LISTA VII MATEMÁTICA I

2013/1º

EXERCÍCIO 1:

1.A $\int (x^5 - 3x)\,dx$

$$\int (x^5 - 3x)\,dx = \int x^5\,dx - \int 3x\,dx$$

$$= \int x^5\,dx - 3\int x\,dx$$

$$\boxed{= \frac{x^6}{6} - \frac{3x^2}{2} + c}$$

1.B $\int (x e^{x^2+9})\,dx$

MÉTODO DA SUBSTITUIÇÃO:

$x^2 + 9 = u$

$\frac{du}{dx} = 2x \therefore dx = \frac{du}{2x}$

$$\int (x e^{x^2+9})\,dx = \int \cancel{x} e^u \frac{du}{2\cancel{x}}$$

$$= \frac{1}{2}\int e^u\,du$$

$$= \frac{1}{2} e^u + c$$

$$\boxed{= \frac{1}{2} e^{x^2+9} + c}$$

1.C $\int \frac{2x}{x^2+3}\,dx$

MÉTODO DA SUBSTITUIÇÃO:

1.A

$x^2 + 3 = u$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\int \frac{2x}{x^2+3} dx = \int \frac{\cancel{2x}}{u} \cdot \frac{du}{\cancel{2x}}$

$= \int \frac{1}{u} \cdot du$

$= \ln|u| + c$

$\boxed{= \ln|x^2+3| + c}$ //

1.D $\int x \ln x \, dx$

MÉTODO DA INTEGRAÇÃO POR PARTES:

$\ln x = u$ $\quad$ $x\,dx = dv$

$\frac{du}{dx} = \frac{1}{x}$ $\quad$ $\int dv = \int x\,dx$

$du = \frac{dx}{x}$ $\quad$ $v = \frac{x^2}{2}$

REGRA DO PRODUTO:

$\int x \ln x \, dx = \int u \, dv$

$= u \cdot v - \int v \, du$

$= \frac{x^2}{2} \ln x - \int \frac{x^{\cancel{2}}}{2} \frac{dx}{\cancel{x}}$

$= \frac{x^2}{2} \ln x - \frac{1}{2} \int x \, dx$

$= \frac{x^2}{2} \ln x - \frac{1}{2} \cdot \frac{x^2}{2} + c$

$\int x \ln x \, dx = \frac{x^2}{2} \ln x - \frac{x^2}{4} + c$

$\boxed{= \frac{x^2}{2} \left(\ln x - \frac{1}{2} \right) + c}$ //

LISTA VIII MATEMÁTICA I

2013/1.º

1.E $\int \sqrt{1-4Y}\, dY$

MÉTODO DA SUBSTITUIÇÃO:

$u = 1-4Y$

$\frac{du}{dY} = -4$

$dY = \frac{du}{-4}$

$$\int \sqrt{1-4Y}\, dY = \int u^{1/2} \frac{du}{-4}$$

$$= -\frac{1}{4} \int u^{1/2}\, du$$

$$= -\frac{1}{4} \frac{u^{3/2}}{3/2} + c$$

$$= -\frac{1}{4_2} \cdot \frac{2^1}{3} (1-4Y)^{3/2} + c$$

$$\boxed{= -\frac{1}{6} (1-4Y)^{3/2} + c}$$

1.F $\int x \sqrt{x^2-9}\, dx$

MÉTODO DA SUBSTITUIÇÃO:

$u = x^2 - 9$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

2.A

$$\int x\sqrt{x^2-9}\,dx = \int \cancel{x}\, u^{1/2}\,\frac{du}{2\cancel{x}}$$

$$= \frac{1}{2}\int u^{1/2}\,du$$

$$= \frac{1}{2}\cdot\frac{u^{3/2}}{3/2} + C$$

$$= \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}\cdot(x^2-9)^{3/2} + C$$

$$\boxed{= \frac{1}{3}(x^2-9)^{3/2} + C}$$

1.G $\displaystyle\int \frac{x}{(x^2+1)^3}\,dx$

MÉTODO DA SUBSTITUIÇÃO:

$u = x^2+1$

$\dfrac{du}{dx} = 2x$

$dx = \dfrac{du}{2x}$

$$\int \frac{x}{(x^2+1)^3}\,dx = \int \frac{\cancel{x}}{u^3}\,\frac{du}{2\cancel{x}}$$

$$= \frac{1}{2}\int u^{-3}\,du$$

$$= \frac{1}{2}\,\frac{u^{-2}}{-2} + C$$

$$\boxed{= -\frac{1}{4}(x^2+1)^{-2} + C}$$

1.H $\displaystyle\int \frac{(x^2+2x)}{\sqrt{x^3+3x^2+1}}\,dx$

MÉTODO DA SUBSTITUIÇÃO:

$u = x^3 + 3x^2 + 1$

$\frac{du}{dx} = 3x^2 + 6x$

$dx = \frac{du}{3(x^2+2x)}$

$\int \frac{(x^2+2x)}{\sqrt{x^3+3x^2+1}}\,dx = \int \frac{\cancel{(x^2+2x)}}{u^{1/2}} \frac{du}{3\cancel{(x^2+2x)}}$

$= \frac{1}{3} \int u^{-1/2}\,du$

$= \frac{1}{3} \frac{u^{1/2}}{1/2} + C$

$\boxed{= \frac{2}{3} \cdot \sqrt{x^3+3x^2+1} + C}$

1.I $\int_0^4 (x^2+x-6)\,dx$

$\int_0^4 (x^2+x-6)\,dx = \int_0^4 x^2\,dx + \int_0^4 x\,dx - \int_0^4 6\,dx$

$= \frac{x^3}{3}\Big|_0^4 + \frac{x^2}{2}\Big|_0^4 - 6\int_0^4 dx$

$= \frac{x^3}{3}\Big|_0^4 + \frac{x^2}{2}\Big|_0^4 - 6x\Big|_0^4$

$= \left[\frac{(4)^3}{3} - \frac{(0)^3}{3}\right] + \left[\frac{(4)^2}{2} - \frac{(0)^2}{2}\right] - 6(4-0)$

$= 21,3 + 8 - 24$

$\boxed{= 5,3}$

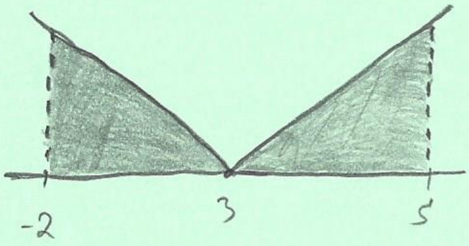
3.A

1.J $\int_{-2}^{5} |x-3|\,dx$

$f(x) = |x-3|$

$f(x) = (x-3)$ SE $x-3 \geq 0 \therefore x \geq 3$

$f(x) = -(x-3)$ SE $x-3 < 0 \therefore x < 3$



$$\int_{-2}^{5} |x-3|\,dx = \int_{-2}^{3} -(x-3)\,dx + \int_{3}^{5} (x-3)\,dx$$

$$= \int_{-2}^{3} -x\,dx + \int_{-2}^{3} 3\,dx + \int_{3}^{5} x\,dx - \int_{3}^{5} 3\,dx$$

$$= (-1)\int_{-2}^{3} x\,dx + 3\int_{-2}^{3} dx + \frac{x^2}{2}\Big|_{3}^{5} - 3\int_{3}^{5} dx$$

$$= (-1)\frac{x^2}{2}\Big|_{-2}^{3} + 3x\Big|_{-2}^{3} + \frac{x^2}{2}\Big|_{3}^{5} - 3x\Big|_{3}^{5}$$

$$= (-1)\left[\frac{(3)^2}{2} - \frac{(-2)^2}{2}\right] + 3[3-(-2)] + \left[\frac{(5)^2}{2} - \frac{(3)^2}{2}\right] - 3(5-3)$$

$$= (-1)\left(\frac{5}{2}\right) + 3(3+2) + \left(\frac{16}{2}\right) - 3(2)$$

$$\boxed{= 14,5}$$

1.K $\int_{1}^{2} t^2\sqrt{t^3+1}\,dt$

MÉTODO DA SUBSTITUIÇÃO:

$u = t^3+1$

$\frac{du}{dt} = 3t^2 \rightarrow dt = \frac{du}{3t^2}$

$$\int_1^2 t^2\sqrt{t^3+1}\,dt = \int_1^2 \cancel{t^2}\, u^{1/2}\,\frac{du}{3\cancel{t^2}}$$

$$= \frac{1}{3}\int_1^2 u^{1/2}\,du$$

$$= \frac{1}{3}\cdot\left.\frac{u^{3/2}}{3/2}\right|_1^2$$

$$= \frac{1}{3}\cdot\frac{2}{3}\left.(t^3+1)^{3/2}\right|_1^2$$

$$= \frac{2}{9}\left\{\left[(2)^3+1\right]^{3/2} - \left[(1)^3+1\right]^{3/2}\right\}$$

$$\boxed{= 5{,}4}$$

1.L $\int_2^3 (e^{2x}+e^{x})\,dx$

$$\int_2^3 (e^{2x}+e^{x})\,dx = \int_2^3 e^{2x}\,dx + \int_2^3 e^{x}\,dx$$

MÉTODO DA SUBSTITUIÇÃO

$u = e^{2x}$

$\frac{du}{dx} = e^{2x}(2x)'$

$= 2e^{2x}$

$dx = \frac{du}{2e^{2x}}$

$= \frac{du}{2u}$

$$\int_2^3 (e^{2x}+e^{x})\,dx = \int_2^3 \cancel{u}\,\frac{du}{2\cancel{u}} + \left. e^{x}\right|_2^3$$

$$= \frac{1}{2}\int_2^3 du + (e^3 - e^2)$$

$$= \frac{1}{2}\left. u\right|_2^3 + (e^3 - e^2)$$

$\int_2^3 (e^{2x} + e^{x})\,dx = \frac{1}{2} e^{2x} \Big|_2^3 + (e^3 - e^2)$ 4.A

$= \frac{1}{2}(e^{2(3)} - e^{2(2)}) + (e^3 - e^2)$

$\boxed{= 187,1}$ //

1.M $\int_e^6 \left(\frac{1}{x} + \frac{1}{1+x}\right) dx$

$\int_e^6 \left(\frac{1}{x} + \frac{1}{1+x}\right) dx = \int_e^6 \frac{1}{x}\,dx + \int_e^6 \frac{1}{1+x}\,dx$

$= \ln|x| \Big|_e^6 + \ln|1+x| \Big|_e^6$

$= (\ln 6 - \ln e) + [\ln(1+6) - \ln(1+e)]$

$\boxed{= 1,4}$ //

1.N $\int_{-1}^{e-2} \left(\frac{1}{x+2}\right) dx$

$\int_{-1}^{e-2} \left(\frac{1}{x+2}\right) dx = \ln|x+2| \Big|_{-1}^{e-2}$

$= \ln[(e-2)+2] - \ln(-1+2)$

$= \ln(e-2+2) - \cancel{\ln(1)}$

$= \ln e + 0$

$\boxed{= 1}$ //

1.O $\int_0^\infty e^{-ax}\,dx$

MÉTODO DA SUBSTITUIÇÃO:

$u = -ax$

$$\frac{du}{dx} = -a \quad \therefore \quad dx = \frac{du}{-a}$$

$$\int_0^\infty e^{-ax}\,dx = \int_0^\infty e^{u}\,\frac{du}{-a}$$

$$= -\frac{1}{a}\int_0^\infty e^{u}\,du$$

$$= -\frac{1}{a}\cdot e^{u}\Big|_0^\infty$$

$$= -\frac{1}{a}\cdot e^{-ax}\Big|_0^\infty$$

$$= -\frac{1}{a}\{e^{-a\infty} - e^{-a(0)}\}$$

$$= -\frac{1}{a}(0-1)$$

$$\boxed{= \frac{1}{a}}$$

1.7 $\int_0^1 x^{-2/3}\,dx$

$$\int_0^1 x^{-2/3}\,dx = \frac{x^{1/3}}{1/3}\Big|_0^1$$

$$= 3x^{1/3}\Big|_0^1$$

$$= 3[(1)^{1/3} - (0)^{1/3}]$$

$$= 3(1-0)$$

$$\boxed{= 3}$$

1.2 $\int_1^3 \left(\frac{1}{x-2}\right) dx$ 3.1

$$\int_1^3 \left(\frac{1}{x-2}\right) dx = \ln|x-2| \Big|_1^3$$

$$= (\ln|3-2| - \ln|1-2|)$$

$$\boxed{= 1,1}$$

1.1 $\int (x^2 10^{x^3}) dx$

MÉTODO DA SUBSTITUIÇÃO:

$u = x^3$

$\frac{du}{dx} = 3x^2$

$dx = \frac{du}{3x^2}$

$$\int (x^2 10^{x^3}) dx = \int \cancel{x^2} 10^u \frac{du}{3\cancel{x^2}}$$

$$= \frac{1}{3} \int 10^u du$$

$$= \frac{1}{3} \cdot \frac{10^u}{\ln 10} + c$$

$$\boxed{= \frac{1}{3} \cdot \frac{10^{x^3}}{\ln 10} + c}$$

1.5 $\int \frac{2^x}{\sqrt{3(2^x)+4}} dx$

MÉTODO DA SUBSTITUIÇÃO:

$u = 3(2^x) + 4$

$$\frac{du}{dx} = 3(2^x)\ln 2$$

$$dx = \frac{du}{3(2^x)\ln 2}$$

$$\int \frac{2^x}{\sqrt{3(2^x)+4}}\,dx = \int \frac{2^x}{u^{1/2}} \cdot \frac{du}{3(2^x)\ln 2}$$

$$= \frac{1}{3\ln 2} \int u^{-1/2} \cdot du$$

$$= \frac{1}{3\ln 2} \frac{u^{1/2}}{1/2} + C$$

$$\boxed{= \frac{2}{3\ln 2} \cdot \sqrt{3(2^x)+4} + C}$$

1.T $\int_2^4 \frac{\ln x}{x}\,dx$

MÉTODO DA SUBSTITUIÇÃO:

$u = \ln x$

$$\frac{du}{dx} = \frac{1}{x}$$

$$\frac{dx}{x} = du$$

$$\int_2^4 \frac{\ln x}{x}\,dx = \int_2^4 \ln x \frac{dx}{x}$$

$$= \int_2^4 u\,du$$

$$= \frac{u^2}{2}\Bigg|_2^4$$

$$= \frac{(\ln x)^2}{2}\Bigg|_2^4$$

$$\int_2^4 \frac{\ln x}{x}\,dx = \frac{(\ln 4)^2}{2} - \frac{(\ln 2)^2}{2}$$

$$\boxed{= 0,7}$$

(1.U) $\int_0^{\frac{\pi}{2}} \operatorname{sen} 2x\, dx$

(6.9)

$$\int_0^{\frac{\pi}{2}} \operatorname{sen} 2x\, dx = -\frac{\cos 2x}{2}\Big|_0^{\frac{\pi}{2}}$$

$$= -\frac{\cos \cancel{2}(\frac{\pi}{\cancel{2}})}{2} - \left(-\frac{\cos 2(0)}{2}\right)$$

$$= -\frac{\cos \pi}{2} + \frac{\cos 0}{2}$$

$$= -\frac{-1}{2} + \frac{1}{2}$$

$$\boxed{= 1}$$

(1.U) $\int \frac{\cos \sqrt{x}}{\sqrt{x}}\, dx$

MÉTODO DA SUBSTITUIÇÃO:

$u = x^{1/2}$

$\frac{du}{dx} = \frac{1}{2} x^{-1/2}$

$\frac{du}{dx} = \frac{1}{2x^{1/2}}$

$\frac{du}{dx} = \frac{1}{2u}$

$dx = 2u\, du$

$$\int \frac{\cos \sqrt{x}}{\sqrt{x}}\, dx = \int \frac{\cos u}{\cancel{u}}\, 2\cancel{u}\, du$$

$$= \int 2 \cos u\, du$$

$$= 2 \int \cos u\, du$$

$$= 2 \operatorname{sen} u + C$$

$$\boxed{= 2 \operatorname{sen} \sqrt{x} + C}$$

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1.w $\int_0^1 \frac{1}{x^3}\,dx$

$$\int_0^1 \frac{1}{x^3}\,dx = \int_0^1 x^{-3}\,dx$$

$$= \frac{x^{-2}}{-2}\Big|_0^1$$

$$= \frac{(1)^{-2}}{-2} - \frac{(0)^{-2}}{2}$$

$$\boxed{= -0{,}5}$$

1.x $\int_0^{\frac{\pi}{2}} 2^{\operatorname{sen} x} \cos x\,dx$

MÉTODO DA SUBSTITUIÇÃO:

$u = \operatorname{sen} x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$$\int_0^{\frac{\pi}{2}} 2^{\operatorname{sen} x} \cos x\,dx = \int_0^{\frac{\pi}{2}} 2^u \cancel{\cos x} \frac{du}{\cancel{\cos x}}$$

$$= \int_0^{\frac{\pi}{2}} 2^u\,du$$

$$= \frac{2^u}{\ln 2}\Big|_0^{\frac{\pi}{2}}$$

$$= \frac{2^{\operatorname{sen} x}}{\ln 2}\Big|_0^{\frac{\pi}{2}}$$

7.A

$$\int_0^{\frac{\pi}{2}} 2^{\operatorname{sen} x}\cos x \, dx = \frac{2^{\operatorname{sen}\frac{\pi}{2}}}{\ln 2} - \frac{2^{\operatorname{sen} 0}}{\ln 2}$$

$$\boxed{= 1,4}$$

1.4 $\int_0^1 x^x(\ln x + 1)\,dx$

$x^x = e^{\ln x^x} \therefore x^x = e^{x\ln x}$

$$\int_0^1 x^x(\ln x+1)\,dx = \int_0^1 e^{x\ln x}(\ln x+1)\,dx$$

MÉTODO DA SUBSTITUIÇÃO:

$u = x\ln x$

$\frac{du}{dx} = x(\ln x)' + \ln x(x)'$

$= \cancel{x}\left(\frac{1}{\cancel{x}}\right) + \ln x(1)$

$= \ln x + 1$

$dx = \frac{du}{\ln x + 1}$

$$= \int_0^1 e^u (\cancel{\ln x+1}) \frac{du}{(\cancel{\ln x+1})}$$

$$= \int_0^1 e^u\,du$$

$$= e^u \Big|_0^1$$

$$= e^{x\ln x}\Big|_0^1$$

$$= e^{(1)\ln 1} - e^{(0)\ln 0}$$

$$= 1 - 1$$

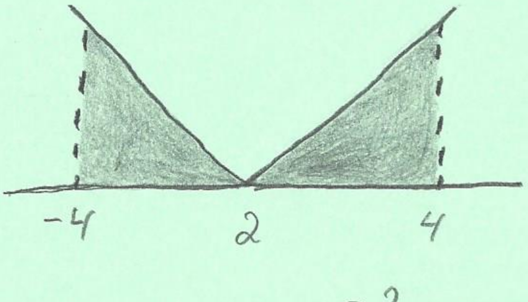
$$\boxed{= 0}$$

1.2 $\int_{-4}^4 |x-2|\,dx$

$f(x) = |x-2|$

$f(x) = (x-2)$ SE $x-2 \geq 0 \therefore x \geq 2$

$f(x) = -(x-2)$ SE $x-2 < 0 \therefore x < 2$



$$\int_{-4}^4 |x-2|\,dx = \int_{-4}^2 -(x-2)\,dx + \int_2^4 (x-2)\,dx$$

$$= \int_{-4}^2 -x\,dx + \int_{-4}^2 2\,dx + \int_2^4 x\,dx + \int_2^4 -2\,dx$$

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$$\int_{-4}^{4}|x-2|\,dx=(-1)\int_{-4}^{2}x\,dx+2\int_{-4}^{2}dx+\frac{x^2}{2}\Big|_2^4+(-2)\int_2^4 dx$$

$$=(-1)\frac{x^2}{2}\Big|_{-4}^{2}+2x\Big|_{-4}^{2}+\frac{x^2}{2}\Big|_2^4-2x\Big|_2^4$$

$$=(-1)\cdot\left[\frac{(2)^2}{2}-\frac{(-4)^2}{2}\right]+2[2-(-4)]+\left[\frac{(4)^2}{2}-\frac{(2)^2}{2}\right]-2(4-2)$$

$$=(-1)\cdot(-6)+2(6)+6-2(2)$$

$$\boxed{=20}$$

1.11 $\int xe^x\,dx$

MÉTODO DA INTEGRAÇÃO POR PARTES:

$u=x$ $\qquad dv=e^x\,dx$

$\frac{du}{dx}=1 \therefore du=dx$ $\qquad \int dv=\int e^x\,dx$

$v=e^x$

REGRA DO PRODUTO:

$$\int xe^x\,dx=\int u\,dv$$

$$=u\cdot v-\int v\,du$$

$$=xe^x-\int e^x\,dx$$

$$=xe^x-e^x+c$$

$$\boxed{=e^x(x-1)+c}$$

(1.33) $\int x^2 e^x dx$ (9.A)

MÉTODO DA INTEGRAÇÃO POR PARTES:

$u = x^2$ $\qquad$ $dv = e^x dx$

$\frac{du}{dx} = 2x \therefore du = 2x\,dx$ $\qquad$ $\int dv = \int e^x dx$

$v = e^x$

$$\int x^2 e^x dx = \int u\,dv$$
$$= u.v - \int v\,du$$
$$= x^2.e^x - \int e^x 2x\,dx$$

APLICANDO O MÉTODO DA INTEGRAÇÃO POR PARTES EM $\int e^x 2x\,dx$:

$u = 2x$ $\qquad$ $dv = e^x dx$

$\frac{du}{dx} = 2 \therefore du = 2\,dx$ $\qquad$ $\int dv = \int e^x dx$

$v = e^x$

$$\int e^x 2x\,dx = \int u\,dv$$
$$= u.v - \int v\,du$$
$$= 2x e^x - \int e^x 2\,dx$$
$$= 2x e^x - 2\int e^x dx$$
$$= 2x e^x - 2e^x$$

LOGO,

$$\int x^2 e^x dx = x^2.e^x - (2xe^x - 2e^x) + c$$
$$= x^2.e^x - 2xe^x + 2e^x + c$$
$$\boxed{= e^x(x^2 - 2x + 2) + c}$$

1.CC $\int a P^{-\epsilon} dP$

$$\int a P^{-\epsilon} dP = a \int P^{-\epsilon} dP$$

$$= a \frac{P^{-\epsilon+1}}{-\epsilon+1} + C$$

$$\boxed{= a \frac{P^{1-\epsilon}}{1-\epsilon} + C}$$

1.DD $\int \frac{x^2-3x-7}{(2x+3)(x+1)^2} dx$

MÉTODO DA INTEGRAÇÃO POR FRAÇÕES PARCIAIS:

$q(x) = (2x+3)(x+1)^2 \therefore q(x) = (2x+3)(x+1)(x+1)$

$$\frac{x^2-3x-7}{(2x+3)(x+1)^2} = \frac{A_1}{(2x+3)} + \frac{A_2}{(x+1)^2} + \frac{A_3}{(x+1)}$$ MULTIPLICANDO TUDO POR $q(x)$

$$x^2-3x-7 = A_1(x+1)^2 + A_2(2x+3) + A_3(2x+3)(x+1)$$

$$= A_1(x^2+2x+1) + A_2 2x + 3A_2 + A_3(2x^2+2x+3x+3)$$

$$= A_1x^2 + A_1 2x + A_1 + A_2 2x + 3A_2 + A_3 2x^2 + A_3 5x + A_3 3$$

$$= (A_1+2A_3)x^2 + (2A_1+2A_2+5A_3)x + A_1+3A_2+3A_3$$

$A_1 + 2A_3 = 1$

$A_1 = 1 - 2A_3$

$2A_1 + 2A_2 + 5A_3 = -3$

$2(1-2A_3) + 2A_2 + 5A_3 = -3$

$2A_2 + A_3 = -5$

$A_3 = -5 - 2A_2$

$A_1 + 3A_2 + 3A_3 = -7$

$(1-2A_3) + 3A_2 + 3A_3 = -7$

$3A_2 + A_3 = -8$

$A_3 = -8 - 3A_2$

LOGO, $-5 - 2A_2 = -8 - 3A_2$

$A_2 = -3 \therefore A_3 = 1 \therefore A_1 = -1$

9.A

$$\int \frac{x^2-3x-1}{(2x+3)(x+1)^2}\,dx = \int \frac{-1}{2x+3}\,dx + \int \frac{-3}{(x+1)^2}\,dx + \int \frac{1}{x+1}\,dx$$

$$= (-1)\int \frac{1}{2x+3}\,dx^{*} + (-3)\int \frac{1}{(x+1)^2}\,dx + \ln|x+1|$$

$$= -\frac{1}{2}\ln|2x+3| + (-3)\int (x+1)^{-2}\,dx + \ln|x+1|$$

$$\boxed{= -\frac{1}{2}\ln|2x+3| + \frac{3}{x+1} + \ln|x+1| + C}$$

*
$u = 2x+3$

$\frac{du}{dx} = 2 \therefore dx = \frac{du}{2}$

$$\int \frac{1}{2x+3}\,dx = \int \frac{1}{u}\,\frac{du}{2} = \frac{1}{2}\int \frac{1}{u}\,du$$

$$\int \frac{1}{2x+3} = \frac{1}{2}\ln|u| = \frac{1}{2}\ln|2x+3|$$

1.ee

$$\int \frac{6x^2-2x-1}{(4x^3-x)}\,dx$$

MÉTODO DA INTEGRAÇÃO POR FRAÇÕES PARCIAIS:

$q(x) = (4x^3-x) \therefore q(x) = x(2x-1)(2x+1)$

$$\frac{6x^2-2x-1}{x(2x-1)(2x+1)} = \frac{A_1}{x} + \frac{A_2}{2x-1} + \frac{A_3}{2x+1}$$

MULTIPLICANDO TUDO POR $q(x)$

$$6x^2-2x-1 = A_1(4x^2-1) + A_2\,x(2x+1) + A_3\,x(2x-1)$$

$$= A_1 4x^2 - A_1 + A_2 2x^2 + A_2 x + A_3 2x^2 - A_3 x$$

$$= (4A_1 + 2A_2 + 2A_3)x^2 + (A_2 - A_3)x - A_1$$

$4A_1 + 2A_2 + 2A_3 = 6$

$4(1) + 2A_2 + 2(A_2+2) = 6$

$4 + 2A_2 + 2A_2 + 4 = 6$

$4A_2 = -2$

$A_2 = -\frac{1}{2}$

$A_2 - A_3 = -2$

$A_3 = A_2 + 2$

$A_3 = -\frac{1}{2} + 2$

$= \frac{3}{2}$

$-A_1 = -1$

$A_1 = 1$

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$$\int \frac{6x^2-2x-1}{(4x^3-1)}\,dx = \int \frac{1}{x}\,dx + \int -\frac{1}{4x-2}\,dx + \int \frac{3}{4x+2}\,dx$$

$$= \ln|x| + \left(-\frac{1}{2}\right)\int \frac{1}{2x-1}\,dx^{*} + \frac{3}{2}\int \frac{1}{2x+1}\,dx$$

$$\boxed{= \ln|x| - \frac{1}{4}\ln|2x-1| + \frac{3}{4}\ln|2x+1| + C}$$

$* \; u = 2x-1$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$$\int \frac{1}{2x-1}\,dx = \int \frac{1}{u}\,\frac{du}{2}$$

$$= \frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{1}{2}\ln|u| = \frac{1}{2}\ln|2x-1|$$

MESMO RACIOCÍNIO PARA:

$$\int \frac{1}{2x+1}\,dx$$

1.FF $\int e^{2x}\,\text{sen}\,3x\,dx$

(1.66) $\int \frac{1}{(x-3)(x+2)}\,dx$ (10.A)

MÉTODO DA INTEGRAÇÃO POR FRAÇÕES PARCIAIS:

$q(x) = (x-3)(x+2)$

$$\frac{1}{(x-3)(x+2)} = \frac{A_1}{x-3} + \frac{A_2}{x+2}$$ MULTIPLICANDO TUDO POR $q(x)$

$1 = A_1(x+2) + A_2(x-3)$

$1 = A_1x + 2A_1 + A_2x - 3A_2$

$1 = (A_1 + A_2)x + 2A_1 - 3A_2$

$0x + 1 = (A_1 + A_2)x + 2A_1 - 3A_2$

$A_1 + A_2 = 0$ $\qquad$ $2A_1 - 3A_2 = 1$

$A_1 = -A_2$ $\qquad$ $2(-A_2) - 3A_2 = 1$

$A_2 = -\frac{1}{5} \therefore A_1 = \frac{1}{5}$

$$\int \frac{1}{(x-3)(x+2)}\,dx = \int -\frac{1}{5x-15}\,dx + \int \frac{1}{5x+10}\,dx$$

$$= -\frac{1}{5}\int \frac{1}{x-3}\,dx + \frac{1}{5}\int \frac{1}{x+2}\,dx$$

$\int \frac{1}{x-3}\,dx$

$u = x-3$

$\frac{du}{dx} = 1$

$dx = du$

$\int \frac{1}{x-3}\,dx = \int \frac{1}{u}\,du$

$= \ln|u|$

$= \ln|x-3|$

$\int \frac{1}{x+2}\,dx$

$u = x+2$

$\frac{du}{dx} = 1$

$dx = du$

$\int \frac{1}{x+2}\,dx = \int \frac{1}{u}\,du$

$= \ln|u|$

$= \ln|x+2|$

PORTANTO,

$$\boxed{\int \frac{1}{(x-3)(x+2)}\,dx = -\frac{1}{5}\ln|x-3| + \frac{1}{5}\ln|x+2| + c}$$

1.HH) $\int \frac{1}{16x^4-1}\,dx$

MÉTODO DA INTEGRAÇÃO POR FRAÇÕES PARCIAIS:

$q(x) = 16x^4 - 1 \therefore q(x) = (4x^2-1)(4x^2+1)$

$$\frac{1}{(4x^2-1)(4x^2+1)} = \frac{A_1}{4x^2-1} + \frac{A_2}{4x^2+1}$$ MULTIPLICANDO TUDO POR $q(x)$

$1 = A_1(4x^2+1) + A_2(4x^2-1)$

$1 = A_1 4x^2 + A_1 + A_2 4x^2 - A_2$

$1 = (4A_1 + 4A_2)x^2 + A_1 - A_2$

$0x^2 + 1 = (4A_1 + 4A_2)x^2 + A_1 - A_2$

$4A_1 + 4A_2 = 0$ $\quad$ $A_1 - A_2 = 1$

$4A_1 = -4A_2$ $\quad$ $-A_2 - A_2 = 1$

$A_1 = -A_2$ $\quad$ $A_2 = -\frac{1}{2} \therefore A_1 = \frac{1}{2}$

$$\int \frac{1}{16x^4-1}\,dx = \int -\frac{1}{8x^2-2}\,dx + \int \frac{1}{8x^2+2}\,dx$$

$$= -\frac{1}{2}\int \frac{1}{4x^2-1}\,dx + \frac{1}{2}\int \frac{1}{4x^2+1}\,dx$$

$$\int \frac{1}{4x^2-1}\,dx$$

M.A

MÉTODO DA INTEGRAÇÃO POR FRAÇÕES PARCIAIS:

$q(x) = 4x^2 - 1 \therefore q(x) = (2x-1)(2x+1)$

$$\frac{1}{(2x-1)(2x+1)} = \frac{A_1}{2x-1} + \frac{A_2}{2x+1}$$

MULTIPLICANDO TUDO POR $q(x)$

$1 = A_1(2x+1) + A_2(2x-1)$

$1 = A_1 2x + A_1 + A_2 2x - A_2$

$1 = (2A_1 + 2A_2)x + A_1 - A_2$

$0x + 1 = (2A_1 + 2A_2)x + A_1 - A_2$

$2A_1 + 2A_2 = 0$

$A_1 = -A_2$

$A_1 - A_2 = 1$

$-A_2 - A_2 = 1$

$A_2 = -\frac{1}{2} \therefore A_1 = \frac{1}{2}$

PORTANTO,

$$\int \frac{1}{4x^2-1}\,dx = \int -\frac{1}{4x-2}\,dx + \int \frac{1}{4x+2}\,dx$$

$$= -\frac{1}{2}\int \frac{1}{2x-1}\,dx + \frac{1}{2}\int \frac{1}{2x+1}\,dx$$

$$\int \frac{1}{2x-1}\,dx$$

$u = 2x-1$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$$\int \frac{1}{2x-1}\,dx = \int \frac{1}{u}\frac{du}{2}$$

$$= \frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{1}{2}\ln|u|$$

$$= \frac{1}{2}\ln|2x-1|$$

$$\int \frac{1}{2x+1}\,dx$$

$u = 2x+1$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$$\int \frac{1}{2x+1}\,dx = \int \frac{1}{u}\frac{du}{2}$$

$$= \frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{1}{2}\ln|u|$$

$$= \frac{1}{2}\ln|2x+1|$$

LOGO,

$$\int \frac{1}{4x^2-1}\,dx = -\frac{1}{4}\ln|2x-1| + \frac{1}{4}\ln|2x+1|$$

12.A

1.II $\int \frac{1}{2x^3 - x} dx$

MÉTODO DA INTEGRAÇÃO POR FRAÇÕES PARCIAIS:

$q(x) = 2x^3 - x \therefore q(x) = x(2x^2 - 1)$

$$\frac{1}{x(2x^2-1)} = \frac{A_1}{x} + \frac{A_2}{2x^2-1}$$

MULTIPLICANDO TUDO POR $q(x)$

$1 = A_1(2x^2 - 1) + A_2 x$

$1 = A_1 2x^2 - A_1 + A_2 x$

$0x^2 + 0x + 1 = A_1 2x^2 + A_2 x - A_1$

$2A_1 = 0 \qquad A_2 = 0 \qquad -A_1 = 1$

$A_1 = 0 \qquad\qquad A_1 = -1$

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1.55 $\int \frac{5x-2}{x^2-4}\,dx$ 13.A

