LISTA VIII MATEMÁTICA I

2013/1.º

$$Y(0) = A_1 e^{1,2(0)} + A_2 e^{-1,7(0)} - 1,3$$

$$4 = A_1 + A_2 - 1,5$$

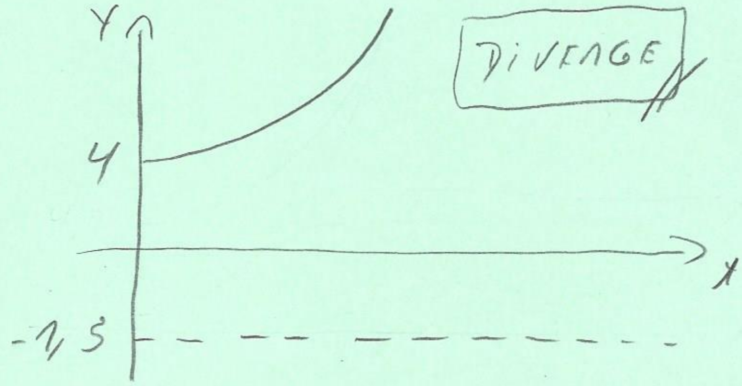
$$A_1 + A_2 = 5,5$$

$$1,4A_2 + A_2 = 5,5$$

$$A_2 = 2,3 \therefore A_1 = 3,2$$

$$\boxed{Y = 3,2e^{1,2t} + 2,3e^{-1,7t} - 1,5}$$ SOLUÇÃO DEFINIDA

$$\lim_{t \to \infty} Y = \infty$$



$\boxed{\text{DIVERGE}}$

12.J) $y'' - 2y' + 5y = 0$ $\quad y\left(\frac{\pi}{2}\right) = 0$ $\quad y'\left(\frac{\pi}{2}\right) = 2$

RAÍZES CARACTERÍSTICAS:

$$r^2 - 2r + 5 = 0$$

FÓRMULA DE BHASKARA:

$\alpha \pm \beta i$

$$\Delta = b^2 - 4ac \qquad r = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$= 4 - 20$$

$$= -16 \qquad = \frac{2 \pm 4i}{2} \to r_1 = 1 + 2i, \quad r_2 = 1 - 2i$$

SOLUÇÃO HOMOGÊNEA ($b=0$):

$$y_H = e^{\alpha t}\{A_1 \cos(\beta t) + A_2 \operatorname{sen}(\beta t)\}$$

30.A

$Y_H = e^{t}\{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\}$

SOLUÇÃO GERAL:

$Y = Y_H$

$$\boxed{= e^{t}\{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\}}$$ //

SOLUÇÃO DEFINIDA:

$Y' = e^{t}\{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\}' + \{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\}(e^{t})'$

$= e^{t}\{-2A_1 \operatorname{sen}(2t) + 2A_2 \cos(2t)\} + \{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\}(e^{t})$

$Y'(\frac{\pi}{2}) = e^{\pi/2}\{-2A_1 \operatorname{sen}(\pi) + 2A_2 \cos(\pi)\} + \{A_1 \cos(\pi) + A_2 \operatorname{sen}(\pi)\}(e^{\pi/2})$

$2 = e^{\pi/2}\{0 - 2A_2\} + \{-A_1 + 0\}e^{\pi/2}$

$2 = -2A_2 e^{\pi/2} - A_1 e^{\pi/2}$

$A_1 e^{\pi/2} + 2A_2 e^{\pi/2} = -2$

$Y(\pi/2) = e^{\pi/2}\{A_1 \cos(\pi) + A_2 \operatorname{sen}(\pi)\}$

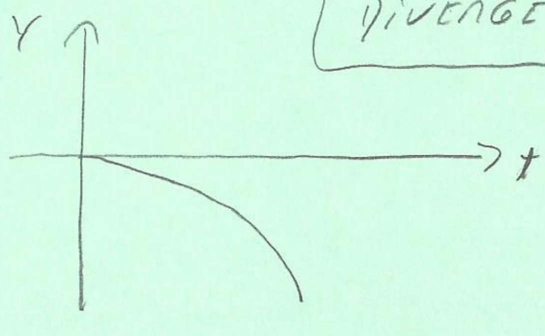
$0 = e^{\pi/2}\{-A_1 + 0\}$

$A_1 = 0 \therefore A_2 = -e^{-\pi/2}$

$$\boxed{Y = e^{t}\{-e^{-\pi/2} \operatorname{sen}(2t)\}}$$ // SOLUÇÃO DEFINIDA

$\boxed{\text{DIVERGE}}$ //

$\lim_{t \to \infty} Y = -\infty$



LISTA VIII MATEMÁTICA I

2013/1-0

EXERCÍCIO 8:

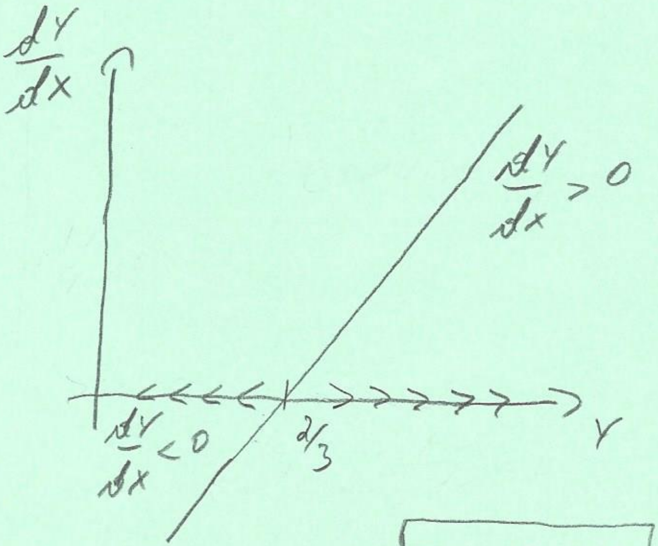
8.A $\dfrac{dY}{dx} - 3Y = -2$

$$\frac{dY}{dx} = 3Y - 2$$

PONTO ESTACIONÁRIO:

$3Y - 2 = 0$

$Y = \dfrac{2}{3}$



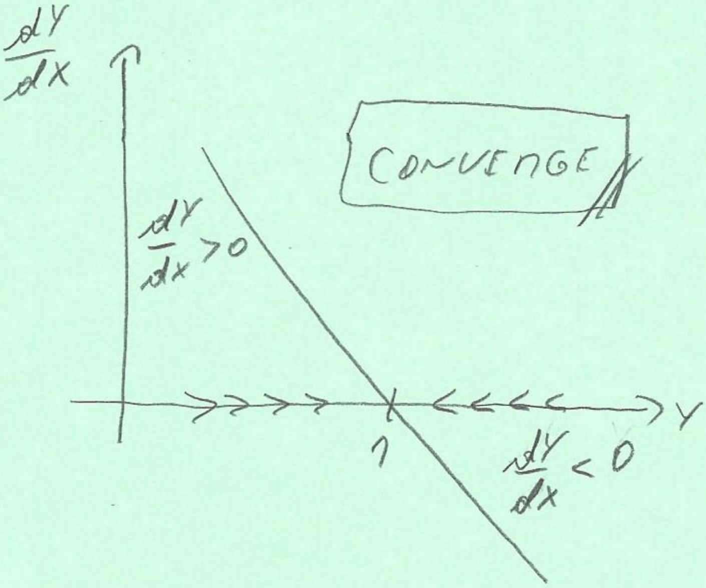
$\boxed{\text{DIVERGE}}$

8.B $\dfrac{dY}{dx} = 1 - Y$

PONTO ESTACIONÁRIO:

$1 - Y = 0$

$Y = 1$



8.C $\dfrac{dY}{dx} = (Y+1)^2 - 12 \qquad Y \geq 0$

PONTO ESTACIONÁRIO:

$(Y+1)^2 - 12 = 0$

$(Y+1)^2 = 12$

$Y^2 + 2Y + 1 = 12$

$\rightarrow Y^2 + 2Y - 11 = 0$

FÓRMULA DE BHASKARA:

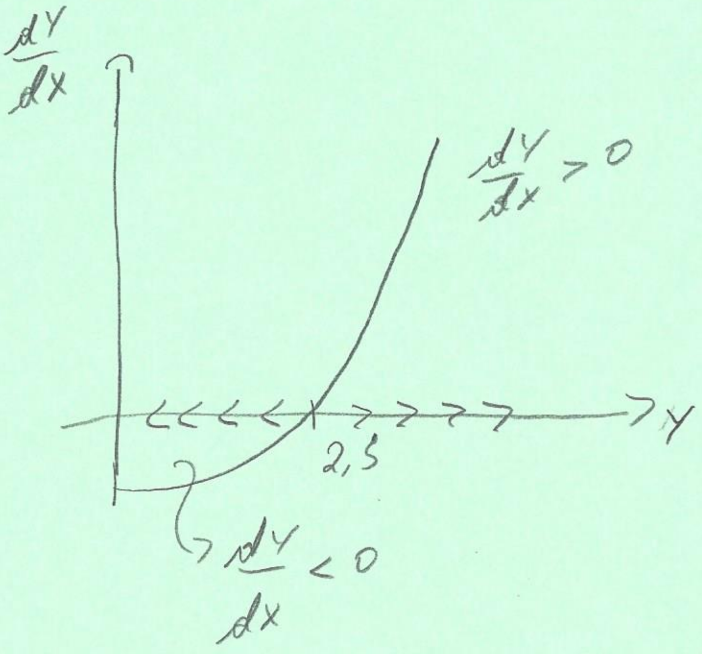
$\Delta = b^2 - 4ac$

$= 4 + 44$

$= 48$

$y = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$= \dfrac{-2 \pm 6,9}{2}$ → $y_1 = 2,5$; → $y_2 = -4,5$ (DESCARTA, POIS O ENUNCIADO DIZ $y > 0$)



