - 4\lambda + 13 = 0$

$\Delta = b^2 - 4ac$

$= (-4)^2 - 4(1)(13)$

$= 16 - 52$

$= -36$

$$\lambda = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{-36}}{2(1)}$$

$$= \frac{4 \pm \sqrt{36(-1)}}{2}$$

$$= \frac{4 \pm 6\sqrt{-1}}{2}$$

$$= \frac{4 \pm 6i}{2} \begin{cases} 2+3i \\ 2-3i \end{cases}$$

$\lambda_1 = 2 + 3i$ E $\lambda_2 = 2 - 3i$

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CALCULANDO OS AUTOVETORES

PARA $r_1 = 2+3i$

$$(A-(2+3i)I)V = \begin{bmatrix} 1-3i & 5 \\ -2 & -1-3i \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1-3i & 5 \\ -2 & -1-3i \end{bmatrix} \times \left(\tfrac{2}{1-3i}\right) + \sim \begin{bmatrix} 1-3i & 5 \\ 0 & 0 \end{bmatrix}$$

$$-1-3i+5\left(\frac{2}{1-3i}\right) = 0$$

$$\frac{(-1-3i)(1-3i)+10=0}{\cancel{(1-3i)}}$$

$$-1+\cancel{3i}-\cancel{3i}+9i^2+10=0$$

$$-1+9i^2+10=0$$

$$-1+[9(\sqrt{-1})^2]+10=0$$

$$-1+[9(-1)]+10=0$$

$$-1-9+10=0$$

$$(1-3i)v_1+5v_2=0$$

$$5v_2=(-1+3i)v_1$$

$$v_2=\frac{(-1+3i)}{5}v_1$$

$$v_1=v_1$$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{-1+3i}{5} \end{pmatrix} v_1$$

FAZENDO $v_1 = 1$ OBTÉM-SE:

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{-1+3i}{5} \end{pmatrix} = \begin{pmatrix} 1 \\ -\frac{1}{5} \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{3}{5} \end{pmatrix} i$$

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PARA $r_2 = 2-3i$

$$(A-(2-3i)I)V = \begin{bmatrix} 1+3i & 5 \\ -2 & -1+3i \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1+3i & 5 \\ -2 & -1+3i \end{bmatrix} \begin{matrix} \times\left(\frac{2}{1+3i}\right) \\ + \end{matrix} \sim \begin{bmatrix} 1+3i & 5 \\ 0 & 0 \end{bmatrix}$$

$$-1+3i+5\left(\frac{2}{1+3i}\right)=0$$

$$-1+3i+\frac{10}{1+3i}=0$$

$$(-1+3i)(1+3i)+10=0$$

$$-1-\cancel{3i}+\cancel{3i}+9i^2+10=0$$

$$-1+\boxed{9(\sqrt{-1})^2}+10=0$$

$$-1+[9(-1)]+10=0$$

$$-1-9+10=0$$

$$(1+3i)v_1+5v_2=0$$

$$5v_2=(-1-3i)v_1$$

$$v_2=\left(\frac{-1-3i}{5}\right)v_1$$

$$v_1=v_1$$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{-1-3i}{5} \end{pmatrix} v_1$$

FAZENDO $v_1 = 1$ OBTÉM-SE:

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{-1-3i}{5} \end{pmatrix} = \begin{pmatrix} 1 \\ -\frac{1}{5} \end{pmatrix} - \begin{pmatrix} 0 \\ \frac{3}{5} \end{pmatrix} i$$

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1.F $\begin{pmatrix} 2 & 4 \\ -2 & -2 \end{pmatrix}$

CALCULANDO OS AUTOVALORES

$$det(A - \lambda I) = \begin{vmatrix} 2-\lambda & 4 \\ -2 & -2-\lambda \end{vmatrix}$$

$$= (2-\lambda)(-2-\lambda) - 4(-2)$$

$$= -4 \cancel{-2\lambda} \cancel{+2\lambda} + \lambda^2 + 8$$

$$= \lambda^2 + 4$$

$$\lambda^2 + 4 = 0$$

$$\lambda^2 = -4$$

$$\lambda = \sqrt{-4} \rightarrow \lambda = \sqrt{4(-1)}$$

$$= \pm 2\sqrt{-1}$$

$$= \pm 2i$$

CALCULANDO OS AUTOVETORES

PARA $\lambda_1 = 2i$

$$(A - 2iI)V = \begin{bmatrix} 2-2i & 4 \\ -2 & -2-2i \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2-2i & 4 \\ -2 & -2-2i \end{bmatrix} \times \left(\frac{2}{2-2i}\right) + \sim \begin{bmatrix} 2-2i & 4 \\ 0 & 0 \end{bmatrix}$$

$$-2-2i + 4\left(\frac{2}{2-2i}\right) = 0$$

$$(-2-2i)(2-2i) + 8 = 0$$

$$-4 \cancel{+4i} \cancel{-4i} + 4i^2 + 8 = 0 \rightarrow 4i^2 + 4 = 0$$

$$4(\sqrt{-1})^2 + 4 = 0$$

$$4(-1) + 4 = 0$$

$$-4 + 4 = 0$$

$(2-2i)v_1 + 4v_2 = 0$

$4v_2 = (-2+2i)v_1$

$v_2 = \left(\dfrac{-2+2i}{4}\right)v_1$

$= \left(\dfrac{-1+i}{2}\right)v_1$

$v_1 = v_1$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \dfrac{-1+i}{2} \end{pmatrix} v_1$$

FAZENDO $v_1 = 1$ OBTÉM-SE:

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \dfrac{-1+i}{2} \end{pmatrix} = \begin{pmatrix} 1 \\ -\frac{1}{2} \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix} i$$

PARA $\lambda_2 = -2i$

$$(A-(-2i)I)V = \begin{bmatrix} 2+2i & 4 \\ -2 & -2+2i \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2+2i & 4 \\ -2 & -2+2i \end{bmatrix} \times \left(\frac{2}{2+2i}\right) + \sim \begin{bmatrix} 2+2i & 4 \\ 0 & 0 \end{bmatrix}$$

$-2+2i + 4\left(\dfrac{2}{2+2i}\right) = 0$

$(2+2i)(2+2i) + 8 = 0$

$-4 - \cancel{4i} + \cancel{4i} + 4i^2 + 8 = 0$

$4i^2 + 4 = 0$

$4(\sqrt{-1})^2 + 4 = 0$

$4(-1) + 4 = 0$

$-4 + 4 = 0$

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$$(2+2i)v_1 + 4v_2 = 0$$

$$4v_2 = (-2-2i)v_1$$

$$v_2 = \left(\frac{-2-2i}{4}\right)v_1$$

$$= \left(\frac{-1-i}{2}\right)v_1 \qquad \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{-1-i}{2} \end{pmatrix} v_1$$

$$v_1 = v_1$$

FAZENDO $v_1 = 1$ OBTÉM-SE:

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{-1-i}{2} \end{pmatrix} = \begin{pmatrix} 1 \\ -\frac{1}{2} \end{pmatrix} - \begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix} i$$

11.6 $\begin{pmatrix} 2 & 5 \\ -1 & 0 \end{pmatrix}$

CALCULANDO OS AUTOVALORES

$$\det(A - \lambda I) = \begin{vmatrix} 2-\lambda & 5 \\ -1 & -\lambda \end{vmatrix}$$

$$= (2-\lambda)(-\lambda) - 5(-1)$$

$$= -2\lambda + \lambda^2 + 5$$

$$= \lambda^2 - 2\lambda + 5$$

FÓRMULA DE BHASKARA:

$$\lambda^2 - 2\lambda + 5 = 0$$

$$\Delta = b^2 - 4ac$$

$$= (-2)^2 - 4(1)(5)$$

$$\Delta = 4 - 20$$

$$= -16$$

$$\lambda = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{-16}}{2(1)}$$

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$$r = \frac{2 \pm \sqrt{16(-1)}}{2}$$

$$= \frac{2 \pm 4\sqrt{-1}}{2}$$

$$= \frac{2 \pm 4i}{2} \begin{cases} \nearrow 1+2i \\ \searrow 1-2i \end{cases}$$

$r_1 = 1+2i$ E $r_2 = 1-2i$

CALCULANDO OS AUTOVETORES

PARA $r_1 = 1+2i$

$$(A-(1+2i)I)V = \begin{bmatrix} 1-2i & 5 \\ -1 & -1-2i \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1-2i & 5 \\ -1 & -1-2i \end{bmatrix} \begin{matrix} \times\left(\frac{1}{1-2i}\right) \\ \hookleftarrow + \end{matrix} \sim \begin{bmatrix} 1-2i & 5 \\ 0 & 0 \end{bmatrix}$$

$$(-1-2i) + 5\left(\frac{1}{1-2i}\right) = 0$$

$$(-1-2i)(1-2i) + 5 = 0$$

$$-1 + \cancel{2i} - \cancel{2i} + 4i^2 + 5 = 0$$

$$4i^2 + 4 = 0$$

$$4(\sqrt{-1})^2 + 4 = 0$$

$$4(-1) + 4 = 0$$

$$-4 + 4 = 0$$

$$(1-2i)v_1 + 5v_2 = 0$$

$$5v_2 = (-1+2i)v_1$$

$$v_2 = \left(\frac{-1+2i}{5}\right)v_1$$

$$v_1 = v_1$$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{-1+2i}{5} \end{pmatrix} v_1$$