MÉTODO DA INTEGRAÇÃO POR FRAÇÕES PARCIAIS:

$q(x) = x^2 - 4 \therefore q(x) = (x-2)(x+2)$

$$\frac{5x-2}{(x-2)(x+2)} = \frac{A_1}{x-2} + \frac{A_2}{x+2}$$

MULTIPLICANDO TUDO POR $q(x)$

$5x - 2 = A_1(x+2) + A_2(x-2)$

$5x - 2 = A_1 x + 2A_1 + A_2 x - 2A_2$

$5x - 2 = (A_1 + A_2)x + 2A_1 - 2A_2$

$A_1 + A_2 = 5$

$A_1 = 5 - A_2$

$2A_1 - 2A_2 = -2$

$A_1 - A_2 = -1$

$A_1 = A_2 - 1$

$5 - A_2 = A_2 - 1$

$2A_2 = 6$

$A_2 = 3 \therefore A_1 = 2$

PORTANTO,

$$\int \frac{5x-2}{x^2-4}\,dx = \int \frac{2}{x-2}\,dx + \int \frac{3}{x+2}\,dx$$

$$= 2\int \frac{1}{x-2}\,dx + 3\int \frac{1}{x+2}\,dx$$

$\int \frac{1}{x-2}\,dx$

$u = x - 2$

$\frac{du}{dx} = 1$

$dx = du$

$\int \frac{1}{x-2}\,dx = \int \frac{1}{u}\,du$

$= \ln|u|$

$= \ln|x-2|$

$\int \frac{1}{x+2}\,dx$

$u = x + 2$

$\frac{du}{dx} = 1$

$dx = du$

$\int \frac{1}{x+2}\,dx = \int \frac{1}{u}\,du$

$= \ln|u|$

$= \ln|x+2|$

SENDO ASSIM, $\boxed{\int \frac{5x-2}{x^2-4}\,dx = 2\ln|x-2| + 3\ln|x+2|}$

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1.KK $\int \frac{x^2-3x-7}{(2x+3)(x+1)^3}\,dx$

14.A

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EXERCÍCIO 2:

13.A

2.A $CF(q) = 20$

$CMg(q) = 10e^{\frac{1}{2}q}$

$CT(q) = \int CMg(q)$

$= \int 10e^{1/2 q}\, dq$

$u = \frac{1}{2}q$

$\frac{du}{dq} = \frac{1}{2}$

$dq = 2du$

$\int 10e^{1/2 q}\, dq = \int 10e^{u}\, 2du$

$= 20\int e^{u}\, du$

$= 20e^{u} + c$

$= 20e^{1/2 q} + c$

PORTANTO,

$CT(q) = 20e^{1/2 q} + c$ $\quad c \Rightarrow$ CUSTO FIXO

$= 20e^{1/2 q} + 20$

CALCULANDO O CUSTO MÉDIO:

$CMe(q) = \frac{CT(q)}{q}$

$= \frac{20e^{1/2 q}}{q} + \frac{20}{q}$

CALCULANDO O CUSTO MÉDIO MÍNIMO:

$$\frac{\partial CMe}{\partial q} = 0$$

$$\left(20e^{1/2q}q^{-1} + 20q^{-1}\right)' = 0$$

$$20e^{1/2q}(q^{-1})' + q^{-1}(20e^{1/2q})' + (20q^{-1})' = 0$$

$$-20e^{1/2q}q^{-2} + 20q^{-1}e^{1/2q}\left(\frac{1}{2}\right) - 20q^{-2} = 0$$

$$\frac{-20e^{1/2q}}{q^2} + \frac{10e^{1/2q}}{q} - \frac{20}{q^2} = 0$$

$$\frac{-20e^{1/2q} + 10e^{1/2q}q - 20}{\cancel{q^2}} = 0$$

$$10e^{1/2q}q = 20 + 20e^{1/2q}$$

$$\boxed{q = \frac{2}{e^{1/2q}} + 2} \quad (1)$$

SUBSTITUINDO (2) EM (1):

$$q = \frac{2}{20} + 2 \therefore \boxed{q = 2,1}$$

2.B) $P = 200$

$\pi = ?$

COMO SE TRATA DE MERCADO EM COMPETIÇÃO PERFEITA O PREÇO DO BEM PRODUZIDO PELA FIRMA É IDÊNTICO AO CUSTO MARGINAL. LOGO,

$$P = CMg$$

$$200 = 10e^{1/2q}$$

$$e^{1/2q} = 20 \quad (2)$$

$$\pi = RT - CT$$

$$= P.q - (20e^{1/2q} + 20)$$

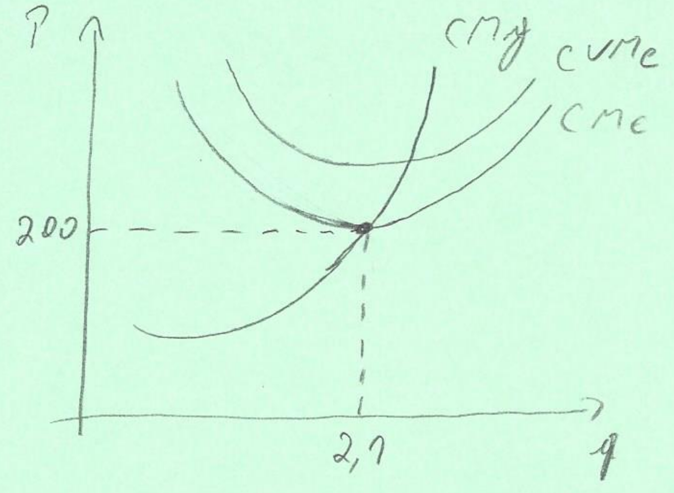
$$= 200(2,1) - [20(20) + 20]$$

$$= 420 - 420$$

$$\boxed{= 0}$$

=> MERCADO EM COMPETIÇÃO PERFEITA, ISTO É, LUCRO ECONÔMICO IGUAL A ZERO

56.A



2.C

$$EC = \int_{150}^{250} (250 - \tfrac{1}{2}P)\, dP$$

$$= \int_{150}^{250} 250\, dP + \int_{150}^{250} -\tfrac{1}{2} P\, dP$$

$$= 250 \int_{150}^{250} dP - \frac{1}{2} \int_{150}^{250} P\, dP$$

$$= 250 P \Big|_{150}^{250} - \frac{1}{2} \cdot \frac{P^2}{2} \Big|_{150}^{250}$$

$$= 250(250-150) - \frac{1}{4}\left[(250)^2 - (150)^2\right]$$

$$= 15.000$$

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Q.D

EXERCÍCIO 6: OBS: OS EXERCÍCIOS DESSE ÍTEM ESTÃO MAIS BEM RESOLVIDOS NAS FOLHAS BRANCAS 17.A

6.A $\frac{dY}{dx} + 5Y = 10$

$\frac{dY}{dx} = -5Y + 10$

$\frac{dY}{dx} = -5(Y-2)$

$\frac{dY}{Y-2} = -5dx$

$\int \frac{1}{Y-2} dY = \int -5 dx$

$\int \frac{1}{Y-2} dY = -5 \int dx$

PORTANTO,

$u = Y - 2$ $\quad \int \frac{1}{Y-2} dY = \int \frac{1}{u} du$

$\frac{du}{dY} = 1$ $\quad = \ln|u|$

$dY = du$ $\quad = \ln|Y-2|$

ENTÃO,

$\int \frac{1}{Y-2} dY = -5 \int dx$

$\ln|Y-2| = -5X + K$

$e^{\ln|Y-2|} = e^{-5X+K}$

$Y - 2 = e^{-5X+K}$

$Y(X) = e^{-5X} \cdot e^{K} + 2 \quad c = e^{K}$

$\boxed{= c \cdot e^{-5X} + 2}$ //

6.B $\frac{dY}{dx} + 2tY = t$

$u = e^{\int 2t\, dt}$ => FATOR INTEGRANTE

$= e^{t^2}$

$e^{t^2} \frac{dY}{dx} + e^{t^2} 2tY = e^{t^2} \cdot t$

18

$$\frac{d(\mu Y)}{dt} = e^{t^2} \cdot t$$

$$d(\mu \cdot Y) = e^{t^2} \cdot t\, dt$$

$$d(e^{t^2} \cdot Y) = e^{t^2} \cdot t\, dt$$

$$\int d(e^{t^2} \cdot Y) = \int e^{t^2} \cdot t\, dt$$

MÉTODO DA SUBSTITUIÇÃO

$$u = t^2$$

$$\frac{du}{dt} = 2t$$

$$dt = \frac{du}{2t}$$

PORTANTO,

$$\int d(e^{t^2} Y) = \int e^{u} \cdot \cancel{t}\, \frac{du}{2\cancel{t}}$$

$$\int d(e^{t^2} \cdot Y) = \frac{1}{2} \int e^{u}\, du$$

$$e^{t^2} \cdot Y = \frac{1}{2} e^{u} + c$$

$$e^{t^2} \cdot Y = \frac{1}{2} e^{t^2} + c$$

$$Y(t) = \frac{1}{2} + \frac{c}{e^{t^2}}$$

$$\boxed{Y(t) = c \cdot e^{-t^2} + \frac{1}{2}}$$

(6.C) $y' + y = t e^{-t} + 1$ (18.A)

FATOR INTEGRANTE:

$\mu = e^{\int dt}$

$= e^{t}$

$e^{t}.y' + e^{t}.y = e^{t}.t.e^{-t} + e^{t}$

$\frac{d(\mu.y)}{dt} = t + e^{t}$

$d(\mu.y) = (t + e^{t})\, dt$

$d(e^{t}.y) = (t + e^{t})\, dt$

$\int d(e^{t}.y) = \int (t + e^{t})\, dt$

$e^{t}.y = \int t\, dt + \int e^{t} dt$

$e^{t}.y = \frac{t^2}{2} + e^{t} + c$

$y = \frac{t^2}{2}.e^{-t} + 1 + c.e^{-t}$

$$\boxed{y(t) = 1 + e^{-t}\left(\frac{t^2}{2} + c\right)}$$

(6.D) $y' - 2y = 3e^{t}$

FATOR INTEGRANTE:

$\mu = e^{\int -2\, dt}$

$= e^{-2t}$

$e^{-2t}.y' - e^{-2t}.y = e^{-2t}.3e^{t}$

$\frac{d(\mu.y)}{dt} = 3e^{-t}$

$d(\mu.y) = 3e^{-t} dt$

$d(e^{-2t}.y) = 3e^{-t} dt$

$\int d(e^{-2t}.y) = \int 3e^{-t} dt$

$e^{-2t}.y = 3e^{-t} + c$

$y = \frac{3e^{-t}}{e^{-2t}} + \frac{c}{e^{-2t}}$

$$\boxed{y(t) = -3e^{t} + c.e^{2t}}$$

6.E $Y' - \frac{1}{t}Y = 3\cos 2t \quad t > 0$

FATOR INTEGRANTE:

$\mu = e^{\int -\frac{1}{t}\,dt}$

$= e^{(-1)\int \frac{1}{t}\,dt}$

$= e^{(-1)\ln|t|} \Rightarrow \mu = e^{\ln|t|^{-1}}$

$= t^{-1}$

$t^{-1}\cdot Y' - t^{-1}\cdot \frac{1}{t}Y = 3t^{-1}\cos 2t$

$t^{-1}\cdot Y' - t^{-2}\cdot Y = 3t^{-1}\cos 2t$

$\frac{d(\mu\cdot Y)}{dt} = 3t^{-1}\cos 2t$

$d(\mu\cdot Y) = 3t^{-1}\cos 2t\,dt$

$d(t^{-1}\cdot Y) = 3t^{-1}\cos 2t\,dt$

$\int d(t^{-1}\cdot Y) = \int 3t^{-1}\cos 2t\,dt$

$t^{-1}\cdot Y = 3\int t^{-1}\cos 2t\,dt$

6.F $2Y' + Y = 3t^2$

19.A

$2Y' + Y = 3t^2 \quad \times (1/2)$

$Y' + \frac{1}{2} Y = \frac{3}{2} t^2$

FATOR INTEGRANTE

$\mu = e^{\int 1/2\, dt}$

$= e^{1/2 \int dt}$

$= e^{1/2 t}$

$e^{1/2 t} \cdot Y' + e^{1/2 t} \cdot \frac{1}{2} Y = e^{1/2 t} \cdot \frac{3}{2} t^2$

$\frac{d(\mu \cdot Y)}{dt} = \frac{3}{2} e^{1/2 t} \cdot t^2$

$d(\mu \cdot Y) = \frac{3}{2} e^{1/2 t} \cdot t^2\, dt$

$d(e^{1/2 t} \cdot Y) = \frac{3}{2} e^{1/2 t} \cdot t^2\, dt$

$\int d(e^{1/2 t} \cdot Y) = \int \frac{3}{2} e^{1/2 t} \cdot t^2\, dt$

$\int d(e^{1/2 t} \cdot Y) = \frac{3}{2} \int e^{1/2 t} \cdot t^2\, dt$ (*)

$e^{1/2 t} \cdot Y = 3t^2 e^{1/2 t} - 12 t e^{1/2 t} + 24 e^{1/2 t} + c$

$\boxed{Y(t) = 3t^2 - 12t + 24 + c}$ //

(*) MÉTODO DA INTEGRAÇÃO POR PARTES:

$\mu = t^2 \qquad dv = e^{1/2 t} dt$

$\frac{d\mu}{dt} = 2t \qquad \int dv = \int e^{1/2 t} dt$

$v = 2e^{1/2 t}$

$\int e^{1/2 t} \cdot t^2 dt = \mu \cdot v - \int v\, d\mu$

$= t^2 2e^{1/2 t} - \int 2e^{1/2 t} 2t\, dt$

$= 2t^2 e^{1/2 t} - 4 \int e^{1/2 t} t\, dt$

$\underline{\int e^{1/2 t} t\, dt}$

$\mu = t \qquad dv = e^{1/2 t} dt$

$\frac{d\mu}{dt} = 1 \qquad \int dv = \int e^{1/2 t} dt$

$v = 2e^{1/2 t}$

$d\mu = dt$

$\int e^{1/2 t} t\, dt = \mu \cdot v - \int v\, d\mu$

$= t 2e^{1/2 t} - \int 2e^{1/2 t} dt$

$= 2t e^{1/2 t} - 2 \int e^{1/2 t} dt$

$= 2t e^{1/2 t} - 4 e^{1/2 t}$

LOGO,

$\int e^{1/2 t} \cdot t^2 dt = 2t^2 e^{1/2 t} - 8t e^{1/2 t} + 16 e^{1/2 t}$

6G $tY' + 2Y = \text{sen}\, t \qquad t > 0$

$tY' + 2Y = \text{sen}\, t \quad \times (1/t)$

$Y' + \frac{2}{t}Y = \frac{\text{sen}\, t}{t}$

FATOR INTEGRANTE:

$\mu = e^{\int \frac{2}{t}\, dt}$

$= e^{2\int \frac{1}{t}\, dt}$

$= e^{2\ln|t|}$

$\mu = e^{\ln t^2}$

$= t^2$

$t^2 Y' + \cancel{t^2}\frac{2}{\cancel{t}}Y = \cancel{t^2}\frac{\text{sen}\, t}{\cancel{t}}$

$t^2 Y' + t2Y = t\, \text{sen}\, t$

$\frac{d(\mu \cdot Y)}{dt} = t\, \text{sen}\, t$

$d(\mu \cdot Y) = t\, \text{sen}\, t\, dt$

$d(t^2 \cdot Y) = t\, \text{sen}\, t\, dt$

$\int d(t^2 \cdot Y) = \int t\, \text{sen}\, t\, dt$ (*)

$t^2 \cdot Y = -t \cdot \cos t + \text{sen}\, t + c$

$Y = -\frac{t \cdot \cos t}{t^{2}} + \frac{\text{sen}\, t}{t^2} + \frac{c}{t^2}$

$\boxed{Y(t) = -t^{-1} \cdot \cos t + t^{-2} \cdot \text{sen}\, t + t^{-2} \cdot c}$ //

(*) MÉTODO DA INTEGRAÇÃO POR PARTES:

$u = t \qquad dv = \text{sen}\, t\, dt$

$\frac{du}{dt} = 1 \qquad \int dv = \int \text{sen}\, t\, dt$

$du = dt \qquad v = -\cos t$

$\int t\, \text{sen}\, t\, dt = u \cdot v - \int v\, du$

$= t(-\cos t) - \int -\cos t\, dt$

$= -t \cdot \cos t - (-1)\int \cos t\, dt$

$= -t \cdot \cos t + \text{sen}\, t + c$

6.H $tY' - Y = t^2 e^{-t}$

$tY' - Y = t^2 e^{-t} \quad \times (1/t) \quad \therefore \quad Y' - \frac{1}{t}Y = t\, e^{-t}$

FATOR INTEGRANTE:

20.A

$$\mu = e^{\int -\frac{1}{t}\,dt}$$
$$= e^{(-1)\int \frac{1}{t}\,dt}$$
$$= e^{(-1)\ln|t|}$$

$$\mu = e^{\ln t^{-1}}$$
$$= t^{-1}$$
$$= \frac{1}{t}$$

$$\frac{1}{t}\cdot Y' - \frac{1}{t}\cdot\frac{1}{t}\cdot Y = \frac{1}{t}\cdot t\cdot e^{-t}$$

$$\frac{Y'}{t} - \frac{1}{t^2}Y = e^{-t}$$

$$\frac{d(\mu\cdot Y)}{dt} = e^{-t}$$

$$d(\mu\cdot Y) = e^{-t}\,dt$$

$$d\left(\frac{1}{t}\cdot Y\right) = e^{-t}\,dt$$

$$\int d\left(\frac{1}{t}\cdot Y\right) = \int e^{-t}\,dt$$

$$\frac{1}{t}\cdot Y = -e^{-t} + c$$

$$\boxed{Y(t) = -t\,e^{-t} + t\,c}$$ //

6. I $\quad Y' - \frac{1}{2}Y = 2\cos t$

FATOR INTEGRANTE:

$$\mu = e^{\int -\frac{1}{2}\,dt}$$
$$= e^{-1/2\int dt}$$
$$= e^{-1/2\,t}$$

$$e^{-1/2\,t}\cdot Y' - \frac{1}{2}\cdot e^{-1/2\,t}\cdot Y = 2e^{-1/2\,t}\cos t$$

$$\frac{d(\mu\cdot Y)}{dt} = 2e^{-1/2\,t}\cos t$$

$$d(\mu\cdot Y) = 2e^{-1/2\,t}\cos t\,dt$$

$$d(e^{-1/2\,t}\cdot Y) = 2e^{-1/2\,t}\cos t\,dt$$

$$\int d(e^{-1/2\,t}\cdot Y) = \int 2e^{-1/2\,t}\cos t\,dt$$

$$\int d(e^{-1/2\,t}\cdot Y) = 2\int e^{-1/2\,t}\cos t\,dt \quad (*)$$

(*) MÉTODO DA INTEGRAÇÃO POR PARTES:

$$u = e^{-1/2\,t} \qquad dv = \cos t\,dt$$

$$\frac{du}{dt} = -\frac{1}{2}e^{-1/2\,t} \qquad \int dv = \int \cos t\,dt$$

$$v = \operatorname{sen} t$$

$$du = -\frac{1}{2}e^{-1/2\,t}\,dt$$

$$\int e^{-1/2\,t}\cos t\,dt = u\cdot v - \int v\,du$$
$$= e^{-1/2\,t}\cdot\operatorname{sen} t - \int \operatorname{sen} t\cdot\left(-\frac{1}{2}e^{-1/2\,t}\right)dt$$
$$= e^{-1/2\,t}\cdot\operatorname{sen} t + \frac{1}{2}\underbrace{\int e^{-1/2\,t}\operatorname{sen} t\,dt}_{(**)}$$

(*) MÉTODO DA INTEGRAÇÃO POR PARTES:

$u = e^{-1/2t}$ $\qquad dv = \text{sen}\, t\, dt$

$\frac{du}{dt} = -\frac{1}{2} e^{-1/2t}$ $\qquad \int dv = \int \text{sen}\, t\, dt$

$\qquad v = -\cos t$

$du = -\frac{1}{2} e^{-1/2t} dt$

$$\int e^{-1/2t} \text{sen}\, t\, dt = u.v - \int v\, du$$
$$= e^{-1/2t}.(-\cos t) - \int -\cos t.\left(-\frac{1}{2} e^{-1/2t} dt\right)$$
$$= -e^{-1/2t}.\cos t - \frac{1}{2} \int \underbrace{e^{-1/2t}.\cos t\, dt}_{\blacktriangle}$$

▲ REPETE-SE O CICLO INDEFINIDAMENTE

(6.5) $y' + t^2 y = 3t^2$

FATOR INTEGRANTE:

$u = e^{\int t^2 dt}$

$= e^{\frac{t^3}{3}}$

$e^{\frac{t^3}{3}}.y' + e^{t^3/3}.t^2.y = 3e^{t^3/3}.t^2$

$\frac{d(u.y)}{dt} = 3e^{t^3/3}.t^2$

$d(u.y) = 3e^{t^3/3}.t^2 dt$

$d(e^{t^3/3}.y) = 3e^{t^3/3}.t^2 dt$

$\int d(e^{t^3/3}.y) = \int 3e^{t^3/3}.t^2 dt$

$\int d(e^{t^3/3}.y) = 3\int e^{t^3/3}.t^2 dt$ (*)

$e^{t^3/3}.y = 3e^{t^3/3} + c$

$$\boxed{y(t) = 3 + e^{-t^3/3}.c}$$

(*) MÉTODO DA SUBSTITUIÇÃO:

(27A)