$\boxed{\text{DIVERGE}}$

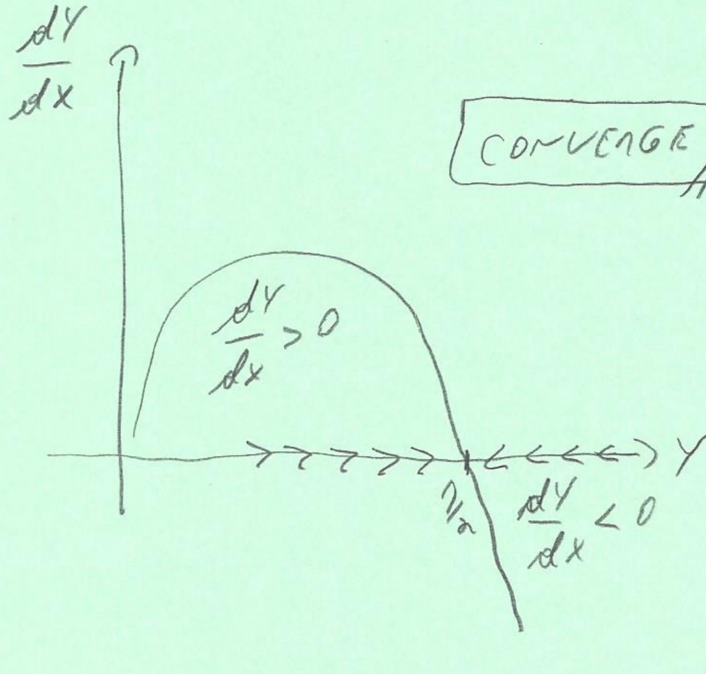
(g.D) $\dfrac{dy}{dx} = -y^2 + \dfrac{1}{2}y$ $\quad y > 0$

PONTO ESTACIONÁRIO:

$-y^2 + \dfrac{1}{2}y = 0$

$-y\left(y - \dfrac{1}{2}\right) = 0$

$y_1 = 0$ E $y_2 = \dfrac{1}{2}$



$\boxed{\text{CONVERGE}}$

EXERCÍCIO 13:

13.A $Y'' + Y' - 2Y = 2t$ $\quad Y(0) = 0 \quad Y'(0) = 1$

RAÍZES CARACTERÍSTICAS:

$r^2 + r - 2 = 0$

FÓRMULA DE BHASKARA:

$\Delta = b^2 - 4ac \qquad r = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$= 1 + 8$

$= 9 \qquad = \dfrac{-1 \pm 3}{2} \begin{cases} r_1 = 1 \\ r_2 = -2 \end{cases}$

SOLUÇÃO HOMOGÊNEA:

$Y_H = A_1 e^{r_1 t} + A_2 e^{r_2 t}$

$= A_1 e^{t} + A_2 e^{-2t}$

SOLUÇÃO GERAL:

$Y = Y_H + Y_P$

$\boxed{= A_1 e^{t} + A_2 e^{-2t} - t - \frac{1}{2}}$ //

SOLUÇÃO PARTICULAR:

$Y_P = A_1 t + A_2 \quad Y_P' = A_1 \quad Y_P'' = 0$

$0 + A_1 - 2(A_1 t + A_2) = 2t$

$A_1 - 2A_1 t - 2A_2 = 2t$

$A_1 - 2A_2 - 2A_1 t = 2t$

$A_1 - 2A_2 = 0 \therefore A_2 = \frac{1}{2} A_1$

$-2A_1 = 2$

$A_1 = -1 \therefore A_2 = -\frac{1}{2}$

LOGO, $Y_P = -t - \frac{1}{2}$

SOLUÇÃO DEFINIDA:

$Y(0) = A_1 e^{0} + A_2 e^{-2(0)} - 0 - \frac{1}{2}$

$0 = A_1 + A_2 - \frac{1}{2}$ $\rightarrow$ $A_1 + A_2 = \frac{1}{2}$

32.A

$Y' = A_1 e^{t} - 2A_2 e^{-2t} - 1$

$Y'(0) = A_1 e^{0} - 2A_2 e^{-2(0)} - 1$

$1 = A_1 - 2A_2 - 1$

$A_1 - 2A_2 = 2$

$$\begin{cases} A_1 + A_2 = 1/2 & \times(2) \\ A_1 - 2A_2 = 2 \end{cases}$$

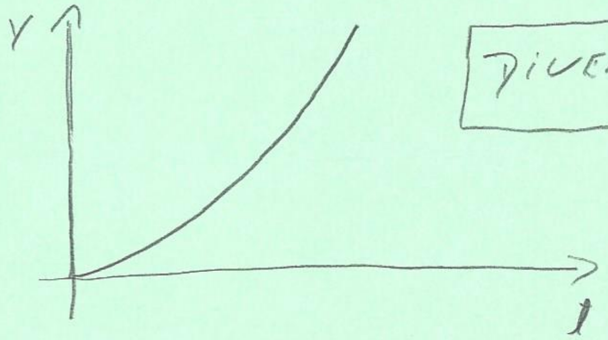
$$\begin{cases} 2A_1 + \cancel{2A_2} = 1 \\ A_1 - \cancel{2A_2} = 2 \end{cases}$$

$3A_1 = 3$

$A_1 = 1 \therefore A_2 = -\frac{1}{2}$

$$\boxed{Y = e^{t} - \frac{1}{2}e^{-2t} - t - \frac{1}{2}}$$ // SOLUÇÃO DEFINIDA

$\lim_{t \to \infty} Y = \infty$



$\boxed{\text{DIVERGE}}$ //

13.B $Y'' + Y' + 2Y = e^{t}$ $\quad Y(0) = 1 \quad Y'(0) = 0$

RAÍZES CARACTERÍSTICAS:

$r^2 + r + 2 = 0$

FÓRMULA DE BHASKARA:

$a \pm \omega i$

$\Delta = b^2 - 4ac$

$= 1 - 8$

$= -7$

$r = \frac{-b \pm \sqrt{\Delta}}{2a}$

$= \frac{-1 \pm 2{,}6i}{2}$ $\quad r_1 = -0{,}5 + 1{,}3i$ $\quad r_2 = -0{,}5 - 1{,}3i$

SOLUÇÃO HOMOGÊNEA:

$Y_H = e^{at}\{A_1 \cos(\omega t) + A_2 \operatorname{sen}(\omega t)\}$

$= e^{-0{,}5t}\{A_1 \cos(1{,}3t) + A_2 \operatorname{sen}(1{,}3t)\}$

LISTA VIII MATEMÁTICA I

2013/1.º

SOLUÇÃO PARTICULAR:

$Y_P = A_1 e^t \quad Y'_P = A_1 e^t \quad Y''_P = A_1 e^t$

$A_1 e^t + A_1 e^t + 2 A_1 e^t = e^t$

$(A_1 + A_1 + 2A_1) e^t = e^t$

$4 A_1 e^t = e^t$

$4 A_1 = 1$

$A_1 = \frac{1}{4} \quad \therefore \quad Y_P = \frac{1}{4} e^t$

SOLUÇÃO GERAL:

$Y = Y_H + Y_P$

$$\boxed{= e^{-0,5t}\{A_1 \cos(1,3t) + A_2 \operatorname{sen}(1,3t)\} + \frac{1}{4} e^t}$$

SOLUÇÃO DEFINIDA:

$Y(0) = e^{-0,5(0)}\{A_1 \cos(1,3(0)) + A_2 \underbrace{\operatorname{sen}(1,3(0))}_{0}\} + \frac{1}{4} e^0$

$1 = A_1 + \frac{1}{4} \quad \therefore \quad A_1 = 0,75$

$Y' = e^{-0,5t}\{A_1 \cos(1,3t) + A_2 \operatorname{sen}(1,3t)\}' + \{A_1 \cos(1,3t) + A_2 \operatorname{sen}(1,3t)\}(e^{-0,5t})' +$

$+ \frac{1}{4} e^t$

$Y' = e^{-0,5t}\{-1,3 A_1 \operatorname{sen}(1,3t) + 1,3 A_2 \cos(1,3t)\} + \{A_1 \cos(1,3t) + A_2 \operatorname{sen}(1,3t)\}(-0,5 e^{-0,5t}) +$

$+ \frac{1}{4} e^t$

$Y'(0) = e^{-0,5(0)}\{-1,3 A_1 \underbrace{\operatorname{sen}(1,3(0))}_{0} + 1,3 A_2 \cos(1,3(0))\} + \{A_1 \cos(1,3(0)) + A_2 \underbrace{\operatorname{sen}(1,3(0))}_{0}\}(\cdot$

13.A

$\cdot(-0{,}5e^{-0,5(0)}) + \frac{1}{4}e^{0}$

$0 = 1{,}3A_2 - 0{,}5A_1 + \frac{1}{4}$

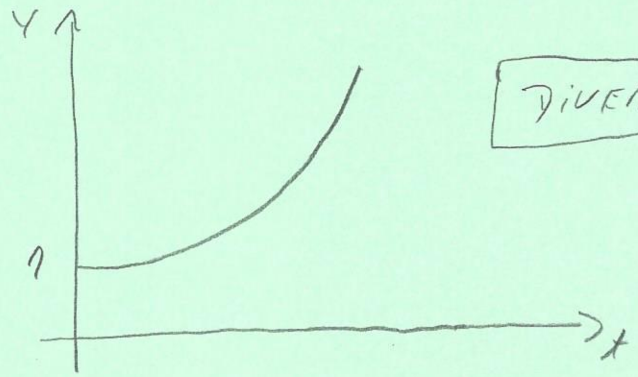
$1{,}3A_2 - 0{,}5A_1 = -0{,}25$

COMO $A_1 = 0{,}75$ TEM-SE:

$1{,}3A_2 - 0{,}5(0{,}75) = -0{,}25 \therefore A_2 = 0{,}1$

$$\boxed{Y = e^{-0,5t}\{0{,}75\cos(1{,}3t) + 0{,}1\,\text{sen}(1{,}3t)\} + \frac{1}{4}e^{t}}$$ // SOLUÇÃO DEFINIDA

$\lim_{t\to\infty} Y = \infty$



$\boxed{\text{DIVERGE}}$ //

13.C $Y'' + 4Y = t^2 + 3e^{t}$ $\quad Y(0) = 0 \quad Y'(0) = 2$

RAÍZES CARACTERÍSTICAS:

$r^2 + 4 = 0$ $\qquad \lambda \pm \mu i$

$r = \pm 2i$

SOLUÇÃO HOMOGÊNEA:

$Y_H = e^{\lambda t}\{A_1\cos(\mu t) + A_2\,\text{sen}(\mu t)\}$

$= e^{(0)t}\{A_1\cos(2t) + A_2\,\text{sen}(2t)\}$

$= \{A_1\cos(2t) + A_2\,\text{sen}(2t)\}$

SOLUÇÃO PARTICULAR:

$Y_P = A_1t^2 + A_2t + A_3 + A_4e^{t}$ $\qquad Y'_P = 2A_1t + A_2 + A_4e^{t}$

$Y''_p = 2A_1 + A_4 e^t$

$2A_1 + A_4 e^t + 4(A_1 t^2 + A_2 t + A_3 + A_4 e^t) = t^2 + 3e^t$

$2A_1 + A_4 e^t + 4A_1 t^2 + 4A_2 t + 4A_3 + 4A_4 e^t = t^2 + 3e^t$

$2A_1 + 5A_4 e^t + 4A_1 t^2 + 4A_2 t + 4A_3 = t^2 + 3e^t$

$4A_1 t^2 + 4A_2 t + 2A_1 + 4A_3 + 5A_4 e^t = t^2 + 3e^t$

$4A_1 t^2 = t^2 \qquad 4A_2 t = 0 \qquad 2A_1 + 4A_3 = 0 \qquad 5A_4 e^t = 3e^t$

$A_1 = 0{,}25 \qquad A_2 = 0 \qquad 2\left(\frac{1}{4}\right) + 4A_3 = 0 \qquad A_4 = 0{,}6$

$A_3 = -0{,}125$

$Y_p = 0{,}25t^2 - 0{,}125 + 0{,}6e^t$

SOLUÇÃO GERAL:

$Y = Y_H + Y_P$

$$\boxed{= \{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\} + 0{,}25t^2 + 0{,}6e^t - 0{,}125} //$$

SOLUÇÃO DEFINIDA:

$Y(0) = \{A_1 \cos(2(0)) + A_2 \cancel{\operatorname{sen}}^{0}(2(0))\} + 0{,}25(0)^2 + 0{,}6e^0 - 0{,}125$

$0 = A_1 + 0{,}6 - 0{,}125$

$A_1 = -0{,}475$

$Y' = \{-2A_1 \operatorname{sen}(2t) + 2A_2 \cos(2t)\} + 0{,}5t + 0{,}6e^t$

$Y'(0) = \{-2A_1 \cancel{\operatorname{sen}}^{0}(2(0)) + 2A_2 \cos(2(0))\} + 0{,}5(0) + 0{,}6e^0$

$2 = 2A_2 + 0,6$

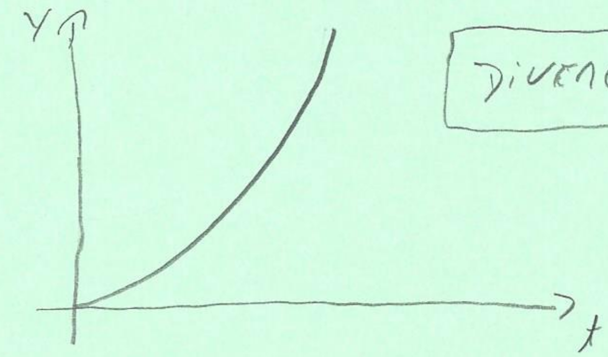
$2A_2 = 1,4$

$A_2 = 0,7$

$$\boxed{Y = \{-0,415\cos(2t) + 0,7\,\mathrm{sen}(2t)\} + 0,25t^2 + 0,6e^t - 0,125}$$

SOLUÇÃO DEFINIDA

$\lim_{t \to \infty} Y = \infty$



$\boxed{\text{DIVERGE}}$

(13.D) $Y'' - 2Y' + Y = te^t + 4 \qquad Y(0) = 1 \qquad Y'(0) = 1$

RAÍZES CARACTERÍSTICAS:

$r^2 - 2r + 1 = 0$

FÓRMULA DE BHASKARA:

$\Delta = b^2 - 4ac$

$= 4 - 4$

$= 0$

$r = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$= \dfrac{2 \pm 0}{2}$ → $r_1 = 1$, $r_2 = 1$

SOLUÇÃO HOMOGÊNEA:

$Y_H = A_1 e^{r_1 t} + A_2 t e^{r_2 t}$

$= A_1 e^t + A_2 t e^t$

SOLUÇÃO PARTICULAR:

$Y_P = A_1 t^3 e^t + A_2 \qquad Y'_P = A_1 t^3 e^t + 3A_1 t^2 e^t$

$$Y''p = A_1t^3e^t + 3A_1t^2e^t + 3A_1t^2e^t + 6A_1te^t$$

$$= A_1t^3e^t + 6A_1t^2e^t + 6A_1te^t$$

$$\cancel{A_1t^3e^t} + \cancel{6A_1t^2e^t} + 6A_1te^t - \cancel{2A_1t^3e^t} - \cancel{6A_1t^2e^t} + \cancel{A_1t^3e^t} + A_2 = te^t + 4$$

$$6A_1te^t + A_2 = te^t + 4$$

$$6A_1te^t = te^t \qquad A_2 = 4$$

$$A_1 = \frac{1}{6}$$

$$Y_p = \frac{1}{6}t^3e^t + 4$$

SOLUÇÃO GERAL:

$$Y = Y_H + Y_p$$

$$\boxed{= A_1e^t + A_2te^t + \frac{1}{6}t^3e^t + 4}$$

SOLUÇÃO DEFINIDA:

$$Y(0) = A_1e^0 + A_2(0)e^0 + \frac{1}{6}(0)^3e^0 + 4$$

$$1 = A_1 + 4$$

$$A_1 = -3$$

$$Y' = A_1e^t + A_2te^t + A_2e^t + \frac{1}{6}t^3e^t + \frac{1}{2}t^2e^t$$

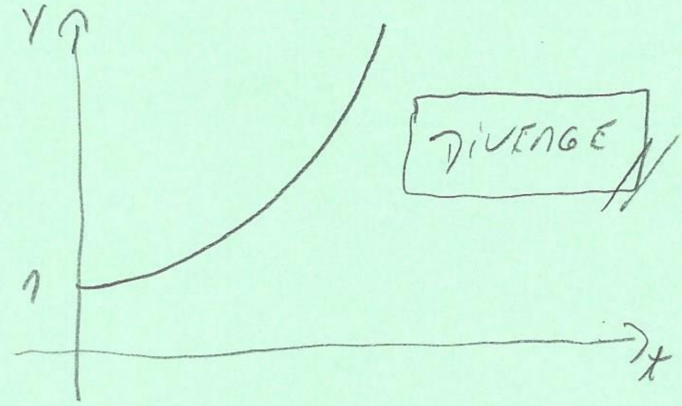
$$Y'(0) = A_1e^0 + A_2(0)e^0 + A_2e^0 + \frac{1}{6}(0)^3e^0 + \frac{1}{2}(0)^2e^0$$

$$1 = A_1 + A_2 \therefore -3 + A_2 = 1 \therefore A_2 = 4$$

35.A

$$\boxed{Y = -3e^{t} + 4te^{t} + \frac{1}{6}t^{3}e^{t} + 4}$$

$$\lim_{t \to \infty} Y = \infty$$



$\boxed{\text{DIVERGE}}$

13.E $Y'' - 2Y' - 3Y = 3te^{2t}$ $\quad Y(0) = 1 \quad Y'(0) = 0$

RAÍZES CARACTERÍSTICAS:

$r^2 - 2r - 3 = 0$

FÓRMULA DE BHASKARA:

$\Delta = b^2 - 4ac \qquad r = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$= 4 + 12$

$= 16 \qquad = \dfrac{2 \pm 4}{2} \nearrow r_1 = 3 \;\; \searrow r_2 = -1$

SOLUÇÃO HOMOGÊNEA:

$Y_H = A_1 e^{r_1 t} + A_2 e^{r_2 t}$

$= A_1 e^{3t} + A_2 e^{-t}$

SOLUÇÃO PARTICULAR:

$Y_p = A_1 t e^{2t} + A_2 e^{2t} \qquad Y_p' = 2A_1 t e^{2t} + A_1 e^{2t} + 2A_2 e^{2t}$

$Y_p'' = 4A_1 t e^{2t} + 2A_1 e^{2t} + 2A_1 e^{2t} + 4A_2 e^{2t}$

$= 4A_1 t e^{2t} + 4A_1 e^{2t} + 4A_2 e^{2t}$

$4A_1 \cancel{t e^{2t}} + \underline{4A_1 e^{2t}} + \cancel{4A_2 e^{2t}} - \cancel{4A_1 t e^{2t}} - \underline{2A_1 e^{2t}} - \cancel{4A_2 e^{2t}} - 3A_1 t e^{2t} - 3A_2 e^{2t} =$

$= 3te^{2t}$

$2A_1e^{2t} - 3A_1te^{2t} - 3A_2e^{2t} = 3te^{2t}$

$2A_1e^{2t} - 3A_2e^{2t} - 3A_1te^{2t} = 3te^{2t}$

$(2A_1 - 3A_2)e^{2t} - 3A_1te^{2t} = 3te^{2t}$

$2A_1 - 3A_2 = 0$ $\qquad$ $-3A_1te^{2t} = 3te^{2t}$

$3A_2 = 2A_1$ $\qquad$ $-3A_1 = 3$

$A_2 = \frac{2}{3}A_1$ $\qquad$ $A_1 = -1 \therefore A_2 = -\frac{2}{3}$

$Y_P = -te^{2t} - 0{,}7e^{2t}$

SOLUÇÃO GERAL:

$Y = Y_H + Y_P$

$\boxed{= A_1e^{3t} + A_2e^{-t} - te^{2t} - 0{,}7e^{2t}}$

SOLUÇÃO DEFINIDA:

$Y(0) = A_1e^{3(0)} + A_2e^{0} - (0)e^{2(0)} - 0{,}7e^{2(0)}$

$1 = A_1 + A_2 - 0{,}7$

$A_1 + A_2 = 1{,}7$

$Y' = 3A_1e^{3t} - A_2e^{-t} - 2te^{2t} - e^{2t} - 1{,}4e^{2t}$

$Y'(0) = 3A_1e^{3(0)} - A_2e^{0} - 2(0)e^{2(0)} - e^{2(0)} - 1{,}4e^{2(0)}$

$0 = 3A_1 - A_2 - 1 - 1{,}4$

$3A_1 - A_2 = 2{,}4$

36.A

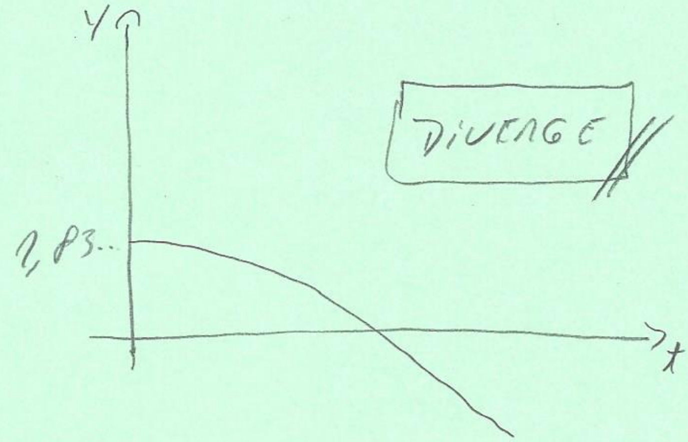
$$\begin{cases} A_1 + \cancel{A_2} = 1,7 \\ 3A_1 - \cancel{A_2} = 2,4 \end{cases}$$