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FAZENDO $v_1 = 1$ OBTÉM-SE:

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{-1+2i}{5} \end{pmatrix} = \begin{pmatrix} 1 \\ -\frac{1}{5} \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{2}{5} \end{pmatrix} i$$

PARA $r_2 = 1-2i$

$$(A-(1-2i)I)V = \begin{bmatrix} 1+2i & 5 \\ -1 & -1+2i \end{bmatrix}$$

$$\begin{bmatrix} 1+2i & 5 \\ -1 & -1+2i \end{bmatrix} \times \left(\frac{1}{1+2i}\right) + \sim \begin{bmatrix} 1+2i & 5 \\ 0 & 0 \end{bmatrix}$$

$$(-1+2i) + 5\left(\frac{1}{1+2i}\right) = 0$$

$$(-1+2i)(1+2i) + 5 = 0$$

$$-1 - \cancel{2i} + \cancel{2i} + 4i^2 + 5 = 0$$

$$4i^2 + 4 = 0$$

$$4(\sqrt{-1})^2 + 4 = 0$$

$$4(-1) + 4 = 0$$

$$-4 + 4 = 0$$

$$(1+2i)v_1 + 5v_2 = 0$$

$$5v_2 = (-1-2i)v_1$$

$$v_2 = \left(\frac{-1-2i}{5}\right)v_1$$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{-1-2i}{5} \end{pmatrix} v_1$$

$$v_1 = v_1$$

FAZENDO $v_1 = 1$ OBTÉM-SE:

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{-1-2i}{5} \end{pmatrix} = \begin{pmatrix} 1 \\ -\frac{1}{5} \end{pmatrix} - \begin{pmatrix} 0 \\ \frac{2}{5} \end{pmatrix} i$$

1.H $\begin{pmatrix} 2 & -5 & 0 \\ 1 & -2 & -3 \\ 0 & 1 & 2 \end{pmatrix}$

CALCULANDO OS AUTOVALORES

$$\det(A-MI)=\begin{bmatrix} 2-M & -5 & 0 \\ 1 & -2-M & -3 \\ 0 & 1 & 2-M \end{bmatrix}$$

$$=(2-M)\begin{vmatrix} -2-M & -3 \\ 1 & 2-M \end{vmatrix}-(-5)\begin{vmatrix} 1 & -3 \\ 0 & 2-M \end{vmatrix}+\cancel{(0)\begin{vmatrix} 1 & -2-M \\ 0 & 1 \end{vmatrix}}$$

$$=(2-M)[(-2-M)(2-M)-(-3)(1)]+5[1(2-M)-(-3)(0)]$$

$$=(2-M)(-4+\cancel{2M}-\cancel{2M}+M^2+3)+5(2-M+0)$$

$$=(2-M)(M^2-1)+5(2-M)$$

$$=(2-M)(M^2-1+5)$$

$$=(2-M)(M^2+4)$$

$2-M=0$

$-M=-2$

$M=2$

$M^2+4=0$

$M^2=-4$

$M=\sqrt{-4} \rightarrow M=\sqrt{4(-1)}$

$=\pm 2\sqrt{-1}$

$=\pm 2i$

$M_1=2$, $M_2=2i$ E $M_3=-2i$

CALCULANDO OS AUTOVETORES

PARA $M_1=2$

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$$(A-2I)V=\begin{bmatrix}0 & -5 & 0\\ 1 & -4 & -3\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}v_1\\ v_2\\ v_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}0 & -5 & 0\\ 1 & -4 & -3\\ 0 & 1 & 0\end{bmatrix}\sim\begin{bmatrix}1 & -4 & -3\\ 0 & -5 & 0\\ 0 & 1 & 0\end{bmatrix}\sim\begin{bmatrix}1 & -4 & -3\\ 0 & 1 & 0\\ 0 & -5 & 0\end{bmatrix}\begin{matrix}+\\ \times(4)\ \times(5)\\ +\end{matrix}\sim$$

$$\sim\begin{bmatrix}1 & 0 & -3\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$v_1 - 3v_3 = 0$ $\qquad$ $v_1 = 3,\ v_2 = 0$ E $v_3 = 1$

$v_1 = 3v_3$

$v_2 = 0$

$$\begin{pmatrix}v_1\\ v_2\\ v_3\end{pmatrix}=\begin{pmatrix}3\\0\\1\end{pmatrix}$$

PARA $r_2 = 2i$

$$(A-2iI)V=\begin{bmatrix}2-2i & -5 & 0\\ 1 & -2-2i & -3\\ 0 & 1 & 2-2i\end{bmatrix}\begin{bmatrix}v_1\\ v_2\\ v_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

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EXERCÍCIO 2:

2.A $\begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix}$

CALCULANDO OS AUTOVALORES

$$\det(A - \lambda I) = \begin{vmatrix} 3-\lambda & 0 \\ 1 & 2-\lambda \end{vmatrix}$$

$$= (3-\lambda)(2-\lambda) - (0)(1)$$

$$= 6 - 3\lambda - 2\lambda + \lambda^2 + 0$$

$$= \lambda^2 - 5\lambda + 6$$

FÓRMULA DE BHASKARA:

$$\lambda^2 - 5\lambda + 6 = 0$$

$$\Delta = b^2 - 4ac$$

$$= (-5)^2 - 4(1)(6)$$

$$= 25 - 24$$

$$= 1$$

$$\lambda = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{1}}{2(1)}$$

$$= \frac{5 \pm 1}{2} \begin{matrix} \nearrow 3 \\ \searrow 2 \end{matrix}$$

$\lambda_1 = 3$ E $\lambda_2 = 2$

CALCULANDO OS AUTOVETORES

PARA $\lambda_1 = 3$

$$(A - 3I)v = \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$v_1 = 1$ E $v_2 = 1$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = v^1$$

PARA $\lambda_2 = 2$

$$(A-2I)v = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$v_1 = 0$ E $v_2 = 1$ $\quad \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = v^2$

OBTENDO UMA MATRIZ NÃO-SINGULAR DE AUTOVETORES

$$P = [v^1 \ v^2]$$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}.$$

OBTENDO UMA MATRIZ DIAGONAL DE AUTOVALORES ($P^{-1}AP = D$)

$$|\tilde{P}| = [P \mid I]$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right] \times(-1) + \sim \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{array}\right]$$

LOGO,

$$P^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$B = P^{-1}A$$

$$= \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1(3)+(0)(1) & 1(0)+(0)(2) \\ (-1)(3)+(1)(1) & (-1)(0)+(1)(2) \end{bmatrix}$$

$$B = \begin{bmatrix} 3+0 & 0+0 \\ -3+1 & 0+2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 \\ -2 & 2 \end{bmatrix}$$

$$D = B\,P$$

$$= \begin{bmatrix} 3 & 0 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3(1)+(0)(1) & 3(0)+(0)(1) \\ (-2)(1)+2(1) & (-2)(0)+2(1) \end{bmatrix}$$

$$= \begin{bmatrix} 3+0 & 0+0 \\ -2+2 & 0+2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

SENDO ASSIM,

$$P = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \quad E \quad D = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

2.B $\begin{pmatrix} 0 & 1 \\ -1 & 3 \end{pmatrix}$

CALCULANDO OS AUTOVALORES

$$\det(A - \lambda I) = \begin{bmatrix} -\lambda & 1 \\ -1 & 3-\lambda \end{bmatrix}$$

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$$\det(A-\lambda I) = (-\lambda)(5-\lambda) - 1(-1)$$
$$= -5\lambda + \lambda^2 + 1$$
$$= \lambda^2 - 5\lambda + 1$$

FÓRMULA DE BHASKARA:

$$\lambda^2 - 5\lambda + 1 = 0$$

$$\Delta = b^2 - 4ac$$
$$= (-5)^2 - 4(1)(1)$$
$$= 25 - 4$$
$$= 21$$

$$\lambda = \frac{-b \pm \sqrt{\Delta}}{2a}$$
$$= \frac{-(-5) \pm \sqrt{21}}{2(1)}$$
$$= \frac{5 \pm \sqrt{21}}{2} \begin{cases} \frac{5+\sqrt{21}}{2} \\ \frac{5-\sqrt{21}}{2} \end{cases}$$

$$\lambda_1 = \frac{5+\sqrt{21}}{2} \text{ E } \lambda_2 = \frac{5-\sqrt{21}}{2}$$

CALCULANDO OS AUTOVETORES

PARA $\lambda_1 = \frac{5+\sqrt{21}}{2}$

$$\left(A - \left(\frac{5+\sqrt{21}}{2}\right)I\right)v = \begin{bmatrix} \frac{-5-\sqrt{21}}{2} & 1 \\ -1 & \frac{5-\sqrt{21}}{2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{-5-\sqrt{21}}{2} & 1 \\ -1 & \frac{5-\sqrt{21}}{2} \end{bmatrix} \times(-1) \sim \begin{bmatrix} \frac{-5-\sqrt{21}}{2} & 1 \\ 1 & \frac{-5+\sqrt{21}}{2} \end{bmatrix}$$

$$\left(\frac{-5-\sqrt{21}}{2}\right) v_1 + v_2 = 0$$

$$v_2 = \left(\frac{5+\sqrt{21}}{2}\right) v_1$$

$$v_1 = v_1 \qquad \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{5+\sqrt{21}}{2} \end{pmatrix} v_1$$

FAZENDO $v_1 = 1$ OBTÉM-SE:

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{5+\sqrt{21}}{2} \end{pmatrix} = v^1$$

PARA $m_2 = \frac{5-\sqrt{21}}{2}$

$$\left(A - \left(\frac{5-\sqrt{21}}{2}\right) I\right) V = \begin{bmatrix} \frac{-5+\sqrt{21}}{2} & 1 \\ -1 & \frac{5+\sqrt{21}}{2} \end{bmatrix}$$

$$\begin{bmatrix} \frac{-5+\sqrt{21}}{2} & 1 \\ -1 & \frac{5+\sqrt{21}}{2} \end{bmatrix} \times(-1) \sim \begin{bmatrix} \frac{-5+\sqrt{21}}{2} & 1 \\ 1 & \frac{-5-\sqrt{21}}{2} \end{bmatrix}$$

$$\left(\frac{-5+\sqrt{21}}{2}\right) v_1 + v_2 = 0$$

$$v_2 = \left(\frac{5-\sqrt{21}}{2}\right) v_1$$

$$v_1 = v_1 \qquad \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{5-\sqrt{21}}{2} \end{pmatrix} v_1$$

FAZENDO $v_1 = 1$ OBTÉM-SE:

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{5-\sqrt{21}}{2} \end{pmatrix} = v^2$$

OBTENDO UMA MATRIZ NÃO-SINGULAR DE AUTOVETORES

$$P = [v^1 \; v^2]$$

$$= \begin{bmatrix} 1 & 1 \\ \frac{5+\sqrt{21}}{2} & \frac{5-\sqrt{21}}{2} \end{bmatrix}$$

OBTENDO UMA MATRIZ DIAGONAL DE AUTOVALORES ($P^{-1}AP = D$)

$$|\tilde{P}| = [P \,|\, I]$$

$$= \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ \frac{5+\sqrt{21}}{2} & \frac{5-\sqrt{21}}{2} & 0 & 1 \end{array}\right] \times\left(-\frac{5+\sqrt{21}}{2}\right) \sim$$

$$\sim \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & -\sqrt{21} & \frac{-5-\sqrt{21}}{2} & 1 \end{array}\right] \times\left(-\frac{1}{\sqrt{21}}\right) \sim \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & \frac{5+\sqrt{21}}{2\sqrt{21}} & -\frac{1}{\sqrt{21}} \end{array}\right] \times(-1) \sim$$

$$\sim \left[\begin{array}{cc|cc} 1 & 0 & \frac{\sqrt{21}-5}{2\sqrt{21}} & \frac{1}{\sqrt{21}} \\ 0 & 1 & \frac{5+\sqrt{21}}{2\sqrt{21}} & -\frac{1}{\sqrt{21}} \end{array}\right]$$

LOGO,

$$P^{-1} = \begin{bmatrix} \frac{\sqrt{21}-5}{2\sqrt{21}} & \frac{1}{\sqrt{21}} \\ \frac{5+\sqrt{21}}{2\sqrt{21}} & -\frac{1}{\sqrt{21}} \end{bmatrix}$$

FAZENDO A RACIONALIZAÇÃO DE CADA ELEMENTO DE P^{-1} OBTÉM-SE

$$\frac{\sqrt{21}-5}{2\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = \frac{21-5\sqrt{21}}{2(21)} = \frac{21-5\sqrt{21}}{42}$$

$$\frac{1}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = \frac{\sqrt{21}}{21}$$

$$\frac{5+\sqrt{21}}{2\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = \frac{5\sqrt{21}+21}{2(21)} = \frac{5\sqrt{21}+21}{42}$$

$$-\frac{1}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = -\frac{\sqrt{21}}{21}$$

ENTÃO,

$$P^{-1} = \begin{bmatrix} \frac{21-5\sqrt{21}}{42} & \frac{\sqrt{21}}{21} \\ \frac{5\sqrt{21}+21}{42} & -\frac{\sqrt{21}}{21} \end{bmatrix}$$

$$B = P^{-1}A$$

$$= \begin{bmatrix} \dfrac{21-5\sqrt{21}}{42} & \dfrac{\sqrt{21}}{21} \\[2ex] \dfrac{5\sqrt{21}+21}{42} & -\dfrac{\sqrt{21}}{21} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} \left(\dfrac{21-5\sqrt{21}}{42}\right)(0)+(-1)\left(\dfrac{\sqrt{21}}{21}\right) & \left(\dfrac{21-5\sqrt{21}}{42}\right)(1)+5\left(\dfrac{\sqrt{21}}{21}\right) \\[2ex] \left(\dfrac{5\sqrt{21}+21}{42}\right)(0)+\left(-\dfrac{\sqrt{21}}{21}\right)(-1) & \left(\dfrac{5\sqrt{21}+21}{42}\right)(1)+\left(-\dfrac{\sqrt{21}}{21}\right)(5) \end{bmatrix}$$

$$= \begin{bmatrix} 0-\dfrac{\sqrt{21}}{21} & \dfrac{21-5\sqrt{21}}{42}+\dfrac{5\sqrt{21}}{21} \\[2ex] 0+\dfrac{\sqrt{21}}{21} & \dfrac{5\sqrt{21}+21}{42}-\dfrac{5\sqrt{21}}{21} \end{bmatrix}$$

$$= \begin{bmatrix} -\dfrac{\sqrt{21}}{21} & \dfrac{21-5\sqrt{21}+10\sqrt{21}}{42} \\[2ex] \dfrac{\sqrt{21}}{21} & \dfrac{5\sqrt{21}+21-10\sqrt{21}}{42} \end{bmatrix}$$

$$= \begin{bmatrix} -\dfrac{\sqrt{21}}{21} & \dfrac{21+5\sqrt{21}}{42} \\[2ex] \dfrac{\sqrt{21}}{21} & \dfrac{21-5\sqrt{21}}{42} \end{bmatrix}$$

$D = B\,P$

$$= \begin{bmatrix} -\dfrac{\sqrt{21}}{21} & \dfrac{21+5\sqrt{21}}{42} \\[2ex] \dfrac{\sqrt{21}}{21} & \dfrac{21-5\sqrt{21}}{42} \end{bmatrix} \begin{bmatrix} 1 & 1 \\[2ex] \dfrac{5+\sqrt{21}}{2} & \dfrac{5-\sqrt{21}}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \left(-\dfrac{\sqrt{21}}{21}\right)(1) + \left(\dfrac{21+5\sqrt{21}}{42}\right)\left(\dfrac{5+\sqrt{21}}{2}\right) & \left(-\dfrac{\sqrt{21}}{21}\right)(1) + \left(\dfrac{21+5\sqrt{21}}{42}\right)\left(\dfrac{5-\sqrt{21}}{2}\right) \\[2ex] \left(\dfrac{\sqrt{21}}{21}\right)(1) + \left(\dfrac{21-5\sqrt{21}}{42}\right)\left(\dfrac{5+\sqrt{21}}{2}\right) & \left(\dfrac{\sqrt{21}}{21}\right)(1) + \left(\dfrac{21-5\sqrt{21}}{42}\right)\left(\dfrac{5-\sqrt{21}}{2}\right) \end{bmatrix}$$

$$= \begin{bmatrix} -\dfrac{\sqrt{21}}{21} + \dfrac{210+46\sqrt{21}}{84} & -\dfrac{\sqrt{21}}{21} + \dfrac{4\sqrt{21}}{84} \\[2ex] \dfrac{\sqrt{21}}{21} - \dfrac{4\sqrt{21}}{84} & \dfrac{\sqrt{21}}{21} + \dfrac{210-46\sqrt{21}}{84} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{-4\sqrt{21}+210+46\sqrt{21}}{84} & \cancel{\dfrac{-4\sqrt{21}+4\sqrt{21}}{84}} \\[4ex] \cancel{\dfrac{4\sqrt{21}-4\sqrt{21}}{84}} & \dfrac{4\sqrt{21}+210-46\sqrt{21}}{84} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{210+42\sqrt{21}}{84} & 0 \\[4ex] 0 & \dfrac{210-42\sqrt{21}}{84} \end{bmatrix}$$

$$D = \begin{bmatrix} \dfrac{\cancel{42}^{1}(5+\sqrt{21})}{\cancel{84}_{2}} & 0 \\ 0 & \dfrac{\cancel{42}^{1}(5-\sqrt{21})}{\cancel{84}_{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{5+\sqrt{21}}{2} & 0 \\ 0 & \dfrac{5-\sqrt{21}}{2} \end{bmatrix}$$

SENDO ASSIM,

$$P = \begin{bmatrix} 1 & 1 \\ \dfrac{5+\sqrt{21}}{2} & \dfrac{5-\sqrt{21}}{2} \end{bmatrix} \text{ E } D = \begin{bmatrix} \dfrac{5+\sqrt{21}}{2} & 0 \\ 0 & \dfrac{5-\sqrt{21}}{2} \end{bmatrix}$$

2-C $\begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$

CALCULANDO OS AUTOVALORES

$$\det(A - \lambda I) = \begin{vmatrix} 1-\lambda & -1 \\ 2 & 4-\lambda \end{vmatrix}$$

$$= (1-\lambda)(4-\lambda) - (-1)(2)$$

$$= 4 - \lambda - 4\lambda + \lambda^2 + 2$$

$$= \lambda^2 - 5\lambda + 6$$

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FÓRMULA DE BHASKARA:

$r^2 - 5r + 6 = 0$

$r = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$\Delta = b^2 - 4ac$

$= (-5)^2 - 4(1)(6)$

$= 25 - 24$

$= 1$

$= \dfrac{-(-5) \pm \sqrt{1}}{2(1)}$

$= \dfrac{5 \pm 1}{2} \to 3, \; 2$

$r_1 = 3$ e $r_2 = 2$

CALCULANDO OS AUTOVETORES

PARA $r_1 = 3$

$$(A - 3I)V = \begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$v_1 = 1$ e $v_2 = -2$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} = v^1$$

PARA $r_2 = 2$

$$(A - 2I)V = \begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$v_1 = 1$ e $v_2 = -1$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} = v^2$$

OBTENDO UMA MATRIZ NÃO-SINGULAR DE AUTOVETORES

$P = [v^1 \; v^2]$

$$= \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$$

OBTENDO UMA MATRIZ DIAGONAL DE AUTOVALORES $(P^{-1}AP = D)$

$$|\tilde{P}| = [P \mid I]$$

$$= \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ -2 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} \times(2) \\ \leftarrow + \end{array} \sim \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{array}\right] \begin{array}{l} \leftarrow + \\ \times(-1) \end{array} \sim$$

$$\sim \left[\begin{array}{cc|cc} 1 & 0 & -1 & -1 \\ 0 & 1 & 2 & 1 \end{array}\right]$$

LOGO,

$$P^{-1} = \begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$$

$$B = P^{-1}A$$

$$= \begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} (-1)(1)+(-1)(2) & (-1)(-1)+(-1)(4) \\ (2)(1)+(1)(2) & (2)(-1)+(1)(4) \end{bmatrix}$$

$$= \begin{bmatrix} -1-2 & 1-4 \\ 2+2 & -2+4 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -3 \\ 4 & 2 \end{bmatrix}$$

$$D = BP$$

$$D = \begin{bmatrix} -3 & -3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} (-3)(1)+(-3)(-2) & (-3)(1)+(-3)(-1) \\ 4(1)+2(-2) & 4(1)+2(-1) \end{bmatrix}$$

$$= \begin{bmatrix} -3+6 & -3+3 \\ 4-4 & 4-2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

SENDO ASSIM,

$$P = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix} \quad E \quad D = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

2.D $\begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$

CALCULANDO OS AUTOVALORES

$$\det(A - \lambda I) = \begin{vmatrix} 3-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 3-\lambda \end{vmatrix}$$

$$= (3-\lambda)\begin{vmatrix} 2-\lambda & -1 \\ -1 & 3-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ 0 & 3-\lambda \end{vmatrix} + (0)\cancel{\begin{vmatrix} -1 & 2-\lambda \\ 0 & -1 \end{vmatrix}}$$

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$$\det(A-\lambda I) = (3-\lambda)[(2-\lambda)(3-\lambda)-(-1)(-1)] + [(-1)(3-\lambda)-(-1)(0)]$$

$$= (3-\lambda)[(2-\lambda)(3-\lambda)-1] + [(-1)(3-\lambda)-0]$$

$$= (3-\lambda)[(2-\lambda)(3-\lambda)-1] - (3-\lambda)$$

$$= (3-\lambda)[(2-\lambda)(3-\lambda)-1-1]$$

$$= (3-\lambda)(6-2\lambda-3\lambda+\lambda^2-2)$$

$$= (3-\lambda)(\lambda^2-5\lambda+4)$$

$3-\lambda=0$

$-\lambda=-3$

$\lambda=3$

FÓRMULA DE BHASKARA:

$\lambda^2-5\lambda+4=0$

$\Delta = b^2-4ac$

$= (-5)^2-4(1)(4)$

$= 25-16$

$= 9$

$$\lambda = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-(-5)\pm\sqrt{9}}{2(1)} = \frac{5\pm 3}{2} \begin{cases} \nearrow 4 \\ \searrow 1 \end{cases}$$

$\lambda_1 = 3$, $\lambda_2 = 4$ E $\lambda_3 = 1$

CALCULANDO OS AUTOVETORES

PARA $\lambda_1 = 3$

$$(A-3I)v = \begin{bmatrix} 0 & -1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$v_1 = 1$, $v_2 = 0$ E $v_3 = -1$

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = v^1$$

PARA $r_2 = 4$

$$(A-4I)V = \begin{bmatrix} -1 & -1 & 0 \\ -1 & -2 & -1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$v_1 = 1$, $v_2 = -1$ E $v_3 = 1$

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = v^2$$

PARA $r_3 = 1$

$$(A-1I)V = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$v_1 = 1$, $v_2 = 2$ E $v_3 = 1$

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = v^3$$

OBTENDO UMA MATRIZ NÃO-SINGULAR DE AUTOVETORES

$P = [v^1 \ v^2 \ v^3]$

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$$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 2 \\ -1 & 1 & 1 \end{bmatrix}$$

OBTENDO UMA MATRIZ DIAGONAL DE AUTOVALORES ($P^{-1}AP = D$)

$$[\tilde{P}] = [P \mid I]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 1 & 0 \\ -1 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \times(1) \\ \quad + \\ \leftarrow \end{array} \sim \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 1 & 0 \\ 0 & 2 & 2 & 1 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \times(-1) \\ \\ \end{array} \sim$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & -1 & 0 \\ 0 & 2 & 2 & 1 & 0 & 1 \end{array}\right] \begin{array}{l} \leftarrow + \\ \times(-1)\ \times(-2) \\ \leftarrow + \end{array} \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 1 & 0 \\ 0 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 6 & 1 & 2 & 1 \end{array}\right] \begin{array}{l} \\ \\ \times(1/6) \end{array} \sim$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 1 & 0 \\ 0 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1/6 & 1/3 & 1/6 \end{array}\right] \begin{array}{l} \leftarrow + \\ \leftarrow + \\ \times(2)\ \times(-3) \end{array} \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 0 & -1/2 \\ 0 & 1 & 0 & 1/3 & -1/3 & 1/3 \\ 0 & 0 & 1 & 1/6 & 1/3 & 1/6 \end{array}\right]$$

LOGO,

$$P^{-1} = \begin{bmatrix} 1/2 & 0 & -1/2 \\ 1/3 & -1/3 & 1/3 \\ 1/6 & 1/3 & 1/6 \end{bmatrix}$$

$$B = P^{-1}A$$

$$= \begin{bmatrix} 1/2 & 0 & -1/2 \\ 1/3 & -1/3 & 1/3 \\ 1/6 & 1/3 & 1/6 \end{bmatrix} \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} (1/2)(1)+(0)(-1)+(-1/2)(0) & (1/2)(-1)+(0)(2)+(-1/2)(-1) & (1/2)(0)+(0)(-1)+(-1/2)(3) \\ (1/3)(1)+(-1/3)(-1)+(1/3)(0) & (1/3)(-1)+(-1/3)(2)+(1/3)(-1) & (1/3)(0)+(-1/3)(-1)+(1/3)(3) \\ (1/6)(1)+(1/3)(-1)+(1/6)(0) & (1/6)(-1)+(1/3)(2)+(1/6)(-1) & (1/6)(0)+(1/3)(-1)+(1/6)(3) \end{bmatrix}$$

$$= \begin{bmatrix} 3/2+0+0 & -1/2+0+1/2 & 0+0-3/2 \\ 1+1/3+0 & -1/3-2/3-1/3 & 0+1/3+1 \\ 1/2-1/3+0 & -1/6+2/3-1/6 & 0-1/3+1/2 \end{bmatrix}$$

$$= \begin{bmatrix} 3/2 & 0 & -3/2 \\ 4/3 & -4/3 & 4/3 \\ 1/6 & 1/3 & 1/6 \end{bmatrix}$$

$$D = BP$$

$$= \begin{bmatrix} 3/2 & 0 & -3/2 \\ 4/3 & -4/3 & 4/3 \\ 1/6 & 1/3 & 1/6 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (3/2)(1)+(0)(0)+(-3/2)(-1) & (3/2)(1)+(0)(-1)+(-3/2)(1) & (3/2)(1)+(0)(2)+(-3/2)(1) \\ (4/3)(1)+(-4/3)(0)+(4/3)(-1) & (4/3)(1)+(-4/3)(-1)+(4/3)(1) & (4/3)(1)+(-4/3)(2)+(4/3)(1) \\ (1/6)(1)+(1/3)(0)+(1/6)(-1) & (1/6)(1)+(1/3)(-1)+(1/6)(1) & (1/6)(1)+(1/3)(2)+(1/6)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 3/2+0+3/2 & 3/2+0-3/2 & 3/2+0-3/2 \\ 4/3+0-4/3 & 4/3+4/3+4/3 & 4/3-8/3+4/3 \\ 1/6+0-1/6 & 1/6-1/3+1/6 & 1/6+2/3+1/6 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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SENDO ASSIM,

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 2 \\ -1 & 1 & 1 \end{bmatrix} \quad E \quad D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2-E
$$\begin{pmatrix} 4 & -2 & -2 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