$u = \frac{t^3}{3}$

$\frac{du}{dt} = t^2$

$dt = \frac{du}{t^2}$

$\int e^{t^3/3} \cdot t^2 \, dt = \int e^u \cdot t^2 \frac{du}{t^2}$

$= \int e^u \, du$

$= e^u$

$= e^{t^3/3}$

(6.K) $y' + y^2 \operatorname{sen} x = 0$

$y' + y^2 \operatorname{sen} x = 0$

$y' = -y^2 \operatorname{sen} x$

$\frac{dy}{dx} = -y^2 \operatorname{sen} x$

$\frac{dy}{y^2} = -\operatorname{sen} x \, dx$

$\int y^{-2} dy = \int -\operatorname{sen} x \, dx$

$-y^{-1} = \cos x + c$

$-\frac{1}{y} = \cos x + c$

$\boxed{y(x) = -\frac{1}{\cos x + c}}$ //

(6.L) $\frac{dy}{dx} = \frac{x - e^{-x}}{y - e^y}$

$\frac{dy}{dx} = \frac{x - e^{-x}}{y - e^y}$

$(y - e^y) dy = (x - e^{-x}) dx$

$\int (y - e^y) dy = \int (x - e^{-x}) dx$

$\int y \, dy - \int e^y dy = \int x \, dx - \int e^{-x} dx$

$\frac{y^2}{2} - e^y = \frac{x^2}{2} - (-e^{-x}) + c$

$\frac{y^2}{2} - e^y = \frac{x^2}{2} + e^{-x} + c$

$\frac{y^2}{2} = \frac{x^2}{2} + e^{-x} + e^y + c$

$y^2 = x^2 + 2(e^{-x} + e^y + c)$

$\boxed{y(x) = [x^2 + 2(e^{-x} + e^y + c)]^{1/2}}$ //

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6.M $XY' = (1-Y^2)^{1/2}$

$XY' = (1-Y^2)^{1/2}$

$$\frac{1}{(1-Y^2)^{1/2}} \, dY = \frac{1}{X} \cdot dX$$

$$\int \frac{1}{(1-Y^2)^{1/2}} \, dY = \int \frac{1}{X} \, dX$$

$\operatorname{arcsen} Y = \ln X + C$

$\boxed{Y = \operatorname{sen}(\ln X + C)}$ // SOLUÇÃO GERAL

6.N $\dfrac{dY}{dX} = \dfrac{X^2 + 3Y^2}{2XY}$

$$\frac{dY}{dX} = \frac{X^2}{2XY} + \frac{3Y^2}{2XY}$$

$$\frac{dY}{dX} = \frac{X}{2Y} + \frac{3Y}{2X}$$

$$\frac{dY}{dX} = \frac{1}{2}\left(\frac{Y}{X}\right)^{-1} + \frac{3}{2}\,\frac{Y}{X}$$

$M = \dfrac{Y}{X} \therefore Y = MX \therefore \dfrac{dY}{dX} = M(X)' + X \cdot \dfrac{dM}{dX} \therefore \dfrac{dY}{dX} = M + X\dfrac{dM}{dX}$

22.A

$$u + x \cdot \frac{du}{dx} = \frac{1}{2}\left(\frac{y}{x}\right)^{-1} + \frac{3}{2} \cdot \frac{y}{x}$$

$$u + x \cdot \frac{du}{dx} = \frac{1}{2} u^{-1} + \frac{3}{2} u$$

$$u + x \cdot \frac{du}{dx} = \frac{1}{2u} + \frac{3}{2} u$$

$$u + x \cdot \frac{du}{dx} = \frac{2 + 6u^2}{4u}$$

$$4u^2 + 4u \cdot x \frac{du}{dx} = 2 + 6u^2$$

$$4u \cdot x \cdot \frac{du}{dx} = 2 + 2u^2$$

$$2u \cdot x \frac{du}{dx} = 1 + u^2$$

$$2u \, du = \frac{1+u^2}{x} dx$$

$$\frac{2u}{1+u^2} du = \frac{1}{x} dx$$

$$\int \frac{2u}{1+u^2} du = \int \frac{1}{x} dx$$

MÉTODO DA SUBSTITUIÇÃO:

$$a = 1 + u^2$$

$$\frac{da}{du} = 2u$$

$$du = \frac{da}{2u}$$

$$\int \frac{\cancel{2u}}{a} \frac{da}{\cancel{2u}} = \ln x + c$$

$$\ln a = \ln x + c$$

$$\ln(1+u^2) = \ln x + c$$

$$e^{\ln(1+u^2)} = e^{\ln x + c}$$

$$1 + u^2 = e^{\ln x} \cdot e^{c}$$

$$1 + u^2 = x \cdot c_1$$

$$\frac{y^2}{x^2} = x \cdot c_1 - 1$$

$$y^2 = x^2 (x \cdot c_1 - 1)$$

$$\boxed{y = x\sqrt{x \cdot c_1 - 1}}$$

SOLUÇÃO GERAL

6.0 $\frac{dY}{dx} = \frac{4Y - 3X}{2X - Y}$

$$\frac{dY}{dx} = \frac{4\frac{Y}{X} - 3}{2 - \frac{Y}{X}}$$

$\frac{Y}{X} = u \therefore Y = uX \therefore \frac{dY}{dx} = u(x)' + X \cdot \frac{du}{dx} \therefore \frac{dY}{dx} = u + X \cdot \frac{du}{dx}$

$$u + X \cdot \frac{du}{dx} = \frac{4u - 3}{2 - u}$$

$$X \cdot \frac{du}{dx} = \frac{4u - 3}{2 - u} - u$$

$$(2 - u) X \cdot \frac{du}{dx} = 4u - 3 - u(2 - u)$$

$$(2 - u) X \frac{du}{dx} = 4u - 3 - 2u + u^2$$

$$(2 - u) X \frac{du}{dx} = u^2 + 2u - 3$$

$$\frac{2 - u}{u^2 + 2u - 3} du = \frac{1}{X} dx$$

$$\int \frac{2 - u}{u^2 + 2u - 3} du = \int \frac{1}{X} dx$$

MÉTODO DA INTEGRAÇÃO POR FRAÇÕES PARCIAIS:

$$\frac{2 - u}{u^2 + 2u - 3} = \frac{A_1}{(u + 1)} + \frac{A_2}{(u - 3)}$$

MULTIPLICANDO TUDO POR: $(u+1)(u-3)$

$$2 - u = A_1(u - 3) + A_2(u + 1)$$

$$2 - u = A_1 u - 3A_1 + A_2 u + A_2$$

$$2 - u = A_2 - 3A_1 + (A_1 + A_2) u$$

23.A

$A_2 - 3A_1 = 2$

$A_2 = 2 + 3A_1$

$A_1 + A_2 = -1$

$A_1 + 2 + 3A_1 = -1$

$4A_1 = -3$

$A_1 = -\frac{3}{4} \therefore A_2 = -\frac{1}{4}$

$$\int \frac{2-u}{u^2+2u-3} = \int -\frac{3}{4(u+1)}du + \int -\frac{1}{4(u-3)}du$$

$$= -\frac{3}{4}\int \frac{1}{(u+1)}du - \frac{1}{4}\int \frac{1}{(u-3)}du$$

$$= -\frac{3}{4}\ln(u+1) - \frac{1}{4}\ln(u-3)$$

$$= -\frac{3}{4}\ln\left(\frac{y}{x}+1\right) - \frac{1}{4}\ln\left(\frac{y}{x}-3\right)$$

$$-\frac{3}{4}\ln\left(\frac{y}{x}+1\right) - \frac{1}{4}\ln\left(\frac{y}{x}-3\right) = \ln x + c$$

$$\ln\left(\frac{y}{x}+1\right)^{-3/4} + \ln\left(\frac{y}{x}-3\right)^{-1/4} = \ln x + c$$

$$e^{\ln(y/x+1)^{-3/4}} + e^{\ln(y/x-3)^{-1/4}} = e^{\ln x + c}$$

$$\left(\frac{y}{x}+1\right)^{-3/4} + \left(\frac{y}{x}-3\right)^{-1/4} = e^{\ln x} \cdot e^{c}$$

$$\boxed{\left(\frac{y}{x}+1\right)^{-3/4} + \left(\frac{y}{x}-3\right)^{-1/4} = x \cdot c_1}$$ SOLUÇÃO GERAL //

6.P $(x^2 + 3xy + y^2)\,dx = x^2\,dy$

$$\frac{dy}{dx} = \frac{x^2+3xy+y^2}{x^2}$$

$$\frac{dy}{dx} = 1 + 3\frac{y}{x} + \left(\frac{y}{x}\right)^2$$

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$$u = \frac{y}{x} \therefore y = ux \therefore \frac{dy}{dx} = u(x)' + x\cdot\frac{du}{dx} \therefore \frac{dy}{dx} = u + x\cdot\frac{du}{dx}$$

$$u + x\cdot\frac{du}{dx} = 1 + 3\frac{y}{x} + \left(\frac{y}{x}\right)^2$$

$$u + x\cdot\frac{du}{dx} = 1 + 3u + u^2$$

$$x\cdot\frac{du}{dx} = u^2 + 2u + 1$$

$$\frac{1}{u^2+2u+1}\,du = \frac{1}{x}\,dx$$

$$\int \frac{1}{u^2+2u+1}\,du = \int \frac{1}{x}\,dx$$

$$\int \frac{1}{(u+1)^2}\,du = \int \frac{1}{x}\,dx$$

MÉTODO DA INTEGRAÇÃO POR SUBSTITUIÇÃO:

$$a = u+1$$

$$\frac{da}{du} = 1$$

$$du = da$$

$$\int \frac{1}{(u+1)^2}\,du = \int \frac{1}{a^2}\,da = -\frac{1}{a} = -\frac{1}{u+1}$$

$$\int \frac{1}{(u+1)^2}\,du = \int \frac{1}{x}\,dx$$

$$-\frac{1}{u+1} = \ln x + c$$

$$u + 1 = -\frac{1}{\ln x + c}$$

$$\frac{y}{x} = -\frac{1}{\ln x + c} - 1$$

$$\boxed{y = -\frac{x}{\ln x + c} - x}$$ SOLUÇÃO GERAL

EXERCÍCIO 12:

12.A $y'' - 2y' + 3y = 2$

RAÍZES CARACTERÍSTICAS:

$r^2 - 2r + 3 = 0$

FÓRMULA DE BHASKARA:

$\alpha \pm \beta i$

$\Delta = b^2 - 4ac \qquad r = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$= 4 - 12$

$= -8$

$= \dfrac{2 \pm 2,8i}{2} \rightarrow r_1 = 1 + 1,4i$

$\qquad\qquad \rightarrow r_2 = 1 - 1,4i$

SOLUÇÃO HOMOGÊNEA ($b=0$):

$y_H = e^{\alpha t}\{A_1 \cos(\beta t) + A_2 \operatorname{sen}(\beta t)\}$

$= e^{t}\{A_1 \cos(1,4t) + A_2 \operatorname{sen}(1,4t)\}$

SOLUÇÃO PARTICULAR ($y'=0$):

$3y_p = 2$

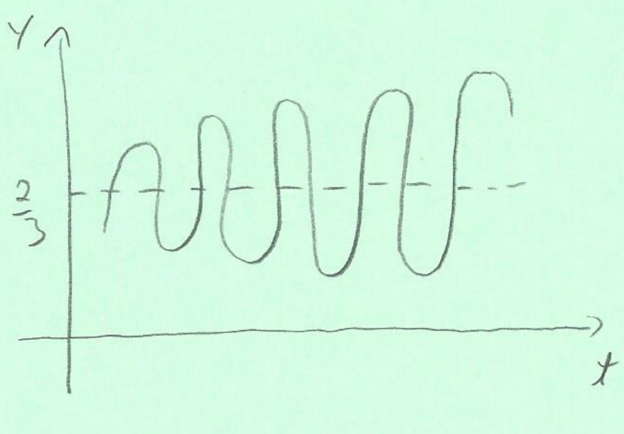
$y_p = \dfrac{2}{3}$

SOLUÇÃO GERAL:

$y = y_H + y_P$

$\boxed{= e^{t}\{A_1 \cos(1,4t) + A_2 \operatorname{sen}(1,4t)\} + \dfrac{2}{3}}$ //

$\lim_{t \to \infty} y = \infty$



$\boxed{\text{DIVERGE}}$ //

12.B $y'' + 3y = 9$

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RAÍZES CARACTERÍSTICAS:

$r^2 + 3 = 0$ $\qquad\qquad$ $R \pm ri$

$r = \pm 1{,}7i$

SOLUÇÃO HOMOGÊNEA ($b=0$)

$Y_H = e^{Rt}\{A_1 \cos(rt) + A_2 \operatorname{sen}(rt)\}$

$= e^{(0)t}\{A_1 \cos(1{,}7t) + A_2 \operatorname{sen}(1{,}7t)\}$

$= \{A_1 \cos(1{,}7t) + A_2 \operatorname{sen}(1{,}7t)\}$

SOLUÇÃO PARTICULAR ($Y'=0$):

$3Y_P = 9$

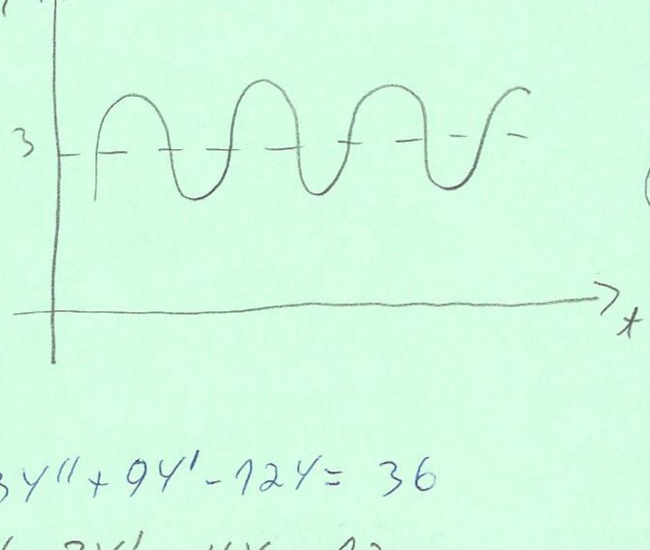
$Y_P = 3$

SOLUÇÃO GERAL:

$Y = Y_H + Y_P$

$$\boxed{= \{A_1 \cos(1{,}7t) + A_2 \operatorname{sen}(1{,}7t)\} + 3}$$

$\lim_{t \to \infty} Y = \{A_1 \cos(1{,}7t) + A_2 \operatorname{sen}(1{,}7t)\} + 3$



$\boxed{\text{NÃO CONVERGE E NEM DIVERGE}}$

12.C $\quad 3Y'' + 9Y' - 12Y = 36$

$Y'' + 3Y' - 4Y = 12$

RAÍZES CARACTERÍSTICAS:

$r^2 + 3r - 4 = 0$

FÓRMULA DE BHASKARA: (25.A)

$\Delta = b^2 - 4ac$

$= 9 + 16$

$= 25$

$r = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$= \dfrac{-3 \pm 5}{2}$ $\nearrow r_1 = 1$ $\searrow r_2 = -4$

SOLUÇÃO HOMOGÊNEA ($b = 0$):

$Y_H = A_1 e^{r_1 t} + A_2 e^{r_2 t}$

$= A_1 e^{t} + A_2 e^{-4t}$

SOLUÇÃO PARTICULAR ($Y' = 0$):

$-4Y_P = 12$

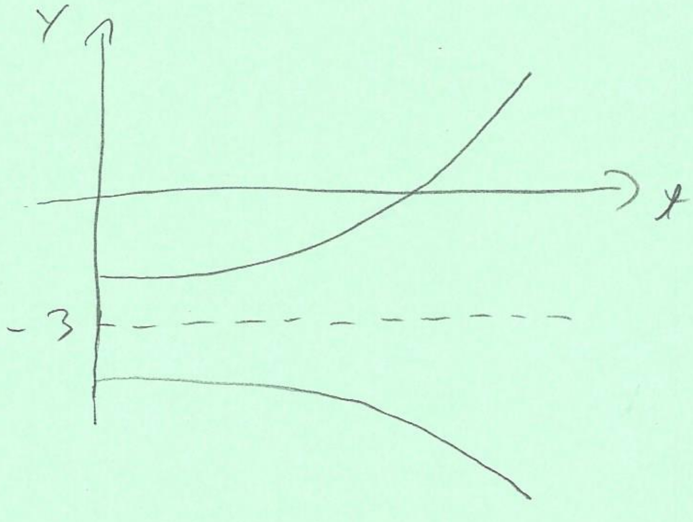
$Y_P = -3$

SOLUÇÃO GERAL:

$Y = Y_H + Y_P$

$\boxed{= A_1 e^{t} + A_2 e^{-4t} - 3}$ SOLUÇÃO GERAL

$\lim_{t \to \infty} Y = \pm\infty$



$\boxed{\text{DIVERGE}}$

(12.D) $Y'' + 4Y' + 8Y = 2$ $\quad Y(0) = 3$ $\quad Y'(0) = 7$

RAÍZES CARACTERÍSTICAS:

$r^2 + 4r + 8 = 0$

FÓRMULA DE BHASKARA:

$\Delta = b^2 - 4ac$

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$\alpha \pm \beta i$

$\Delta = 16 - 32$

$= -16$

$r = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$= \dfrac{-4 \pm 4i}{2}$ $\nearrow r_1 = -2+2i$ $\searrow r_2 = -2-2i$

SOLUÇÃO HOMOGÊNEA (b=0):

$Y_H = e^{\alpha t}\{A_1 \cos(\beta t) + A_2 \operatorname{sen}(\beta t)\}$

$= e^{-2t}\{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\}$

SOLUÇÃO PARTICULAR (Y'=0):

$8Y_P = 2$

$Y_P = \dfrac{1}{4}$

SOLUÇÃO GERAL:

$Y = Y_H + Y_P$

$\boxed{= e^{-2t}\{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\} + \dfrac{1}{4}}$ //

SOLUÇÃO DEFINIDA:

$Y' = e^{-2t}\{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\}' + \{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\}(e^{-2t})'$

$= e^{-2t}\{-2A_1 \operatorname{sen}(2t) + 2A_2 \cos(2t)\} + \{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\}(-2e^{-2t})$

$Y'(0) = e^{-2(0)}\{-2A_1 \cancelto{0}{\operatorname{sen}(2(0))} + 2A_2 \cos(2(0))\} + \{A_1 \cos(2(0)) + A_2 \cancelto{0}{\operatorname{sen}(2(0))}\}(-2e^{-2(0)})$

$7 = 2A_2 - 2A_1$

$2A_2 - 2A_1 = 7$

$Y(0) = e^{-2(0)}\{A_1 \cos(2(0)) + A_2 \cancelto{0}{\operatorname{sen}(2(0))}\} + \dfrac{1}{4}$

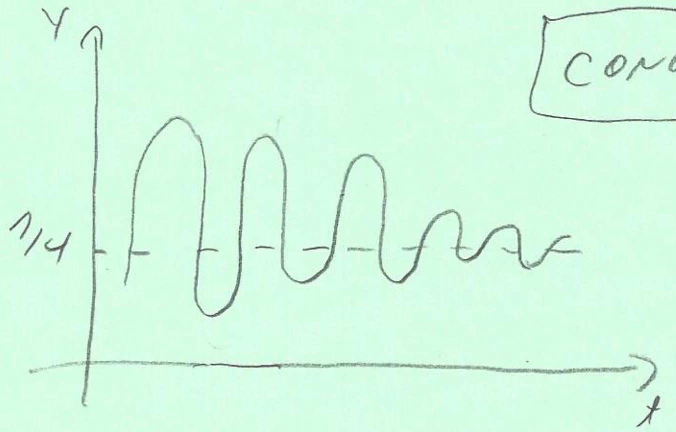
$3 = A_1 + \dfrac{1}{4}$

$A_1 = \dfrac{11}{4} \therefore A_2 = \dfrac{25}{4}$

26.A

$$\boxed{Y = e^{-2x}\{2{,}75\cos(2x) + 6{,}25\,\mathrm{sen}(2x)\} + \frac{1}{4}}$$ SOLUÇÃO DEFINIDA

$$\lim_{x\to\infty} Y = \frac{1}{4}$$



CONVERGE

12.E $Y'' + 9Y = 3 \qquad Y(0) = 1 \qquad Y'(0) = 3$

RAÍZES CARACTERÍSTICAS:

$r^2 + 9 = 0 \qquad \alpha \pm \beta i$

$r = \pm 3i$

SOLUÇÃO HOMOGÊNEA ($b=0$):

$Y_H = e^{\alpha x}\{A_1\cos(\beta x) + A_2\,\mathrm{sen}(\beta x)\}$

$= e^{(0)x}\{A_1\cos(3x) + A_2\,\mathrm{sen}(3x)\}$

$= \{A_1\cos(3x) + A_2\,\mathrm{sen}(3x)\}$

SOLUÇÃO PARTICULAR ($Y'=0$):

$9Y_p = 3$

$Y_p = \frac{1}{3}$

SOLUÇÃO GERAL:

$Y = Y_H + Y_P$

$$\boxed{= \{A_1\cos(3x) + A_2\,\mathrm{sen}(3x)\} + \frac{1}{3}}$$

SOLUÇÃO DEFINIDA:

$Y' = \{-3A_1\,\mathrm{sen}(3x) + 3A_2\cos(3x)\}$

$Y'(0) = \{-3A_1\,\overset{0}{\cancel{\mathrm{sen}}}(3(0)) + 3A_2\cos(3(0))\}$

$3 = 3A_2$

$A_2 = 1$

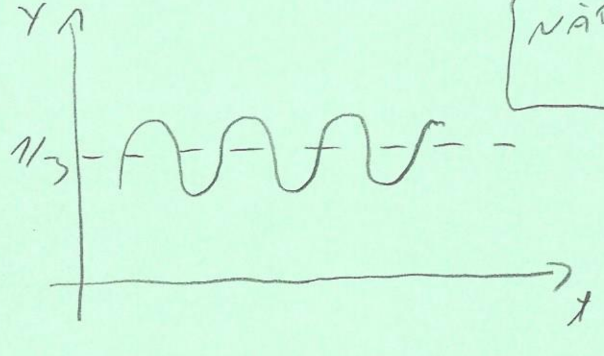
$Y(0) = \{A_1 \cos(3(0)) + A_2 \cancel{sen(3(0))}^{0}\} + \frac{1}{3}$

$1 = A_1 + \frac{1}{3}$

$A_1 = \frac{2}{3}$

$$\boxed{Y = \left\{\frac{2}{3}\cos(3t) + sen(3t)\right\} + \frac{1}{3}}$$ SOLUÇÃO DEFINIDA

$$\lim_{t \to \infty} Y = \left\{\frac{2}{3}\cos(3t) + sen(3t)\right\} + \frac{1}{3}$$



NÃO CONVERGE E NEM DIVERGE

12.F $2Y'' - 12Y' + 20Y = 40 \quad Y(0) = 4 \quad Y'(0) = 5$

$Y'' - 6Y' + 10Y = 20$

RAÍZES CARACTERÍSTICAS:

$r^2 - 6r + 10 = 0$

FÓRMULA DE BHASKARA:

$\Delta = b^2 - 4ac$
$= 36 - 40$
$= -4$

$r = \frac{-b \pm \sqrt{\Delta}}{2a}$
$= \frac{6 \pm 2i}{2}$

$r_1 = 3 + i$
$r_2 = 3 - i$

$a \pm ri$

SOLUÇÃO HOMOGÊNEA ($b=0$):

$Y_H = e^{\lambda t}\{A_1 \cos(\omega t) + A_2 \operatorname{sen}(\omega t)\}$

$= e^{3t}\{A_1 \cos t + A_2 \operatorname{sen} t\}$

07.A

SOLUÇÃO PARTICULAR ($Y'=0$):

$10Y_p = 20$

$Y_p = 2$

SOLUÇÃO GERAL:

$Y = Y_H + Y_p$

$\boxed{= e^{3t}\{A_1 \cos t + A_2 \operatorname{sen} t\} + 2}$ //

SOLUÇÃO DEFINIDA:

$Y' = e^{3t}\{A_1 \cos t + A_2 \operatorname{sen} t\}' + \{A_1 \cos t + A_2 \operatorname{sen} t\}(e^{3t})'$

$= e^{3t}\{-A_1 \operatorname{sen} t + A_2 \cos t\} + \{A_1 \cos t + A_2 \operatorname{sen} t\}(3e^{3t})$

$Y'(0) = e^{3(0)}\{-A_1 \overset{0}{\cancel{\operatorname{sen}(0)}} + A_2 \cos(0)\} + \{A_1 \cos(0) + A_2 \overset{0}{\cancel{\operatorname{sen}(0)}}\}(3e^{3(0)})$

$5 = A_2 + 3A_1$

$3A_1 + A_2 = 5$

$Y(0) = e^{3(0)}\{A_1 \cos(0) + A_2 \overset{0}{\cancel{\operatorname{sen}(0)}}\} + 2$

$4 = A_1 + 2$

$A_1 = 2 \therefore A_2 = -1$

$\boxed{Y = e^{3t}\{2\cos t - \operatorname{sen} t\} + 2}$ // $\lim_{t \to \infty} Y = \infty$

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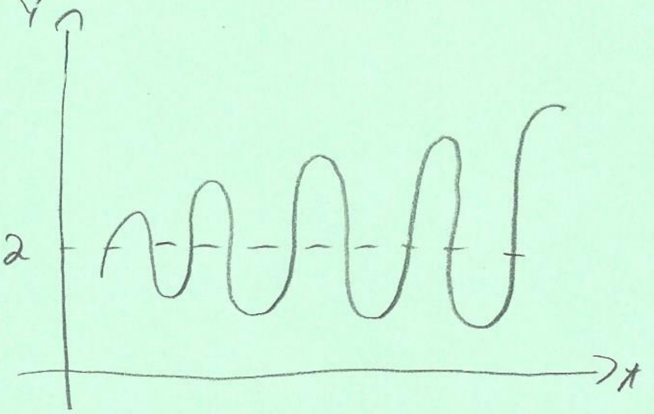


$\boxed{\text{DIVERGE}}$ //

12.6 $y'' + y' - 2y = 0 \qquad y(0) = 1 \qquad y'(0) = 1$

RAÍZES CARACTERÍSTICAS:

$r^2 + r - 2 = 0$

FÓRMULA DE BHASKARA:

$\Delta = b^2 - 4ac \qquad r = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$= 1 + 8$

$= 9 \qquad = \dfrac{-1 \pm 3}{2} \begin{cases} r_1 = 1 \\ r_2 = -2 \end{cases}$

SOLUÇÃO HOMOGÊNEA $(b=0)$:

$y_H = A_1 e^{r_1 x} + A_2 e^{r_2 x}$

$= A_1 e^{x} + A_2 e^{-2x}$

SOLUÇÃO GERAL:

$y = y_H$

$\boxed{= A_1 e^{x} + A_2 e^{-2x}}$ //

SOLUÇÃO DEFINIDA:

$y' = A_1 e^{x} - 2A_2 e^{-2x}$

28.A

$Y'(0) = A_1 e^{0} - 2A_2 e^{-2(0)}$

$1 = A_1 - 2A_2$

$A_1 - 2A_2 = 1$

$Y(0) = A_1 e^{0} + A_2 e^{-2(0)}$

$1 = A_1 + A_2$

$A_1 + A_2 = 1$

$$\begin{cases} A_1 - 2A_2 = 1 \\ A_1 + A_2 = 1 \quad \times(2) \end{cases}$$

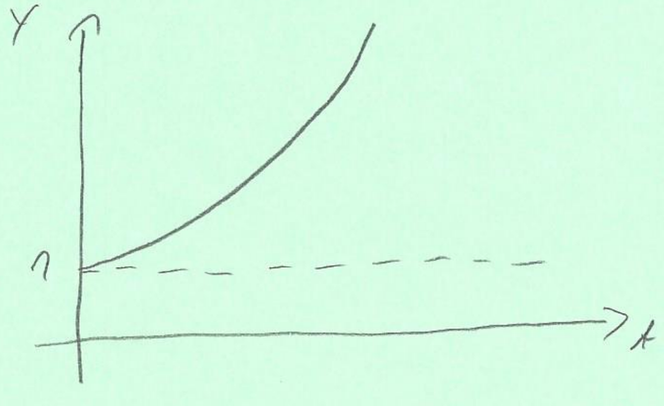
$$\begin{cases} A_1 - 2\cancel{A_2} = 1 \\ 2A_1 + 2\cancel{A_2} = 2 \end{cases}$$

$3A_1 = 3$

$A_1 = 1 \therefore A_2 = 0$

$\boxed{Y = e^{t}}$ SOLUÇÃO DEFINIDA

$\lim_{t \to \infty} Y = \infty$



$\boxed{\text{DIVERGE}}$

12.H $4Y'' - Y = 0$ $\quad Y(-2) = 1 \quad Y'(-2) = -1$

RAÍZES CARACTERÍSTICAS:

$4r^2 - 1 = 0$

$r = \pm \frac{1}{2}$

SOLUÇÃO HOMOGÊNEA $(b=0)$:

$Y_H = A_1 e^{r_1 t} + A_2 e^{r_2 t}$

$= A_1 e^{1/2\,t} + A_2 e^{-1/2\,t}$

LISTA VIII MATEMÁTICA I

(29)

2013/1.º

SOLUÇÃO GERAL:

$$Y = Y_H$$

$$\boxed{= A_1 e^{1/2 x} + A_2 e^{-1/2 x}}$$

SOLUÇÃO DEFINIDA:

$$Y' = \frac{1}{2} A_1 e^{1/2 x} - \frac{1}{2} A_2 e^{-1/2 x}$$

$$Y'(-2) = \frac{1}{2} A_1 e^{-1} - \frac{1}{2} A_2 e^{1}$$

$$-1 = 0,18 A_1 - 1,36 A_2$$

$$0,18 A_1 - 1,36 A_2 = -1$$

$$Y(-2) = A_1 e^{-1} + A_2 e^{1}$$

$$1 = 0,37 A_1 + 2,72 A_2$$

$$0,36 A_1 + 2,72 A_2 = 1$$

$$\begin{cases} 0,18 A_1 - 1,36 A_2 = -1 & \times(-2) \\ 0,36 A_1 + 2,72 A_2 = 1 \end{cases}$$

$$\begin{cases} -\cancel{0,36 A_1} + 2,72 A_2 = 2 \\ \cancel{0,36 A_1} + 2,72 A_2 = 1 \end{cases}$$

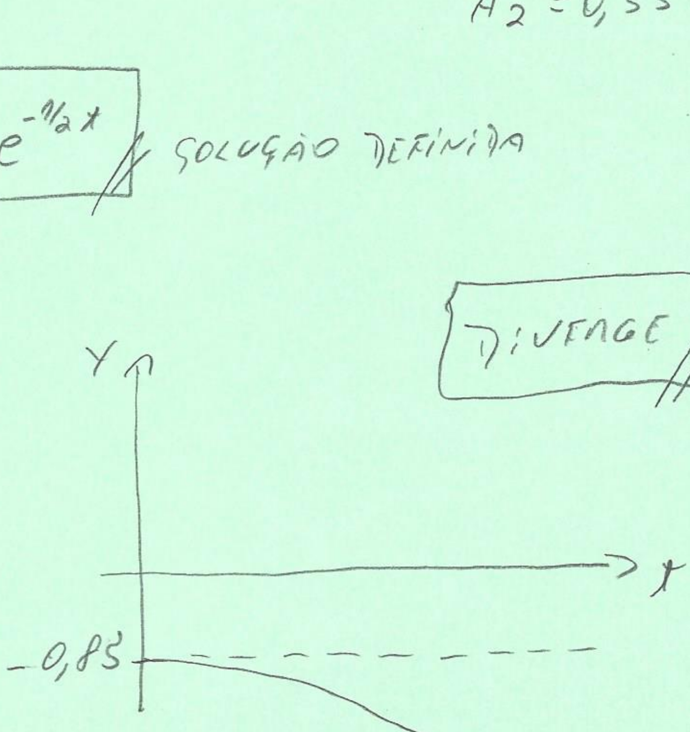
$$5,44 A_2 = 3$$

$$A_2 = 0,55 \therefore A_1 = -1,4$$

$$\boxed{Y = -1,4 e^{1/2 x} + 0,55 e^{-1/2 x}}$$ SOLUÇÃO DEFINIDA

$$\lim_{x \to \infty} Y = -\infty$$

$\boxed{\text{DIVERGE}}$



12.I $2Y'' + Y' - 4Y = 6 \quad Y(0) = 4 \quad Y'(0) = 0$ 29.A

$$Y'' + \frac{1}{2}Y' - 2Y = 3$$

RAÍZES CARACTERÍSTICAS:

$$r^2 + \frac{1}{2}r - 2 = 0$$

FÓRMULA DE BHASKARA:

$$\Delta = b^2 - 4ac \qquad r = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$= \frac{1}{4} + 8$$

$$= \frac{-0{,}5 \pm 2{,}9}{2} \quad \begin{cases} r_1 = 1{,}2 \\ r_2 = -1{,}7 \end{cases}$$

$$= \frac{33}{4}$$

SOLUÇÃO HOMOGÊNEA ($b=0$):

$$Y_H = A_1 e^{r_1 t} + A_2 e^{r_2 t}$$

$$= A_1 e^{1{,}2t} + A_2 e^{-1{,}7t}$$

SOLUÇÃO PARTICULAR ($Y'' = Y' = 0$):

$$-2Y_p = 3$$

$$Y_p = -1{,}5$$

SOLUÇÃO GERAL:

$$Y = Y_H + Y_P$$

$$\boxed{= A_1 e^{1{,}2t} + A_2 e^{-1{,}7t} - 1{,}5}$$ //

SOLUÇÃO DEFINIDA:

$$Y' = 1{,}2A_1 e^{1{,}2t} - 1{,}7A_2 e^{-1{,}7t}$$

$$Y'(0) = 1{,}2A_1 e^{1{,}2(0)} - 1{,}7A_2 e^{-1{,}7(0)}$$

$$0 = 1{,}2A_1 - 1{,}7A_2$$

$$1{,}2A_1 = 1{,}7A_2$$

$$A_1 = 1{,}4A_2$$