$4A_1 = 4,1$

$A_1 = 1,025 \therefore A_2 = 0,675$

$$\boxed{Y = 1,025e^{3t} + 0,675e^{-t} - te^{2t} - 0,7e^{2t}}$$ // SOLUÇÃO DEFINIDA

$$\lim_{t \to \infty} Y = -\infty$$



$\boxed{\text{DIVERGE}}$ //

13.F $Y'' + 4Y = 3\,\text{sen}\,2t$ $\quad Y(0) = 2 \quad Y'(0) = -1$

RAÍZES CARACTERÍSTICAS:

$r^2 + 4 = 0 \qquad \lambda \pm \mu i$

$r = \pm 2i$

SOLUÇÃO HOMOGÊNEA:

$Y_H = e^{\lambda t}\{A_1 \cos(\mu t) + A_2\,\text{sen}(\mu t)\}$

$= e^{(0)t}\{A_1 \cos(2t) + A_2\,\text{sen}(2t)\}$

$= \{A_1 \cos(2t) + A_2\,\text{sen}(2t)\}$

SOLUÇÃO PARTICULAR:

$Y_P = At\cos 2t \qquad Y_P' = -2At\,\text{sen}\,2t + A\cos 2t$

$Y_P'' = -4At\cos 2t - 2A\,\text{sen}\,2t - 2A\,\text{sen}\,2t$

$= -4At\cos 2t - 4A\,\text{sen}\,2t$

$-4At\cancel{\cos}2t - 4A\,\text{sen}\,2t + 4At\cancel{\cos}\,2t = 3\,\text{sen}\,2t$

$-4A\,\text{sen}\,2t = 3\,\text{sen}\,2t$

$-4A = 3$

$A = -0{,}75$

$Y_P = -0{,}75\,t\cos 2t$

SOLUÇÃO GERAL:

$Y = Y_H + Y_P$

$$\boxed{= \{A_1\cos(2t) + A_2\,\text{sen}(2t)\} - 0{,}75\,t\cos(2t)}$$ //

SOLUÇÃO DEFINIDA:

$Y(0) = \{A_1\cos(2(0)) + A_2\,\underbrace{\text{sen}(2(0))}_{0}\} - 0{,}75(0)\cos(2(0))$

$2 = A_1 \therefore A_1 = 2$

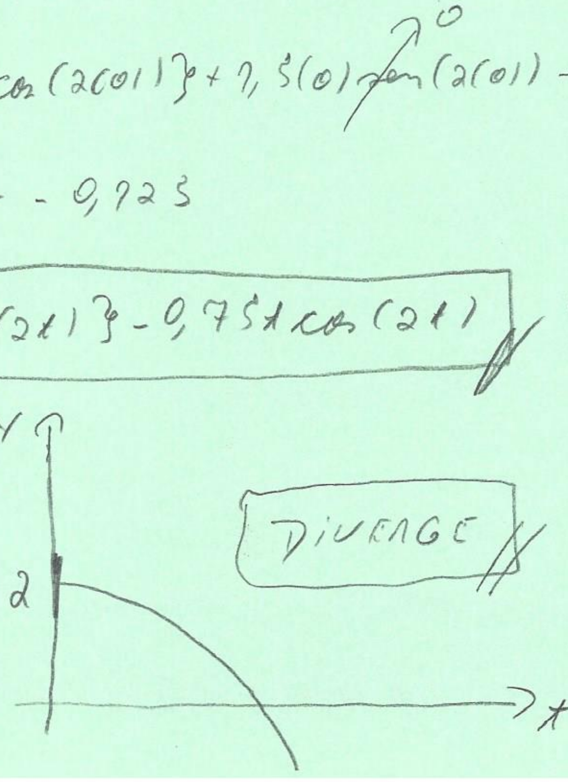
$Y' = \{-2A_1\,\text{sen}(2t) + 2A_2\cos(2t)\} + 1{,}5t\,\text{sen}(2t) - 0{,}75\cos(2t)$

$Y'(0) = \{-2A_1\,\underbrace{\text{sen}(2(0))}_{0} + 2A_2\cos(2(0))\} + 1{,}5(0)\,\underbrace{\text{sen}(2(0))}_{0} - 0{,}75\cos(2(0))$

$-1 = 2A_2 - 0{,}75 \therefore A_2 = -0{,}125$

$$\boxed{Y = \{2\cos(2t) - 0{,}125\,\text{sen}(2t)\} - 0{,}75\,t\cos(2t)}$$ // SOLUÇÃO DEFINIDA

$\lim_{t\to\infty} Y = -\infty$



DIVERGE //

13.6 $y'' + 2y' + 5y = 4e^{t}\cos 2t \quad y(0) = 1 \quad y'(0) = 1$ 37.A

RAÍZES CARACTERÍSTICAS:

$r^2 + 2r + 5 = 0$

FÓRMULA DE BHASKARA:

$\Delta = b^2 - 4ac \qquad r = \dfrac{-b \pm \sqrt{\Delta}}{2a} \qquad \lambda \pm \mu i$

$= 4 - 20$

$= -16 \qquad = \dfrac{-2 \pm 4i}{2} \quad \begin{matrix} \nearrow r_1 = -1 + 2i \\ \searrow r_2 = -1 - 2i \end{matrix}$

SOLUÇÃO HOMOGÊNEA:

$y_H = e^{\lambda t}\{A_1 \cos(\mu t) + A_2 \operatorname{sen}(\mu t)\}$

$= e^{-t}\{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\}$

SOLUÇÃO PARTICULAR:

$y_P = (A_1 \cos 2t + A_2 \operatorname{sen} 2t) A_3 e^{t}$

$y'_P = (A_1 \cos 2t + A_2 \operatorname{sen} 2t) A_3 e^{t} + A_3 e^{t}(A_1 \cos 2t + A_2 \operatorname{sen} 2t)'$

$= (A_1 \cos 2t + A_2 \operatorname{sen} 2t) A_3 e^{t} + (-2A_1 \operatorname{sen} 2t + 2A_2 \cos 2t) A_3 e^{t}$

$y''_P = (A_1 \cos 2t + A_2 \operatorname{sen} 2t) A_3 e^{t} + A_3 e^{t}(A_1 \cos 2t + A_2 \operatorname{sen} 2t)' + (-2A_1 \operatorname{sen} 2t +$

$+ 2A_2 \cos 2t) A_3 e^{t} + A_3 e^{t}(-2A_1 \operatorname{sen} 2t + 2A_2 \cos 2t)'$

$= (A_1 \cos 2t + A_2 \operatorname{sen} 2t) A_3 e^{t} + (-2A_1 \operatorname{sen} 2t + 2A_2 \cos 2t) + (-2A_1 \operatorname{sen} 2t +$

$+ 2A_2 \cos 2t) A_3 e^{t} + A_3 e^{t}(-4A_1 \cos 2t - 4A_2 \operatorname{sen} 2t)$

$= (-3A_1 \cos 2t - 3A_2 \operatorname{sen} 2t) A_3 e^{t} + 2(-2A_1 \operatorname{sen} 2t + 2A_2 \cos 2t) A_3 e^{t}$

$(-3A_1 \cos 2t - 3A_2 \operatorname{sen} 2t) A_3 e^{t} + (-4A_1 \operatorname{sen} 2t + 4A_2 \cos 2t) A_3 e^{t} + (2A_1 \cos 2t +$

$2A_2 \operatorname{sen} 2t) A_3 e^{t} + (-4A_1 \operatorname{sen} 2t + 4A_2 \cos 2t) A_3 e^{t} + (5A_1 \cos 2t + 5A_2 \operatorname{sen} 2t) A_3 e^{t} =$

$= 4e^{t} \cos 2t$

LISTA VIII MATEMÁTICA I

2013/1.º

(38)

$(4A_1 \cos 2t + 4A_2 \operatorname{sen} 2t)A_3 e^t + (-8A_1 \operatorname{sen} 2t + 8A_2 \cos 2t)A_3 e^t =$

EXERCÍCIO 14:

38.A

14.A $t^2Y''+2tY'-1=0 \qquad t>0$

REDUÇÃO DE ORDEM:

$u=Y' \therefore u'=Y''$

$t^2u'+2tu-1=0$

$t^2u'+2tu=1$

$u'+2t^{-1}u=t^{-2}$

FATOR INTEGRANTE:

$\mu=e^{\int a\,dt}$

$=e^{\int 2t^{-1}dt}$

$=e^{2\int \frac{1}{t}dt}$

$=e^{2\ln t}$

$\mu=e^{\ln t^2}$

$=t^2$

MULTIPLICANDO AMBOS OS LADOS POR t^2:

$t^2u'+2tu=1$

$\frac{d(\mu\cdot u)}{dt}=1$

$d(t^2\cdot u)=dt$

$\int d(t^2\cdot u)=\int dt$

$t^2\cdot u=t+c_1$

$\boxed{u=t^{-1}+c_1\cdot t^{-2}}$

UMA VEZ QUE $u=Y'$ TEM-SE

$Y'=t^{-1}+c_1\cdot t^{-2}$

$\frac{dY}{dt}=t^{-1}+c_1\cdot t^{-2}$

$dY=(t^{-1}+c_1\cdot t^{-2})dt$

$\int dY=\int(t^{-1}+c_1\cdot t^{-2})dt$

$Y=\int\frac{1}{t}dt+\int c_1 t^{-2}dt$

$=\ln t+c_1\int t^{-2}dt$

$\boxed{=\ln t-c_1\cdot t^{-1}+c_2}$ SOLUÇÃO GERAL

14.B $tY''+Y'=1 \qquad t>0$

REDUÇÃO DE ORDEM:

$u=Y' \therefore u'=Y''$

$$t u' + u = 1$$

$$u' + \frac{1}{t} u = \frac{1}{t}$$

$$u' + t^{-1} u = t^{-1}$$

FATOR INTEGRANTE:

$$\mu = e^{\int a\, dt} = e^{\int \frac{1}{t} dt} = e^{\ln t} = t$$

MULTIPLICANDO AMBOS OS LADOS POR t:

$$t \cdot u' + u = 1$$

$$\frac{d(\mu \cdot u)}{dt} = 1$$

$$d(\mu \cdot u) = dt$$

$$d(t \cdot u) = dt$$

$$\int d(t \cdot u) = \int dt$$

$$t \cdot u = t + c_1$$

$$\boxed{u = 1 + c_1 \cdot t^{-1}}$$

UMA VEZ QUE $u = Y'$ TEM-SE QUE:

$$Y' = 1 + c_1 \cdot t^{-1}$$

$$\frac{dY}{dt} = 1 + c_1 \cdot t^{-1}$$

$$dY = (1 + c_1 \cdot t^{-1})\, dt$$

$$\int dY = \int (1 + c_1 \cdot t^{-1})\, dt$$

$$Y = \int dt + \int c_1 \cdot \frac{1}{t} dt$$

$$= t + c_2 + c_1 \int \frac{1}{t} dt$$

$$= t + c_2 + c_1 \cdot \ln t + c_3$$

$$\boxed{= t + c_1 \cdot \ln t + C}$$ SOLUÇÃO GERAL

EM QUE $C = c_2 + c_3$

14.C $Y'' + Y' = e^{-t}$

REDUÇÃO DE ORDEM:

$$u = Y' \therefore u' = Y''$$

$$u' + u = e^{-t}$$

FATOR INTEGRANTE:

$$\mu = e^{\int a\, dt} = e^{\int dt} = e^{t}$$

39.A

MULTIPLICANDO AMBOS OS LADOS POR e^t:

$e^t.u' + e^t.u = 1$

$\dfrac{d(u.u)}{dt} = 1$

$d(e^t.u) = dt$

$\int d(e^t.u) = \int dt$

$e^t.u = t + c_1$

$\boxed{u = t.e^{-t} + c_1.e^{-t}}$

UMA VEZ QUE $u = y'$ TEM-SE QUE:

$y' = t.e^{-t} + c_1.e^{-t}$

$\dfrac{dy}{dt} = t.e^{-t} + c_1.e^{-t}$

$dy = (t.e^{-t} + c_1.e^{-t})\,dt$

$\int dy = \int (t.e^{-t} + c_1.e^{-t})\,dt$

$y = \int te^{-t}dt + \int c_1.e^{-t}dt$

$= \int te^{-t}dt + c_1 \int e^{-t}dt$

MÉTODO DA INTEGRAÇÃO POR PARTES:

$u = t$

$\dfrac{du}{dt} = 1$

$du = dt$

$dv = e^{-t}dt$

$\int dv = \int e^{-t}dt$

$v = -e^{-t} + c$

$\int te^{-t}dt = u.v - \int v.du$

$= t.(-e^{-t}) - \int -e^{-t}dt$

$= -t.e^{-t} - e^{-t}$

LOGO,

$y = -t.e^{-t} - e^{-t} + c_1(-e^{-t}) + c_2$

$\boxed{= -t.e^{-t} - e^{-t} - c_1.e^{-t} + c_2}$ SOLUÇÃO GERAL

EXERCÍCIO 15:

15.A $YY'' + (Y')^2 = 0$

SUBSTITUIÇÃO:

$u = Y' \therefore u' = Y''$

$Yu' + u^2 = 0$

$Y \cdot \frac{du}{dY} \cdot u + u^2 = 0$

$Y \cdot \frac{du}{dY} \cdot u = -u^2$

$Y \cdot \frac{du}{dY} = -\frac{u^2}{u}$

$Y \cdot \frac{du}{dY} = -u$

$-\frac{1}{u} \cdot du = \frac{1}{Y} \cdot dY$

$\int -\frac{1}{u}\, du = \int \frac{1}{Y}\, dY$

$-\ln u = \ln Y + c$

$e^{-\ln u} = e^{\ln Y + c}$

$e^{\ln u^{-1}} = e^{\ln Y} \cdot e^{c}$

$u^{-1} = Y \cdot C$

$u = Y^{-1} \cdot C$

$u = \frac{1}{Y} \cdot C$

UMA VEZ QUE $u = Y'$ TEM-SE QUE:

$Y' = \frac{1}{Y} \cdot C$

$\frac{dY}{dt} = \frac{1}{Y} \cdot C$

$Y \cdot dY = C \cdot dt$

$\int Y \cdot dY = \int C\, dt$

$\frac{Y^2}{2} = C \int dt$

$\frac{Y^2}{2} = C \cdot t + c_1$

$Y^2 = 2(C \cdot t + c_1)$

$Y^2 = A_1 t + A_2$

$\boxed{Y = (A_1 t + A_2)^{1/2}}$ //

13.B $Y'' + Y(Y')^3 = 0$

40.A

SUBSTITUIÇÃO:

$u = Y' \therefore u' = Y''$

$u' + Y \cdot u^3 = 0$

$\frac{du}{dY} \cdot u + Y \cdot u^3 = 0$

$\frac{du}{dY} \cdot u = -Y \cdot u^3$

$du \cdot \frac{u}{u^3} = -Y \cdot dY$

$u^{-2} \cdot du = -Y \cdot dY$

$\int u^{-2} \cdot du = \int -Y \cdot dY$

$-\frac{1}{u} = -\frac{Y^2}{2} + c$

$u = \frac{2}{Y^2 - 2c}$

UMA VEZ QUE $u = Y'$ TEM-SE QUE:

$Y' = \frac{2}{Y^2 - 2c}$

$\frac{dY}{dt} = \frac{2}{Y^2 - 2c}$

$(Y^2 - 2c)\, dY = 2 dt$

$\int (Y^2 - 2c)\, dY = \int 2 dt$

$\int Y^2 dY - \int 2c\, dY = 2 \int dt$

$\frac{Y^3}{3} - 2c \int dY = 2t + c_1$

$\frac{Y^3}{3} - 2cY = 2t + c_1$

$Y\left(\frac{Y^2}{3} - 2c\right) = 2t + c_1$

$Y = \frac{2t + c_1}{\frac{Y^2}{3} - 2c}$

$\boxed{= \frac{6t + 3c_1}{Y^2 - 6c}}$

13.C $Y'' + (Y')^2 = 2e^{-Y}$

SUBSTITUIÇÃO:

$u = Y' \therefore u' = Y''$

LISTA VIII MATEMÁTICA I

2023/1.º

(41)

$u' + u^2 = 2e^{-y}$

$\frac{du}{dy} \cdot u + u^2 = 2e^{-y}$

$(u^2 + u)\,du = 2e^{-y}\,dy$

$\int (u^2 + u)\,du = \int 2e^{-y}\,dy$

$\int u^2\,du + \int u\,du = 2\int e^{-y}\,dy$

$\frac{u^3}{3} + \frac{u^2}{2} = -2e^{-y} + c$

$2u^3 + 3u^2 = -12e^{-y} + 6c$

$u^2(2u + 3) = -12e^{-y} + 6c$

$u^2 = \frac{-12e^{-y} + 6c}{2u+3}$

15.7) $y'' + y = 0$

SUBSTITUIÇÃO:

$u = y' \therefore u' = y''$

$u' + y = 0$

$\frac{du}{dy} \cdot u = -y$

$u \cdot du = -y \cdot dy$

$\int u\, du = \int -y \cdot dy$

$\frac{u^2}{\cancel{2}} = -\frac{y^2}{\cancel{2}} + c$

$u^2 = -y^2 + 2c$

$u = (-y^2 + 2c)^{1/2}$

UMA VEZ QUE $u = y'$ TEM-SE QUE:

$y' = (-y^2 + 2c)^{1/2}$

$\frac{dy}{dt} = (-y^2 + 2c)^{1/2}$

$\frac{1}{(-y^2+2c)^{1/2}} \cdot dy = dt$

$\int \frac{1}{(-y^2+2c)^{1/2}}\, dy = \int dt$

$\arcsin y = t + c$

$\boxed{y = \operatorname{sen}(t + c)}$

EXERCÍCIO 16:

$y'' - y' - 2y = e^{-t} \qquad y(0) = 0 \qquad y'(0) = 2$

RAÍZES CARACTERÍSTICAS:

$r^2 - r - 2 = 0$

FÓRMULA DE BHASKARA:

$\Delta = b^2 - 4ac$

$= 1 + 8$

$= 9$

$r = \frac{-b \pm \sqrt{\Delta}}{2a}$

$= \frac{1 \pm 3}{2}$ $\nearrow r_1 = 2$ $\searrow r_2 = -1$

SOLUÇÃO HOMOGÊNEA:

$$y_H = A_1 e^{r_1 t} + A_2 e^{r_2 t}$$
$$= A_1 e^{2t} + A_2 e^{-t}$$

SOLUÇÃO PARTICULAR:

$$y_p = Ate^{-t}$$

$$y'_p = -Ate^{-t} + Ae^{-t}$$

$$y''_p = Ate^{-t} - Ae^{-t} - Ae^{-t}$$
$$= Ate^{-t} - 2Ae^{-t}$$

$$\cancel{Ate^{-t}} - 2Ae^{-t} + \cancel{Ate^{-t}} - Ae^{-t} - \cancel{2Ate^{-t}} = e^{-t}$$

$$-3Ae^{-t} = e^{-t}$$

$$-3A = 1$$

$$A = -\frac{1}{3}$$

$$y_p = -\frac{1}{3} te^{-t}$$

SOLUÇÃO GERAL:

$$y = y_H + y_p$$

$$\boxed{= A_1 e^{2t} + A_2 e^{-t} - \frac{1}{3} te^{-t}} \;//$$

SOLUÇÃO DEFINIDA:

$$y(0) = A_1 e^{2(0)} + A_2 e^{0} - \frac{1}{3}(0)e^{0}$$

$$0 = A_1 + A_2$$

$$y' = 2A_1 e^{2t} - A_2 e^{-t} + \frac{1}{3} te^{-t} - \frac{1}{3} e^{-t}$$

$$y'(0) = 2A_1 e^{2(0)} - A_2 e^{0} + \cancel{\frac{1}{3}(0)e^{0}} - \frac{1}{3} e^{0}$$

$$2 = 2A_1 - A_2 - \frac{1}{3} \therefore 2A_1 - A_2 = \frac{7}{3}$$

(42.A)

$$\begin{cases} A_1 + A_2 = \alpha \\ 2A_1 - A_2 = \frac{7}{3} \end{cases}$$

$$3A_1 = \alpha + \frac{7}{3}$$

$$A_1 = \frac{3\alpha + 7}{9} \therefore A_2 = \frac{6\alpha - 7}{9}$$

$$\boxed{Y = \left(\frac{1}{3}\alpha + \frac{7}{9}\right)e^{2t} + \left(\frac{2}{3}\alpha - \frac{7}{9}\right)e^{-t} - \frac{1}{3} t e^{-t}}$$ SOLUÇÃO DEFINIDA

$$\lim_{t \to \infty} Y = 0$$

$$Y = \left(\frac{1}{3}\alpha + \frac{7}{9}\right)e^{2t}$$

$$0 = \left(\frac{1}{3}\alpha + \frac{7}{9}\right)e^{2t}$$

$$\frac{1}{3}\alpha + \frac{7}{9} = 0$$

$$\frac{1}{3}\alpha = -\frac{7}{9}$$

$$\boxed{\alpha = -\frac{7}{3}}$$

EXERCÍCIO 20:

(20.A) $\dot{X} = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} X$

$$A = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \therefore \dot{X} = AX \therefore \dot{X} - AX = 0$$

SOLUÇÃO HOMOGÊNEA:

AUTOVALORES:

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 3-\lambda & -2 \\ 2 & -2-\lambda \end{vmatrix} = 0$$

LISTA VIII MATEMÁTICA I

2013/1.°

$(3-\lambda)(-2-\lambda)-(-2)(2)=0$

$-6-3\lambda+2\lambda+\lambda^2+4=0$

$\lambda^2-\lambda-2=0$

FÓRMULA DE BHASKARA:

$\Delta = b^2-4ac \qquad \lambda = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$= 1+8$

$= 9$

$= \dfrac{1 \pm 3}{2}$ $\nearrow \lambda_1 = 2$ $\searrow \lambda_2 = -1$

AUTOVETORES:

PARA $\lambda_1 = 2$:

$[A - 2I]V_1 = 0$

$$\begin{bmatrix} 1 & -2 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 2 & -4 \end{bmatrix} \begin{matrix} \times(-2) \\ \hookleftarrow + \end{matrix} \sim \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$$

$x_1 - 2x_2 = 0$

$x_1 = 2x_2$

$x_2 = x_2$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} x_2$$

FAZENDO $x_2 = 1$ TEM-SE QUE:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} = V_1$$

PARA $\lambda_2 = -1$

$[A-(-1)\lambda]V_2 = 0$

$$\begin{bmatrix} 4 & -2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -2 \\ 2 & -1 \end{bmatrix} \times (1/4) \sim \begin{bmatrix} 1 & -1/2 \\ 2 & -1 \end{bmatrix} \begin{matrix} \times(-2) \\ \hookleftarrow + \end{matrix} \sim \begin{bmatrix} 1 & -1/2 \\ 0 & 0 \end{bmatrix}$$

(43.A)

$x_1 - \frac{1}{2}x_2 = 0$

$x_1 = \frac{1}{2}x_2$

$x_2 = x_2$

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1 \end{pmatrix} x_2$

FAZENDO $x_2 = 1$ TEM-SE QUE:

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1 \end{pmatrix} = V_2$

SOLUÇÃO GERAL:

$x(t) = V_1 \cdot c_1 \cdot e^{\lambda_1 t} + V_2 \cdot c_2 \cdot e^{\lambda_2 t}$

$$\boxed{= \begin{pmatrix} 2 \\ 1 \end{pmatrix} c_1 \cdot e^{2t} + \begin{pmatrix} 1/2 \\ 1 \end{pmatrix} \cdot c_2 \cdot e^{-t}}$$

$$\begin{cases} x_1(t) = 2c_1 \cdot e^{2t} + 1/2 \cdot c_2 \cdot e^{-t} \\ x_2(t) = c_1 \cdot e^{2t} + c_2 \cdot e^{-t} \end{cases}$$

$\lim_{t \to \infty} x_1(t) = \infty$ $\quad \lim_{t \to \infty} x_2(t) = \infty$

$\boxed{\text{DIVERGE}}$

(20.B) $\dot{x} = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} x$

$A = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \therefore \dot{x} = Ax \therefore \dot{x} - Ax = 0$

SOLUÇÃO HOMOGÊNEA:

AUTOVALORES:

$|A - \lambda I| = 0$

$\begin{vmatrix} 1-\lambda & 1 \\ 4 & -2-\lambda \end{vmatrix} = 0$

$(1-\lambda) \cdot (-2-\lambda) - (1)(4) = 0$

$-2 - \lambda + 2\lambda + \lambda^2 - 4 = 0$

$\lambda^2 + \lambda - 6 = 0$

LISTA VIII MATEMÁTICA I

2013/1º

FÓRMULA DE BHASKARA:

$$\Delta = b^2 - 4ac \qquad \lambda = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$= 1 + 24$$

$$= 25 \qquad = \frac{-1 \pm 5}{2} \begin{array}{l} \nearrow \lambda_1 = 2 \\ \searrow \lambda_2 = -3 \end{array}$$

AUTOVETORES:

PARA $\lambda_1 = 2$:

$$[A - 2I]V_1 = 0$$

$$\begin{bmatrix} -1 & 1 \\ 4 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 \\ 4 & -4 \end{bmatrix} \times(-1) \sim \begin{bmatrix} 1 & -1 \\ 4 & -4 \end{bmatrix} \times(-4) + \sim \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

$$x_1 - x_2 = 0 \qquad \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} x_2$$

$$x_1 = x_2$$

$$x_2 = x_2$$

FAZENDO $x_2 = 1$ TEM-SE QUE:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = V_1$$

PARA $\lambda_2 = -3$

$$[A - (-3)I]V_2 = 0$$

$$\begin{bmatrix} 4 & 1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 1 \\ 4 & 1 \end{bmatrix} \times(1/4) \sim \begin{bmatrix} 1 & 1/4 \\ 4 & 1 \end{bmatrix} \times(-4) + \sim \begin{bmatrix} 1 & 1/4 \\ 0 & 0 \end{bmatrix}$$

44.A

$x_1 + \frac{1}{4} x_2 = 0$ $\qquad \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -1/4 \\ 1 \end{pmatrix} x_2$

$x_1 = -\frac{1}{4} x_2$

FAZENDO $x_2 = 1$ TEM-SE QUE:

$x_2 = x_2$ $\qquad V_2 = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -1/4 \\ 1 \end{pmatrix}$

SOLUÇÃO GERAL:

$$x(t) = V_1 \cdot c_1 \cdot e^{\lambda_1 t} + V_2 \cdot c_2 \cdot e^{\lambda_2 t}$$

$$= \begin{pmatrix} 1 \\ 1 \end{pmatrix} \cdot c_1 \cdot e^{2t} + \begin{pmatrix} -1/4 \\ 1 \end{pmatrix} \cdot c_2 \cdot e^{-3t}$$

$$\begin{cases} x_1(t) = c_1 \cdot e^{2t} - 1/4 \cdot c_2 \cdot e^{-3t} \\ x_2(t) = c_1 \cdot e^{2t} + c_2 \cdot e^{-3t} \end{cases}$$

$\lim_{t \to \infty} x_1(t) = \infty$ $\qquad \lim_{t \to \infty} x_2(t) = \infty$

$\boxed{\text{DIVERGE}}$

20.C $\dot{X} = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} X$

$A = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} \therefore \dot{X} = AX \therefore \dot{X} - AX = 0$

SOLUÇÃO HOMOGÊNEA:

AUTOVALORES:

$|A - \lambda I| = 0$

$$\begin{vmatrix} 3-\lambda & -2 \\ 4 & -1-\lambda \end{vmatrix} = 0$$

$(3-\lambda) \cdot (-1-\lambda) - (-2) \cdot (4) = 0$

$-3 - 3\lambda + \lambda + \lambda^2 + 8 = 0$

$\lambda^2 - 2\lambda + 5 = 0$

FÓRMULA DE BHASKARA:

$\Delta = b^2 - 4ac$

$= 4 - 20$

$= -16$

$\lambda = \frac{-b \pm \sqrt{\Delta}}{2a}$

$= \frac{2 \pm 4i}{2}$

$\lambda_1 = 1 + 2i$

$\lambda_2 = 1 - 2i$

$\lambda = \alpha \pm \beta i$

LISTA VIII MATEMÁTICA I

2013/1º

13

AUTOVETORES:

PARA $\lambda_1 = 1+2i$:

$$[A-(1+2i)I]V_1 = 0$$

$$\begin{bmatrix} 2-2i & -2 \\ 4 & -2-2i \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2-2i & -2 \\ 4 & -2-2i \end{bmatrix} \times\left(\frac{-4}{2-2i}\right) \sim \begin{bmatrix} 2-2i & -2 \\ 0 & 0 \end{bmatrix}$$

$$-2\left(\frac{-4}{2-2i}\right) + (-2-2i) = 0$$

$$\frac{8}{2-2i} + (-2-2i) = 0$$

$$8 + (-2-2i)(2-2i) = 0$$

$$8 + (-4+\cancel{4i}-\cancel{4i}+4i^2) = 0$$

$$8 + [-4+4(\sqrt{-1})^2] = 0$$

$$8 + (-4-4) = 0$$

$$0 = 0$$

$$(2-2i)x_1 - 2x_2 = 0$$

$$2x_2 = (2-2i)x_1$$

$$x_2 = (1-i)x_1$$

$$x_1 = x_1$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1-i \end{pmatrix} x_1$$

FAZENDO $x_1 = 1$ TEM-SE:

$$V_1 = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} i$$

PARA $\lambda_2 = 1+2i$

UMA VEZ QUE AS RAÍZES SÃO NÚMEROS CONJUGADOS COMPLEXOS, ENTÃO, V_2 SERÁ DADO POR:

$$V_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} i$$

$\mu = u \pm wi \quad u = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad w = \pm\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (45.A)

SOLUÇÃO GERAL:

$$x(t) = e^{\alpha t}\left[(c_1 \cdot u - c_2 \cdot w)\cos(\beta t) - (c_2 u + c_1 w)\,\text{sen}(\beta t)\right]$$

$$= e^{t}\left[\left(c_1 \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} - c_2\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right)\cos(2t) - \left(c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_1\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right)\text{sen}(2t)\right]$$

$$x_1(t) = e^{t}\left[c_1 \cdot \cos 2t - c_2 \cdot \text{sen}\, 2t\right]$$

$$x_2(t) = e^{t}\left[(c_1 - c_2)\cos(2t) - (c_2 + c_1)\,\text{sen}(2t)\right]$$

$\lim_{t \to \infty} x_1(t) = \infty$ $\boxed{\text{DIVERGE CICLICAMENTE}}$ //

$\lim_{t \to \infty} x_2(t) = \infty$ $\boxed{\text{DIVERGE CICLICAMENTE}}$ //

(20.D) $\dot{X} = \begin{pmatrix} 1 & 2 \\ -5 & -1 \end{pmatrix} X$

$A = \begin{pmatrix} 1 & 2 \\ -5 & -1 \end{pmatrix} \therefore \dot{X} = AX \therefore \dot{X} - AX = 0$