CALCULANDO OS AUTOVALORES

$$\det(A - \lambda I) = \begin{vmatrix} 4-\lambda & -2 & -2 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & 1-\lambda \end{vmatrix}$$

$$= -(0)\begin{vmatrix} -2 & -2 \\ 0 & 1-\lambda \end{vmatrix} + (1-\lambda)\begin{vmatrix} 4-\lambda & -2 \\ 1 & 1-\lambda \end{vmatrix} - (0)\begin{vmatrix} 4-\lambda & -2 \\ 1 & 0 \end{vmatrix}$$

$$= (1-\lambda)\left[(4-\lambda)(1-\lambda) - (-2)(1)\right]$$

$$= (1-\lambda)(4 - 4\lambda - \lambda + \lambda^2 + 2)$$

$$= (1-\lambda)(\lambda^2 - 5\lambda + 6)$$

$1 - \lambda = 0$

$-\lambda = -1$

$\lambda = 1$

FÓRMULA DE BHASKARA

$\lambda^2 - 5\lambda + 6 = 0$

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$\Delta = b^2 - 4ac$

$= (-5)^2 - 4(1)(6)$

$= 25 - 24$

$= 1$

$r = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$= \dfrac{-(-5) \pm \sqrt{1}}{2(1)}$

$= \dfrac{5 \pm 1}{2} \begin{cases} \nearrow 3 \\ \searrow 2 \end{cases}$

$r_1 = 1,\ r_2 = 3$ E $r_3 = 2$

CALCULANDO OS AUTOVETORES

PARA $r_1 = 1$

$$(A - 1I)V = \begin{bmatrix} 3 & -2 & -2 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$v_1 = 0,\ v_2 = 1$ E $v_3 = -1$

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = v^1$$

PARA $r_2 = 3$

$$(A - 3I)V = \begin{bmatrix} 1 & -2 & -2 \\ 0 & -2 & 0 \\ 1 & 0 & -2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$v_1 = 2,\ v_2 = 0$ E $v_3 = 1$

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = v^2$$

PARA $\lambda_3 = 2$

$$(A-2I)V = \begin{bmatrix} 2 & -2 & -2 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$v_1 = 1,\ v_2 = 0 \text{ E } v_3 = 1$$

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = v^3$$

OBTENDO UMA MATRIZ NÃO-SINGULAR DE AUTOVETORES

$$P = [v^1\ v^2\ v^3]$$

$$= \begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix}$$

OBTENDO UMA MATRIZ DIAGONAL DE AUTOVALORES $(P^{-1}AP = D)$

$$|\tilde{P}| = [P \mid I]$$

$$= \left[\begin{array}{ccc|ccc} 0 & 2 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ -1 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 2 & 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \times(1) \\ + \\ \end{array} \sim$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 1 & 0 & 0 \end{array}\right] \begin{array}{l} \\ \times(-2) \\ + \end{array} \sim$$

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$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 & -2 & -2 \end{array}\right] \times(-1) \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 & 2 & 2 \end{array}\right] \begin{array}{l} + \\ \times(-1) \end{array} \sim$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & -1 & 2 & 2 \end{array}\right]$$

LOGO,

$$P^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & -1 \\ -1 & 2 & 2 \end{bmatrix}$$

$B = P^{-1}A$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & -1 \\ -1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 4 & -2 & -2 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (0)(4)+(1)(0)+(0)(1) & (0)(-2)+(1)(1)+(0)(0) & (0)(-2)+(1)(0)+(0)(1) \\ (1)(4)+(-1)(0)+(-1)(1) & (1)(-2)+(-1)(1)+(-1)(0) & (1)(-2)+(-1)(0)+(-1)(1) \\ (-1)(4)+(2)(0)+(2)(1) & (-1)(-2)+(2)(1)+(2)(0) & (-1)(-2)+(2)(0)+(2)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+0 & 0+1+0 & 0+0+0 \\ 4+0-1 & -2-1+0 & -2+0-1 \\ -4+0+2 & 2+2+0 & 2+0+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 3 & -3 & -3 \\ -2 & 4 & 4 \end{bmatrix}$$

$D = B\,P$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 3 & -3 & -3 \\ -2 & 4 & 4 \end{bmatrix} \begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (0)(0)+(1)(1)+(0)(-1) & (0)(2)+(1)(0)+(0)(1) & (0)(1)+(1)(0)+(0)(1) \\ (3)(0)+(-3)(1)+(-3)(-1) & (3)(2)+(-3)(0)+(-3)(1) & (3)(1)+(-3)(0)+(-3)(1) \\ (-2)(0)+(4)(1)+(4)(-1) & (-2)(2)+(4)(0)+(4)(1) & (-2)(1)+(4)(0)+(4)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 0+1+0 & 0+0+0 & 0+0+0 \\ 0-3+3 & 6+0-3 & 3+0-3 \\ 0+4-4 & -4+0+4 & -2+0+4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

SENDO ASSIM,

$$P = \begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix} \quad E \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

EXERCÍCIO 3:

3.A $\begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix}$

CALCULANDO OS AUTOVALORES

$$\det(A-\lambda I)=\begin{vmatrix} 3-\lambda & 1 \\ -1 & 1-\lambda \end{vmatrix}$$

$$=(3-\lambda)(1-\lambda)-(1)(-1)$$

$$=3-3\lambda-\lambda+\lambda^2+1$$

$$=\lambda^2-4\lambda+4$$

FÓRMULA DE BHASKARA:

$$\lambda^2-4\lambda+4=0$$

$$\Delta=b^2-4ac$$

$$=(-4)^2-4(1)(4)$$

$$=16-16$$

$$=0$$

$$\lambda=\frac{-b\pm\sqrt{\Delta}}{2a}$$

$$=\frac{-(-4)\pm\sqrt{0}}{2(1)}$$

$$=\frac{4\pm 0}{2} \nearrow 2 \quad \searrow 2$$

$$\lambda_1=\lambda_2=2$$

CALCULANDO OS AUTOVETORES

PARA $\lambda_1=\lambda_2=2$

$$(A-2I)V=\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$v_1=1 \text{ E } v_2=-1$$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}=\begin{pmatrix} 1 \\ -1 \end{pmatrix}=v^1$$

OBTENDO A MATRIZ P QUE QUASE DIAGONALIZA A:

$$(A-2I)V=\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}=\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$v_1 = 1$ e $v_2 = 0$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = v^2$$

LOGO,

$$P = [v^1 \; v^2]$$

$$= \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$$

3.B $\begin{pmatrix} -5 & 2 \\ -2 & -1 \end{pmatrix}$

CALCULANDO OS AUTOVALORES

$$\det(A - \lambda I) = \begin{vmatrix} -5-\lambda & 2 \\ -2 & -1-\lambda \end{vmatrix}$$

$$= (-5-\lambda)(-1-\lambda) - (2)(-2)$$

$$= (5 + 5\lambda + \lambda + \lambda^2) + 4$$

$$= \lambda^2 + 6\lambda + 9$$

FÓRMULA DE BHASKARA:

$\lambda^2 + 6\lambda + 9 = 0$

$\Delta = b^2 - 4ac$

$= (6)^2 - 4(1)(9)$

$= 36 - 36$

$= 0$

$\lambda = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$= \dfrac{-6 \pm \sqrt{0}}{2(1)}$

$= \dfrac{-6 \pm 0}{2} \nearrow -3, \searrow -3$

$\lambda_1 = \lambda_2 = -3$

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CALCULANDO OS AUTOVETORES

PARA $m_1 = m_2 = -3$

$$(A-(-3)I)V = \begin{bmatrix} -2 & 2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$v_1 = 1 \text{ E } v_2 = 1$$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = v^1$$

OBTENDO UMA MATRIZ P QUE QUASE DIAGONALIZA A

$$(A-(-3)I)V = \begin{bmatrix} -2 & 2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$v_1 = 0 \text{ E } v_2 = \frac{1}{2}$$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1/2 \end{pmatrix} = v^2$$

LOGO,

$$P = [\, v^1 \;\; v^2 \,]$$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1/2 \end{bmatrix}$$

3.C $\begin{pmatrix} 3 & 3 \\ -3 & -3 \end{pmatrix}$

CALCULANDO OS AUTOVALORES

$$\det(A - rI) = \begin{vmatrix} 3-r & 3 \\ -3 & -3-r \end{vmatrix}$$

$$= (3-r)(-3-r) - (3)(-3)$$

$$= \cancel{-9} - 3r + 3r + r^2 + \cancel{9}$$

$$= r^2$$

$r^2 = 0$

$r = \sqrt{0}$ $\qquad r_1 = r_2 = 0$

$= \pm 0$

CALCULANDO OS AUTOVETORES

PARA $r_1 = r_2 = 0$

$$(A - 0I)V = \begin{bmatrix} 3 & 3 \\ -3 & -3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$v_1 = 1$ E $v_2 = -1$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} = v^1$$

OBTENDO UMA MATRIZ QUE QUASE DIAGONALIZA A

$$(A - 0I)V = \begin{bmatrix} 3 & 3 \\ -3 & -3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$v_1 = 1$ E $v_2 = -\frac{2}{3}$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -2/3 \end{pmatrix} = v^2$$

LOGO,

$$P = [v^1 \; v^2]$$

$$= \begin{bmatrix} 1 & 1 \\ -1 & -2/3 \end{bmatrix}$$

3.7) $\begin{pmatrix} 3 & 1 & 1 \\ 1 & 2 & 1 \\ -1 & -1 & 1 \end{pmatrix}$

CALCULANDO OS AUTOVALORES

$$\det(A - \lambda I) = \begin{bmatrix} 3-\lambda & 1 & 1 \\ 1 & 2-\lambda & 1 \\ -1 & -1 & 1-\lambda \end{bmatrix}$$

$$= (3-\lambda)\begin{vmatrix} 2-\lambda & 1 \\ -1 & 1-\lambda \end{vmatrix} - (1)\begin{vmatrix} 1 & 1 \\ -1 & 1-\lambda \end{vmatrix} + (1)\begin{vmatrix} 1 & 2-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda)[(2-\lambda)(1-\lambda) - (1)(-1)] - [(1)(1-\lambda) - (1)(-1)] + $$
$$+ [(1)(-1) - (2-\lambda)(-1)]$$

$$= (3-\lambda)[(2-\lambda)(1-\lambda) + 1] - (1-\lambda+1) + (-1+2-\lambda)$$

$$= (3-\lambda)[(2-\lambda)(1-\lambda) + 1] - (2-\lambda) + (1-\lambda)$$

$$= (3-\lambda)(2-\lambda)(1-\lambda) + (3-\lambda) - (2-\lambda) + (1-\lambda)$$

$$= (3-\lambda)(2-\lambda)(1-\lambda) + 3 - \cancel{\lambda} - 2 + \cancel{\lambda} + 1 - \lambda$$

$$= (3-\lambda)(2-\lambda)(1-\lambda) + (2-\lambda)$$

$$= (2-\lambda)[(3-\lambda)(1-\lambda) + 1]$$

$$= (2-\lambda)(3 - 3\lambda - \lambda + \lambda^2 + 1)$$

$$v_1 = 1,\ v_2 = 1 \text{ E } v_3 = -1$$

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = v^2$$

$$(A - 2I)V = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$v_1 = 1,\ v_2 = 0 \text{ E } v_3 = 0$$

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = v^3$$

LOGO,

$$P = [\, v^1 \ v^2 \ v^3 \,]$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix}$$

3.E $\begin{pmatrix} 4 & 1 & 0 \\ -1 & 2 & 0 \\ 1 & 1 & 3 \end{pmatrix}$

CALCULANDO OS AUTOVALORES

$$\det(A - \lambda I) = \begin{bmatrix} 4-\lambda & 1 & 0 \\ -1 & 2-\lambda & 0 \\ 1 & 1 & 3-\lambda \end{bmatrix}$$

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$$\det(A-\lambda I) = (0)\cancel{\begin{vmatrix} -1 & 2-\lambda \\ 1 & 1 \end{vmatrix}} - (0)\cancel{\begin{vmatrix} 4-\lambda & 1 \\ 1 & 1 \end{vmatrix}} + (3-\lambda)\begin{vmatrix} 4-\lambda & 1 \\ -1 & 2-\lambda \end{vmatrix}$$

$$= (3-\lambda)[(4-\lambda)(2-\lambda)-(1)(-1)]$$

$$= (3-\lambda)(8-4\lambda-2\lambda+\lambda^2+1)$$

$$= (3-\lambda)(\lambda^2-6\lambda+9)$$

$3-\lambda=0$

$-\lambda=-3$

$\lambda=3$

FÓRMULA DE BHASKARA:

$\lambda^2-6\lambda+9=0$

$$\lambda = \frac{-b\pm\sqrt{\Delta}}{2a}$$

$\Delta = b^2-4ac$

$$= \frac{-(-6)\pm\sqrt{0}}{2(1)}$$

$=(-6)^2-4(1)(9)$

$=36-36$

$=0$

$$= \frac{6\pm 0}{2} \begin{matrix} \nearrow 3 \\ \searrow 3 \end{matrix}$$

$\lambda_1=\lambda_2=\lambda_3=3$

CALCULANDO OS AUTOVETORES

PARA $\lambda_1=\lambda_2=\lambda_3=3$

$$(A-3I)v = \begin{bmatrix} 1 & 1 & 0 \\ -1 & -1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$v_1=1,\ v_2=-1$ E $v_3=1$

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = v^1$$

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EXERCÍCIO 4:

4.A $\begin{pmatrix} 2 & 4 \\ 4 & 2 \end{pmatrix}$

CALCULANDO OS AUTOVALORES

$$\det(A - \lambda I) = \begin{vmatrix} 2-\lambda & 4 \\ 4 & 2-\lambda \end{vmatrix}$$

$$= (2-\lambda)(2-\lambda) - 4(4)$$

$$= 4 - 2\lambda - 2\lambda + \lambda^2 - 16$$

$$= \lambda^2 - 4\lambda - 12$$

FÓRMULA DE BHASKARA:

$\lambda^2 - 4\lambda - 12 = 0$

$\Delta = b^2 - 4ac$

$= (-4)^2 - 4(1)(-12)$

$= 16 + 48$

$= 64$

$\lambda = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$= \dfrac{-(-4) \pm \sqrt{64}}{2(1)}$

$= \dfrac{4 \pm 8}{2} \begin{matrix} \nearrow 6 \\ \searrow -2 \end{matrix}$

$\lambda_1 = 6$ E $\lambda_2 = -2$

CALCULANDO OS AUTOVETORES

PARA $\lambda_1 = 6$

$$(A - 6I)v = \begin{bmatrix} -4 & 4 \\ 4 & -4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$v_1 = 1$ E $v_2 = 1$ $\quad \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = v^1$

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PARA $\lambda_2 = -2$

$$(A-(-2)I)V = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$v_1 = 1$ E $v_2 = -1$ $\quad \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} = v^2$

OBTENDO UMA MATRIZ ORTOGONAL QUE DIAGONALIZA A

$$\|v^1\| = \sqrt{v_1^2 + v_2^2} = \sqrt{1^2+1^2} = \sqrt{1+1} = \sqrt{2}$$

$$\|v^2\| = \sqrt{v_1^2 + v_2^2} = \sqrt{1^2+(-1)^2} = \sqrt{1+1} = \sqrt{2}$$

$$u_1 = \frac{1}{\|v^1\|} v^1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$$u_2 = \frac{1}{\|v^2\|} v^2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$$

LOGO,

$$P = [u_1 \quad u_2] = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

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4.B $\begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix}$

CALCULANDO OS AUTOVALORES

$$\det(A - \lambda I) = \begin{vmatrix} 4-\lambda & 2 \\ 2 & 1-\lambda \end{vmatrix}$$

$$= (4-\lambda)(1-\lambda) - 2(2)$$

$$= \cancel{4} - 4\lambda - \lambda + \lambda^2 - \cancel{4}$$

$$= \lambda^2 - 5\lambda$$

$$= \lambda(\lambda - 5)$$

$\lambda = 0 \quad E \quad \lambda - 5 = 0$

$\lambda = 5 \qquad \lambda_1 = 0 \ E \ \lambda_2 = 5$

CALCULANDO OS AUTOVETORES

PARA $\lambda_1 = 0$

$$(A - 0I)V = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$v_1 = 1 \ E \ v_2 = -2$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} = v^1$$

PARA $\lambda_2 = 5$

$$(A - 5I)V = \begin{bmatrix} -1 & 2 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$v_1 = 2 \ E \ v_2 = 1$

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$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} = v^2$$

OBTENDO UMA MATRIZ ORTOGONAL QUE DIAGONALIZA A

$$\|v^1\| = \sqrt{v_1^2 + v_2^2} = \sqrt{1^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5}$$

$$\|v^2\| = \sqrt{v_1^2 + v_2^2} = \sqrt{2^2 + 1^2} = \sqrt{4+1} = \sqrt{5}$$

$$u_1 = \frac{1}{\|v^1\|} v^1 = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{5} \\ -2/\sqrt{5} \end{pmatrix}$$

$$u_2 = \frac{1}{\|v^2\|} v^2 = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \end{pmatrix}$$

LOGO,

$$P = [u_1 \quad u_2] = \begin{bmatrix} 1/\sqrt{5} & 2/\sqrt{5} \\ -2/\sqrt{5} & 1/\sqrt{5} \end{bmatrix}$$

4.C $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

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CALCULANDO OS AUTOVALORES

$$\det(A-\lambda I)=\begin{vmatrix} 2-\lambda & 1 & 1 \\ 1 & 1-\lambda & 0 \\ 1 & 0 & 1-\lambda \end{vmatrix}$$

$$= -(1)\begin{vmatrix} 1 & 1 \\ 0 & 1-\lambda \end{vmatrix} + (1-\lambda)\begin{vmatrix} 2-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} - (0)\cancel{\begin{vmatrix} 2-\lambda & 1 \\ 1 & 0 \end{vmatrix}}$$

$$= -[(1)(1-\lambda)-(1)(0)]+(1-\lambda)[(2-\lambda)(1-\lambda)-(1)(1)]$$

$$= -[(1-\lambda)-0]+(1-\lambda)[(2-\lambda)(1-\lambda)-1]$$

$$= (1-\lambda)[(2-\lambda)(1-\lambda)-1]-(1-\lambda)$$

$$= (1-\lambda)(2-2\lambda-\lambda+\lambda^2-1)-(1-\lambda)$$

$$= (1-\lambda)(\lambda^2-3\lambda+1)-(1-\lambda)$$

$$= (1-\lambda)[(\lambda^2-3\lambda+1)-1]$$

$$= (1-\lambda)(\lambda^2-3\lambda+1-1)$$

$$= (1-\lambda)(\lambda^2-3\lambda)$$

$1-\lambda=0$ $\qquad$ $\lambda^2-3\lambda=0$

$-\lambda=-1$ $\qquad$ $\lambda(\lambda-3)=0$

$\lambda=1$

$\lambda=0$ E $\lambda-3=0$ $\qquad$ $\lambda_1=1,\ \lambda_2=0$ E $\lambda_3=3$

$\lambda=3$

CALCULANDO OS AUTOVETORES

PARA $\lambda_1=1$

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$$(A-1I)V=\begin{bmatrix}1&1&1\\1&0&0\\1&0&0\end{bmatrix}\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$v_1=0,\ v_2=1$ E $v_3=-1$

$$\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}=\begin{pmatrix}0\\1\\-1\end{pmatrix}=v^1$$