SOLUÇÃO HOMOGÊNEA:

AUTOVALORES:

$|A - \lambda I| = 0$

$\begin{vmatrix} 1-\lambda & 2 \\ -5 & -1-\lambda \end{vmatrix} = 0$

$(1-\lambda).(-1-\lambda) - (2)(-5) = 0$

$-1 - \lambda + \lambda + \lambda^2 + 10 = 0$

$\lambda^2 + 9 = 0 \quad \alpha \pm \beta i$

$\lambda^2 = -9$

$\lambda = \pm\sqrt{-9}$

$\lambda_1 = 3i$

$\lambda_2 = -3i$

AUTOVETORES:

PARA $\lambda_1 = 3i$

$$[A - 3i\,I]\,V_1 = 0$$

$$\begin{pmatrix} 1-3i & 2 \\ -5 & -1-3i \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1-3i & 2 \\ -5 & -1-3i \end{bmatrix} \times \left(\frac{5}{1-3i}\right) + \sim \begin{bmatrix} 1-3i & 2 \\ 0 & 0 \end{bmatrix}$$

$$2\cdot\left(\frac{5}{1-3i}\right) + (-1-3i) = 0$$

$$\frac{10}{1-3i} + (-1-3i) = 0$$

$$10 + (-1-3i)\cdot(1-3i) = 0$$

$$10 + (-1 + \cancel{3i} - \cancel{3i} + 9i^2) = 0$$

$$10 + [-1 + 9(-1)] = 0$$

$$10 + (-10) = 0$$

$$0 = 0$$

$$(1-3i)x_1 + 2x_2 = 0$$

$$2x_2 = (-1+3i)x_1$$

$$x_2 = \frac{-1+3i}{2}\,x_1$$

$$x_1 = x_1$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{-1+3i}{2} \end{pmatrix} x_1$$

FAZENDO $x_1 = 1$ TEM-SE:

$$V_1 = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1/2 \end{pmatrix} + \begin{pmatrix} 0 \\ 3/2 \end{pmatrix} i$$

PARA $\lambda_2 = -3i$

UMA VEZ QUE AS RAÍZES SÃO NÚMEROS CONJUGADOS COMPLEXOS, ENTÃO, V_2 SERÁ DADO POR:

$$V_2 = \begin{pmatrix} 1 \\ -1/2 \end{pmatrix} - \begin{pmatrix} 0 \\ -3/2 \end{pmatrix} i$$ (16.A)

$$v = u \pm w \qquad u = \begin{pmatrix} 1 \\ -1/2 \end{pmatrix} \qquad w = \pm \begin{pmatrix} 0 \\ 3/2 \end{pmatrix}$$

SOLUÇÃO GERAL:

$$x(t) = e^{\alpha t}\left[(c_1 \cdot u - c_2 w)\cos(\beta t) - (c_2 u + c_1 w)\,\text{sen}(\beta t)\right]$$

$$= e^{(0)t}\left[\left(c_1\begin{pmatrix} 1 \\ -1/2 \end{pmatrix} - c_2\begin{pmatrix} 0 \\ 3/2 \end{pmatrix}\right)\cos 3t - \left(c_2\begin{pmatrix} 1 \\ -1/2 \end{pmatrix} + c_1\begin{pmatrix} 0 \\ 3/2 \end{pmatrix}\right)\text{sen } 3t\right]$$

$$= \left[\left(c_1\begin{pmatrix} 1 \\ -1/2 \end{pmatrix} - c_2\begin{pmatrix} 0 \\ 3/2 \end{pmatrix}\right)\cos 3t - \left(c_2\begin{pmatrix} 1 \\ -1/2 \end{pmatrix} + c_1\begin{pmatrix} 0 \\ 3/2 \end{pmatrix}\right)\text{sen } 3t\right]$$

$$x_1(t) = [c_1 \cdot \cos 3t - c_2\,\text{sen } 3t]$$

$$x_2(t) = \left[\left(-\tfrac{1}{2}c_1 - 3/2\,c_2\right)\cos 3t - \left(-\tfrac{1}{2}c_2 + 3/2\,c_1\right)\text{sen } 3t\right]$$

$\alpha = 0 \therefore$ PONTO DE SELA, ISTO É, NÃO CONVERGE E NEM DIVERGE.

20.E $\dot{x} = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} x$

$$A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} \therefore \dot{X} = AX \therefore \dot{X} - AX = 0$$

SOLUÇÃO HOMOGÊNEA:

AUTOVALORES:

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 3-\lambda & -4 \\ 1 & -1-\lambda \end{vmatrix} = 0$$

FÓRMULA DE BHASKARA:

$$\lambda^2 - 2\lambda + 1 = 0 \qquad \lambda = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$\Delta = b^2 - 4ac$$
$$= 4 - 4$$
$$= 0$$

$$= \frac{2 \pm 0}{2} \begin{cases} \lambda_1 = 1 \\ \lambda_2 = 1 \end{cases}$$

$(3-\lambda).(-1-\lambda)-(-4)(1)=0$

$-3-3\lambda+\lambda+\lambda^2+4=0$

$\lambda^2-2\lambda+1=0$

AUTOVETORES:

PARA $\lambda_1=1$:

$[A-(1)I]V_1=0$

$$\begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 \\ 2 & -4 \end{bmatrix} \times(-2) \sim \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$$

$x_1-2x_2=0$

$x_1=2x_2$

$x_2=x_2$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=\begin{pmatrix} 2 \\ 1 \end{pmatrix}x_2$$

FAZENDO $x_2=1$ TEM-SE QUE:

$$V_1=\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

PARA $\lambda_1=1$: (VETOR GENERALIZADO)

$[A-(1)I]V_2=V_1$

$$\begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 2 & -4 & 2 \\ 1 & -2 & 1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 2 & -4 & 2 \end{array}\right] \times(-2) \sim \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

4/17.A

$x_1 - 2x_2 = 1$

$x_1 = 1 + 2x_2$

$x_2 = x_2$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} x_2 + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

FAZENDO $x_2 = 0$ TEM-SE:

$$v_2 = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

SOLUÇÃO GERAL:

$$x(t) = v_1 (c_1 + c_2 t) e^{\lambda t} + v_2 c_2 e^{\lambda t}$$

$$= \begin{pmatrix} 2 \\ 1 \end{pmatrix} (c_1 + c_2 t) e^{t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} c_2 e^{t}$$

$$\begin{cases} x_1(t) = 2(c_1 + c_2 t) e^{t} + c_2 e^{t} \\ x_2(t) = (c_1 + c_2 t) e^{t} \end{cases}$$

$\lim_{t \to \infty} x_1(t) = \infty$ $\lim_{t \to \infty} x_2(t) = \infty$ $\boxed{\text{DIVERGE}}$

20.F $\dot{x} = \begin{pmatrix} 4 & -2 \\ 8 & -4 \end{pmatrix} x$

$A = \begin{pmatrix} 4 & -2 \\ 8 & -4 \end{pmatrix} \therefore \dot{x} = Ax \therefore \dot{x} - Ax = 0$

SOLUÇÃO HOMOGÊNEA:

AUTOVALORES:

$|A - \lambda I| = 0$

$$\begin{vmatrix} 4-\lambda & -2 \\ 8 & -4-\lambda \end{vmatrix} = 0$$

$(4-\lambda).(-4-\lambda) - (-2).(8) = 0$

$-\cancel{16} - \cancel{4\lambda} + \cancel{4\lambda} + \lambda^2 + \cancel{16} = 0$

$\lambda^2 = 0 \to \lambda_1 = 0,\ \lambda_2 = 0$

LISTA VIII MATEMÁTICA I

2013/1º

AUTOVETORES:

PARA $\lambda_1 = 0$:

$[A - (0)I]\, V_1 = 0$

$$\begin{bmatrix} 4 & -2 \\ 8 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -2 \\ 8 & -4 \end{bmatrix} \times (1/4) \sim \begin{bmatrix} 1 & -1/2 \\ 8 & -4 \end{bmatrix} \times (-8) + \sim \begin{bmatrix} 1 & -1/2 \\ 0 & 0 \end{bmatrix}$$

$x_1 - \frac{1}{2} x_2 = 0$

$x_1 = \frac{1}{2} x_2$

$x_2 = x_2$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1 \end{pmatrix} x_2$$

FAZENDO $x_2 = 1$ TEM-SE:

$$V_1 = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1 \end{pmatrix}$$

PARA $\lambda_2 = 0$: (AUTOVETOR GENERALIZADO)

$[A - (0)I]\, V_2 = V_1$

$$\begin{bmatrix} 4 & -2 \\ 8 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 4 & -2 & 1/2 \\ 8 & -4 & 1 \end{array}\right] \times (1/4) \sim \left[\begin{array}{cc|c} 1 & -1/2 & 1/8 \\ 8 & -4 & 1 \end{array}\right] \times (-8) + \sim \left[\begin{array}{cc|c} 1 & -1/2 & 1/8 \\ 0 & 0 & 0 \end{array}\right]$$

$x_1 - \frac{1}{2} x_2 = \frac{1}{8}$

$x_1 = \frac{1}{8} + \frac{1}{2} x_2$

$x_2 = x_2$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1 \end{pmatrix} x_2 + \begin{pmatrix} 1/8 \\ 0 \end{pmatrix}$$

FAZENDO $x_2 = 0$ TEM-SE

$$V_2 = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1/8 \\ 0 \end{pmatrix}$$

SOLUÇÃO GERAL: (48.A)

$$x(t) = V_1 (c_1 + c_2 t) e^{\lambda t} + V_2 c_2 e^{\lambda t}$$

$$= \begin{pmatrix} 1/2 \\ 1 \end{pmatrix} (c_1 + c_2 t) e^{(0)t} + \begin{pmatrix} 1/8 \\ 0 \end{pmatrix} c_2 e^{(0)t}$$

$$= \begin{pmatrix} 1/2 \\ 1 \end{pmatrix} (c_1 + c_2 t) + \begin{pmatrix} 1/8 \\ 0 \end{pmatrix} c_2$$

$$\begin{cases} x_1(t) = \frac{1}{2}(c_1 + c_2 t) + \frac{1}{8} c_2 \\ x_2(t) = (c_1 + c_2 t) \end{cases}$$

$$\lim_{t \to \infty} x_1(t) = \infty \qquad \lim_{t \to \infty} x_2(t) = \infty$$