PARA $\lambda_2=0$

$$(A-0I)V=\begin{bmatrix}2&1&1\\1&1&0\\1&0&1\end{bmatrix}\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$v_1=1,\ v_2=-1$ E $v_3=-1$

$$\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}=\begin{pmatrix}1\\-1\\-1\end{pmatrix}=v^2$$

PARA $\lambda_3=3$

$$(A-3I)V=\begin{bmatrix}-1&1&1\\1&-2&0\\1&0&-2\end{bmatrix}\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$v_1=2,\ v_1=1$ E $v_3=1$

$$\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}=\begin{pmatrix}2\\1\\1\end{pmatrix}=v^3$$

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OBTENDO UMA MATRIZ ORTOGONAL QUE DIAGONALIZA A

$$\|v^1\| = \sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{0^2 + 1^2 + (-1)^2} = \sqrt{0+1+1} = \sqrt{2}$$

$$\|v^2\| = \sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{1^2 + (-1)^2 + (-1)^2} = \sqrt{1+1+1} = \sqrt{3}$$

$$\|v^3\| = \sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{2^2 + 1^2 + 1^2} = \sqrt{4+1+1} = \sqrt{6}$$

$$u_1 = \frac{1}{\|v^1\|} v^1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$$

$$u_2 = \frac{1}{\|v^2\|} v^2 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ -1/\sqrt{3} \end{pmatrix}$$

$$u_3 = \frac{1}{\|v^3\|} v^3 = \frac{1}{\sqrt{6}}\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2/\sqrt{6} \\ 1/\sqrt{6} \\ 1/\sqrt{6} \end{pmatrix}$$

LOGO,

$$P = [u_1 \ u_2 \ u_3] = \begin{bmatrix} 0 & 1/\sqrt{3} & 2/\sqrt{6} \\ 1/\sqrt{2} & -1/\sqrt{3} & 1/\sqrt{6} \\ -1/\sqrt{2} & -1/\sqrt{3} & 1/\sqrt{6} \end{bmatrix}$$

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4.1 $\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$

CALCULANDO OS AUTOVALORES

$$\det(A-\lambda I) = \begin{vmatrix} 2-\lambda & -1 & -1 \\ -1 & 2-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$

$$= (2-\lambda)\begin{vmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + (-1)\begin{vmatrix} -1 & 2-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (2-\lambda)[(2-\lambda)(2-\lambda)-(-1)(-1)] + [(-1)(2-\lambda)-(-1)(-1)] - [(-1)(-1)-(2-\lambda)(-1)]$$

$$= (2-\lambda)[(2-\lambda)(2-\lambda)-1] + [(-1)(2-\lambda)-1] - [1 + (2-\lambda)]$$

$$= (2-\lambda)[(2-\lambda)(2-\lambda)-1] + (-2+\lambda-1) - (1+2-\lambda)$$

$$= (2-\lambda)[(2-\lambda)(2-\lambda)-1] + (\lambda-3) - (3-\lambda)$$

$$= (2-\lambda)[(2-\lambda)(2-\lambda)-1] + \lambda-3-3+\lambda$$

$$= (2-\lambda)[(2-\lambda)(2-\lambda)-1] + 2\lambda-6$$

$$= (2-\lambda)(4-2\lambda-2\lambda+\lambda^2-1) + 2\lambda-6$$

$$= (2-\lambda)(\lambda^2-4\lambda+3) + 2\lambda-6$$

$$= 2\lambda^2-8\lambda+\cancel{6}-\lambda^3+4\lambda^2-3\lambda+2\lambda-\cancel{6}$$

$$= -\lambda^3+6\lambda^2-9\lambda$$

$$= -\lambda(\lambda^2-6\lambda+9)$$

LISTA III MATEMÁTICA I

2013/1º

$-r = 0$

$r = 0$

FÓRMULA DE BHASKARA:

$r^2 - 6r + 9 = 0$

$r = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$\Delta = b^2 - 4ac$

$= (-6)^2 - 4(1)(9)$

$= 36 - 36$

$= 0$

$= \dfrac{-(-6) \pm \sqrt{0}}{2(1)}$

$= \dfrac{6 \pm 0}{2} \begin{matrix} \nearrow 3 \\ \searrow 3 \end{matrix}$

$r_1 = 0$ E $r_2 = r_3 = 3$

CALCULANDO OS AUTOVETORES

PARA $r_1 = 0$

$$(A - 0I)V = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$v_1 = 1$, $v_2 = 1$ E $v_3 = 1$

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = v^1$$

PARA $r_2 = 3$

$$(A - 3I)V = \begin{bmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

LISTA III MATEMÁTICA I

2013/1°

$v_1 = 1,\ v_2 = 0$ e $v_3 = -1$

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = v^2$$

PARA $\lambda_3 = 3$

$$(A - 3I)v = \begin{bmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$v_1 = 1,\ v_2 = 1$ e $v_3 = -2$

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} = v^3$$

OBTENDO UMA MATRIZ ORTOGONAL QUE DIAGONALIZA A

$$\|v^1\| = \sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{1+1+1} = \sqrt{3}$$

$$\|v^2\| = \sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{1^2 + 0^2 + (-1)^2} = \sqrt{1+0+1} = \sqrt{2}$$

$$\|v^3\| = \sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{1^2 + 1^2 + (-2)^2} = \sqrt{1+1+4} = \sqrt{6}$$

$$u_1 = \frac{1}{\|v^1\|} v^1 \qquad u_2 = \frac{1}{\|v^2\|} v^2 \qquad u_3 = \frac{1}{\|v^3\|} v^3$$

$$= \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \qquad = \frac{1}{\sqrt{6}}\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix} \qquad = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{pmatrix} \qquad = \begin{pmatrix} 1/\sqrt{6} \\ 1/\sqrt{6} \\ -2/\sqrt{6} \end{pmatrix}$$

LOGO,

$$P = [u_1 \ u_2 \ u_3]$$

$$= \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0 & 1/\sqrt{6} \\ 1/\sqrt{3} & -1/\sqrt{2} & -2/\sqrt{6} \end{bmatrix}$$

4.E $\begin{pmatrix} 2 & 0 & -1 \\ 0 & 4 & 0 \\ -1 & 0 & 2 \end{pmatrix}$

CALCULANDO OS AUTOVALORES

$$\det(A - \lambda I) = \begin{vmatrix} 2-\lambda & 0 & -1 \\ 0 & 4-\lambda & 0 \\ -1 & 0 & 2-\lambda \end{vmatrix}$$

$$= -(0)\cancel{\begin{vmatrix} 0 & -1 \\ 0 & 2-\lambda \end{vmatrix}} + (4-\lambda)\begin{vmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (0)\cancel{\begin{vmatrix} 2-\lambda & 0 \\ -1 & 0 \end{vmatrix}}$$

$$\det(A - \lambda I) = (4-\lambda)[(2-\lambda)(2-\lambda) - (-1)(-1)]$$
$$= (4-\lambda)(4 - 2\lambda - 2\lambda + \lambda^2 - 1)$$
$$= (4-\lambda)(\lambda^2 - 4\lambda + 3)$$

$4 - \lambda = 0$

$-\lambda = -4$

$\lambda = 4$

FÓRMULA DE BHASKARA:

$\lambda^2 - 4\lambda + 3 = 0$

$\Delta = b^2 - 4ac$

$= (-4)^2 - 4(1)(3)$

$= 16 - 12$

$= 4$

$\lambda = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$= \dfrac{-(-4) \pm \sqrt{4}}{2(1)}$

$= \dfrac{4 \pm 2}{2}$ $\nearrow 3$, $\searrow 2$

$\lambda_1 = 4$, $\lambda_2 = 3$ e $\lambda_3 = 1$

CALCULANDO OS AUTOVETORES

PARA $\lambda_1 = 4$

$$(A - 4I)V = \begin{bmatrix} -2 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & -2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = v^1$$

$v_1 = 0$, $v_2 = 1$ E $v_3 = 0$

PARA $\lambda_2 = 3$

$$(A - 3I)V = \begin{bmatrix} -1 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$v_1 = 1,\ v_2 = 0$ E $v_3 = -1$

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = v^2$$

PARA $\lambda_3 = 1$

$$(A - 2I)V = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 3 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$v_1 = 1,\ v_2 = 0$ E $v_3 = 1$

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = v^3$$

OBTENDO UMA MATRIZ ORTOGONAL QUE DIAGONALIZA A

$$\|v^1\| = \sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{0^2 + 1^2 + 0^2} = \sqrt{0 + 1 + 0} = \sqrt{1} = 1$$

$$\|v^2\| = \sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{1^2 + 0^2 + (-1)^2} = \sqrt{1 + 0 + 1} = \sqrt{2}$$

$$\|v^3\| = \sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{1^2 + 0^2 + 1^2} = \sqrt{1 + 0 + 1} = \sqrt{2}$$

$$u_1 = \frac{1}{\|v^1\|} v^1 \qquad u_2 = \frac{1}{\|v^2\|} v^2 \qquad u_3 = \frac{1}{\|v^3\|} v^3$$

$$= \frac{1}{1}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \qquad = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{pmatrix} \qquad = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$$

LOGO,

$$P = [u_1 \quad u_2 \quad u_3]$$

$$= \begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & 0 & 0 \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

EXERCÍCIO 5:

5. A $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

CALCULANDO OS AUTOVALORES

$$\det(A - \lambda I) = \begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix}$$

$$= (2-\lambda)(2-\lambda) - (1)(1)$$

$$= 4 - 2\lambda - 2\lambda + \lambda^2 - 1$$

$$= \lambda^2 - 4\lambda + 3$$

FÓRMULA DE BHASKARA:

$r^2-4r+3=0$

$\Delta = b^2-4ac$

$=(-4)^2-4(1)(3)$

$=16-12$

$=4$

$r = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$= \dfrac{-(-4) \pm \sqrt{4}}{2(1)}$

$= \dfrac{4 \pm 2}{2} \begin{cases} \nearrow 3 \\ \searrow 1 \end{cases}$

$r_1 = 3$ E $r_2 = 1$

CALCULANDO OS AUTOVETORES

PARA $r_1 = 3$

$(A-3I)V = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$v_1 = 1$ E $v_2 = 1$

$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = v^1$

PARA $r_2 = 2$

$(A-2I)V = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$v_1 = 1$ E $v_2 = -1$

$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} = v^2$

OBTENDO UMA MATRIZ ORTONORMAL QUE DIAGONALIZA A

$$\|v^1\| = \sqrt{v_1^2 + v_2^2} = \sqrt{1^2 + 1^2} = \sqrt{1+1} = \sqrt{2}$$

$$\|v^2\| = \sqrt{v_1^2 + v_2^2} = \sqrt{1^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$$

$$u_1 = \frac{1}{\|v^1\|} v^1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$$u_2 = \frac{1}{\|v^2\|} v^2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$$

LOGO,

$$P = [u_1 \ u_2] = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

5.B $\begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}$

CALCULANDO OS AUTOVALORES

$$\det(A - \lambda I) = \begin{vmatrix} 1-\lambda & 3 \\ 3 & 1-\lambda \end{vmatrix}$$

$$= (1-\lambda)(1-\lambda) - 3(3)$$

$$= 1 - \lambda - \lambda + \lambda^2 - 9$$

$\det(A - \lambda I) = \lambda^2 - 2\lambda - 8$

FÓRMULA DE BHASKARA

$\lambda^2 - 2\lambda - 8 = 0$

$\Delta = b^2 - 4ac$

$= (-2)^2 - 4(1)(-8)$

$= 4 + 32$

$= 36$

$\lambda = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$= \dfrac{-(-2) \pm \sqrt{36}}{2(1)}$

$= \dfrac{2 \pm 6}{2} \begin{cases} \nearrow 4 \\ \searrow -2 \end{cases}$

$\lambda_1 = 4$ E $\lambda_2 = -2$

CALCULANDO OS AUTOVETORES

PARA $\lambda_1 = 4$

$(A - 4I)V = \begin{bmatrix} -3 & 3 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$v_1 = 1$ E $v_2 = 1$

$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = v^1$

PARA $\lambda_2 = -2$

$(A - (-2)I)V = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$v_1 = 1$ E $v_2 = -1$

$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} = v^2$

OBTENDO UMA MATRIZ ORTONORMAL QUE DIAGONALIZA A

LISTA III MATEMÁTICA I

2013/1.°

$$\|v^1\| = \sqrt{v_1^2 + v_2^2} = \sqrt{1^2 + 1^2} = \sqrt{1+1} = \sqrt{2}$$

$$\|v^2\| = \sqrt{v_1^2 + v_2^2} = \sqrt{1^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$$

$$u_1 = \frac{1}{\|v^1\|} v^1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$$u_2 = \frac{1}{\|v^2\|} v^2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$$

LOGO,

$$P = [u_1 \quad u_2] = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

5.C $\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$

CALCULANDO OS AUTOVALORES

$$\det(A - \lambda I) = \begin{vmatrix} 2-\lambda & -1 & -1 \\ -1 & 2-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$

$$= (2-\lambda) \begin{vmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + (-1) \begin{vmatrix} -1 & 2-\lambda \\ -1 & -1 \end{vmatrix}$$

$\det(A - \lambda I) = (2-\lambda)[(2-\lambda)(2-\lambda) - (-1)(-1)] + [(-1)(2-\lambda) - (-1)(-1)] -$
$- [(-1)(-1) - (2-\lambda)(-1)]$
$= (2-\lambda)[(2-\lambda)(2-\lambda) - 1] + (-2+\lambda-1) - (1+2-\lambda)$
$= (2-\lambda)(4-2\lambda-2\lambda+\lambda^2-1) + (\lambda-3) - (3-\lambda)$
$= (2-\lambda)(\lambda^2-4\lambda+3) + \lambda - 3 - 3 + \lambda$
$= 2\lambda^2 - 8\lambda + \cancel{6} - \lambda^3 + 4\lambda^2 - 3\lambda + 2\lambda - \cancel{6}$
$= -\lambda^3 + 6\lambda^2 - 9\lambda$
$= -\lambda(\lambda^2 - 6\lambda + 9)$

$-\lambda = 0$
$\lambda = 0$

FÓRMULA DE BHASKARA:

$\lambda^2 - 6\lambda + 9 = 0$

$\Delta = b^2 - 4ac$
$= (-6)^2 - 4(1)(9)$
$= 36 - 36$
$= 0$

$\lambda = \dfrac{-b \pm \sqrt{\Delta}}{2a}$
$= \dfrac{-(-6) \pm \sqrt{0}}{2(1)}$
$= \dfrac{6 \pm 0}{2} \to 3, \ 3$

$\lambda_1 = 0$ E $\lambda_2 = \lambda_3 = 3$

CALCULANDO OS AUTOVETORES

PARA $\lambda_1 = 0$

$(A - 0I)V = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = v^1$

$v_1 = 1, \ v_2 = 1$ E $v_3 = 1$

PARA $\lambda_2 = 3$

$$(A-3I)V = \begin{bmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$v_1 = 1,\ v_2 = 0$ E $v_3 = -1$

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = v^2$$

PARA $\lambda_3 = 3$

$$(A-3I)V = \begin{bmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$v_1 = 1,\ v_2 = -2$ E $v_3 = 1$

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = v^3$$

OBTENDO UMA MATRIZ ORTONORMAL QUE DIAGONALIZA A

$$\|v^1\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$
$$= \sqrt{1^2 + 1^2 + 1^2}$$
$$= \sqrt{1+1+1}$$
$$= \sqrt{3}$$

$$\|v^2\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$
$$= \sqrt{1^2 + 0^2 + (-1)^2}$$
$$= \sqrt{1+0+1}$$
$$= \sqrt{2}$$

LISTA IV MATEMÁTICA I

2013/1º

$$\|v^3\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$
$$= \sqrt{1^2 + (-2)^2 + 1^2}$$
$$= \sqrt{1+4+1}$$
$$= \sqrt{6}$$

$$u_1 = \frac{1}{\|v^1\|} v^1 \qquad u_2 = \frac{1}{\|v^2\|} v^2 \qquad u_3 = \frac{1}{\|v^3\|} v^3$$

$$= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \qquad = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix} \qquad = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{pmatrix} \qquad = \begin{pmatrix} 1/\sqrt{6} \\ -2/\sqrt{6} \\ 1/\sqrt{6} \end{pmatrix}$$

LOGO,

$$P = [u_1 \quad u_2 \quad u_3]$$

$$= \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0 & -2/\sqrt{6} \\ 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \end{bmatrix}$$

LISTA III MATEMÁTICA I

2023/1°

$$\det(A - \lambda I) = (2-\lambda)(\lambda^2 - 4\lambda + 4)$$

$$2 - \lambda = 0$$
$$-\lambda = -2$$
$$\lambda = 2$$

FÓRMULA DE BHASKARA:

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\Delta = b^2 - 4ac$$
$$= (-4)^2 - 4(1)(4)$$
$$= 16 - 16$$
$$= 0$$

$$\lambda = \frac{-b \pm \sqrt{\Delta}}{2a}$$
$$= \frac{-(-4) \pm \sqrt{0}}{2(1)}$$
$$= \frac{4 \pm 0}{2} \nearrow 2 \searrow 2$$

$$\lambda_1 = \lambda_2 = \lambda_3 = 2$$

CALCULANDO OS AUTOVETORES

PARA $\lambda_1 = \lambda_2 = \lambda_3 = 2$

$$(A - 2I)V = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$v_1 = 1, \; v_2 = 0 \text{ E } v_3 = -1$$

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = v^1$$

OBTENDO UMA MATRIZ QUE QUASE DIAGONALIZA A

$$(A - 2I)V = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

LISTA III MATEMÁTICA I

2013/1º

OBTENDO UMA MATRIZ QUE QUASE DIAGONALIZA A:

$$(A-3I)V = \begin{bmatrix} 1 & 1 & 0 \\ -1 & -1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$v_1 = 1,\ v_2 = 0 \text{ E } v_3 = 1$$

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = v^2$$

$$(A-3I)V = \begin{bmatrix} 1 & 1 & 0 \\ -1 & -1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ -1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right] \begin{array}{l} \times(1)\ \times(-1) \\ \leftarrow + \\ \leftarrow + \end{array} \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

SISTEMA SEM SOLUÇÃO