20.G $\dot{x} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} x$

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \therefore \dot{x} = Ax \therefore \dot{x} - Ax = 0$$

SOLUÇÃO HOMOGÊNEA:

AUTOVALORES:

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -\lambda & 1 & 1 \\ 1 & -\lambda & 1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$$a_{11} \Delta_{11} + a_{12} \Delta_{12} + a_{13} \Delta_{13} = 0$$

$$a_{11} (-1)^2 \Delta_{11} + a_{12} (-1)^3 \Delta_{12} + a_{13} (-1)^4 \Delta_{13} = 0$$

$$-\lambda \begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} + (1)(-1)\begin{vmatrix} 1 & 1 \\ 1 & -\lambda \end{vmatrix} + (1)\begin{vmatrix} 1 & -\lambda \\ 1 & 1 \end{vmatrix} = 0$$

$$-\lambda(\lambda^2 - 1) + (-1)(-\lambda - 1) + (1)(1 + \lambda) = 0$$

$$-\lambda(\lambda^2 - 1) + \lambda + 1 + 1 + \lambda = 0$$

$$-\lambda^3 + \lambda + \lambda + 1 + 1 + \lambda = 0$$

$$\lambda^3 - 3\lambda - 2 = 0$$

LISTA VIII MATEMÁTICA I

2013/1º

50

20.M $\dot{x} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{pmatrix} x$

$A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{pmatrix} \therefore \dot{x} = Ax \therefore \dot{x} - Ax = 0$

SOLUÇÃO HOMOGÊNEA:

AUTOVALORES:

$|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 0 & 0 \\ 2 & 1-\lambda & -2 \\ 3 & 2 & 1-\lambda \end{vmatrix} = 0$$

$a_{11} \cdot A_{11} + a_{12} A_{12} + a_{13} A_{13} = 0$

$a_{11} (-1)^2 \Delta_{11} + a_{12} (-1)^3 \Delta_{12} + a_{13} (-1)^4 \Delta_{13} = 0$

$$(1-\lambda) \begin{vmatrix} 1-\lambda & -2 \\ 2 & 1-\lambda \end{vmatrix} + (0)(-1) \cancelto{0}{\begin{vmatrix} 2 & -2 \\ 3 & 1-\lambda \end{vmatrix}} + (0) \cancelto{0}{\begin{vmatrix} 2 & 1-\lambda \\ 3 & 2 \end{vmatrix}} = 0$$

$(1-\lambda)[(1-\lambda)(1-\lambda) - (-2)(2)] = 0$

$(1-\lambda)(1-\lambda-\lambda+\lambda^2+4) = 0$

$(1-\lambda)(\lambda^2-2\lambda+5) = 0$

$1-\lambda = 0$

$\lambda_1 = 1$

FÓRMULA DE BHASKARA:

$\lambda^2 - 2\lambda + 5 = 0$

$\Delta = b^2 - 4ac$

$= 4 - 20$

$= -16$

$\lambda = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$= \dfrac{2 \pm 4i}{2}$

$\lambda_2 = 1 + 2i$

$\lambda_3 = 1 - 2i$

AUTOVETORES:

PARA $\lambda_1 = 1$:

$$[A - (1)I]V_1 = 0$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & -2 \\ 3 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & -2 \\ 3 & 2 & 0 \end{bmatrix} \sim \begin{bmatrix} 3 & 2 & 0 \\ 2 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 & -2 \\ 3 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times (1/2) \sim$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times(-3) \sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix} \times(1/2) \sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 3/2 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 - x_3 = 0$ $\quad$ $x_2 + \frac{3}{2}x_3 = 0$ $\quad$ $x_3 = x_3$

$x_1 = x_3$ $\quad$ $x_2 = -\frac{3}{2}x_3$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -3/2 \\ 1 \end{pmatrix} x_3$$

FAZENDO $x_3 = 1$ TEM-SE

$$V_1 = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -3/2 \\ 1 \end{pmatrix}$$

PARA $\lambda_2 = 1 + 2i$

$$[A - (1+2i)I]V_2 = 0$$

$$\begin{bmatrix} -2i & 0 & 0 \\ 2 & -2i & -2 \\ 3 & 2 & -2i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

LISTA VIII MATEMÁTICA I

2013/1°

$$\begin{bmatrix} -2i & 0 & 0 \\ 2 & -2i & -2 \\ 3 & 2 & -2i \end{bmatrix} \times\left(-\frac{1}{2i}\right) \sim \begin{bmatrix} 1 & 0 & 0 \\ 2 & -2i & -2 \\ 3 & 2 & -2i \end{bmatrix} \begin{matrix} \times(-2) \ \times(-3) \\ + \quad + \\ \end{matrix} \sim$$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2i & -2 \\ 0 & 2 & -2i \end{bmatrix} \times\left(-\frac{1}{2i}\right) \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & i^{-1} \\ 0 & 2 & -2i \end{bmatrix} \begin{matrix} \\ \times(-2) \\ + \end{matrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & i^{-1} \\ 0 & 0 & 4 \end{bmatrix} \begin{matrix} \\ \\ \times(1/4) \end{matrix} \sim$$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & i^{-1} \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} \\ + \\ \times(-i^{-1}) \end{matrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$x_1 = 0$; $x_2 = 0$ e $x_3 = 0$

$$V_2 = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

LISTA VIII MATEMÁTICA I

2013/1.º

52

EXERCÍCIO 21:

21.A $\dot{x}_1 = 5x_1 - x_2$ $x_1(0) = 2$ $x_2(0) = -1$

$\dot{x}_2 = 3x_1 + x_2$

$$\dot{X} = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} X \qquad A = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} \therefore \dot{X} = AX \therefore \dot{X} - AX = 0$$

SOLUÇÃO HOMOGÊNEA:

AUTOVALORES:

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 5-\lambda & -1 \\ 3 & 1-\lambda \end{vmatrix} = 0$$

$$(5-\lambda)(1-\lambda) - (-1)(3) = 0$$

$$5 - 5\lambda - \lambda + \lambda^2 + 3 = 0$$

$$\lambda^2 - 6\lambda + 8 = 0$$

FÓRMULA DE BHASKARA:

$$\Delta = b^2 - 4ac \qquad \lambda = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$= 36 - 32$$

$$= 4$$

$$= \frac{6 \pm 2}{2} \begin{cases} \lambda_1 = 4 \\ \lambda_2 = 2 \end{cases}$$

AUTOVETORES:

PARA $\lambda_1 = 4$:

$$[A - 4I]v_1 = 0$$

$$\begin{bmatrix} 1 & -1 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 3 & -3 \end{bmatrix} \begin{array}{l} \times(-3) \\ \leftarrow + \end{array} \sim \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

$x_1 - x_2 = 0$

$x_1 = x_2$

$x_2 = x_2$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} x_2$$

FAZENDO $x_2 = 1$ TEM-SE:

$$V_1 = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

5'2.A

PARA $\lambda_2 = 2$

$[A - 2I]V_2 = 0$

$$\begin{bmatrix} 3 & -1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -1 \\ 3 & -1 \end{bmatrix} \times (1/3) \sim \begin{bmatrix} 1 & -1/3 \\ 3 & -1 \end{bmatrix} \times(-3) + \sim \begin{bmatrix} 1 & -1/3 \\ 0 & 0 \end{bmatrix}$$

$x_1 - \frac{1}{3} x_2 = 0$

$x_1 = \frac{1}{3} x_2$

$x_2 = x_2$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1/3 \\ 1 \end{pmatrix} x_2$$

FAZENDO $x_2 = 1$ TEM-SE:

$$V_2 = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1/3 \\ 1 \end{pmatrix}$$

SOLUÇÃO GERAL:

$$x(t) = V_1 c_1 e^{\lambda_1 t} + V_2 c_2 e^{\lambda_2 t}$$

$$= \begin{pmatrix} 1 \\ 1 \end{pmatrix} c_1 e^{4t} + \begin{pmatrix} 1/3 \\ 1 \end{pmatrix} c_2 e^{2t}$$

$$\boxed{\begin{aligned} x_1(t) &= c_1 e^{4t} + \frac{1}{3} c_2 e^{2t} \\ x_2(t) &= c_1 e^{4t} + c_2 e^{2t} \end{aligned}}$$

SOLUÇÃO DEFINIDA:

LISTA VIII MATEMÁTICA I

$2013/1^{o}$

53

$$x_1(0) = c_1 e^{4(0)} + \frac{1}{3} c_2 e^{2(0)}$$

$$2 = c_1 + \frac{1}{3} c_2 \quad \therefore \quad c_1 + \frac{1}{3} c_2 = 2$$

$$x_2(0) = c_1 e^{4(0)} + c_2 e^{2(0)}$$

$$-1 = c_1 + c_2 \quad \therefore \quad c_1 + c_2 = -1$$

$$\begin{cases} c_1 + \frac{1}{3} c_2 = 2 \\ c_1 + c_2 = -1 \quad \times(-1) \end{cases} \sim \begin{cases} \cancel{c_1} + \frac{1}{3} c_2 = 2 \\ -\cancel{c_1} - c_2 = 1 \end{cases}$$

$$-\frac{2}{3} c_2 = 3$$

$$c_2 = -4,5 \quad \therefore \quad c_1 = 3,5$$

$$x_1 = 3,5 e^{4t} + \frac{1}{3}(-4,5) e^{2t}$$

$$= 3,5 e^{4t} - 1,5 e^{2t}$$

$$x_2 = 3,5 e^{4t} - 4,5 e^{2t}$$

$$\boxed{\begin{aligned} x_1 &= 3,5 e^{4t} - 1,5 e^{2t} \\ x_2 &= 3,5 e^{4t} - 4,5 e^{2t} \end{aligned}}$$ SOLUÇÃO DEFINIDA

21.3 $\dot{x}_1 = x_1 - 5x_2$ $\quad$ $x_1(0) = 1 \quad x_2(0) = 1$

$\dot{x}_2 = x_1 - 3x_2$

$$\dot{x} = \begin{pmatrix} 1 & -5 \\ 1 & -3 \end{pmatrix} x$$

$$A = \begin{pmatrix} 1 & -5 \\ 1 & -3 \end{pmatrix} \therefore \dot{x} = Ax \therefore \dot{x} - Ax = 0$$

53.1

SOLUÇÃO HOMOGÊNEA:

AUTOVALORES:

$$|A-\lambda I|=0$$

$$\begin{vmatrix} 1-\lambda & -5 \\ 1 & -3-\lambda \end{vmatrix}=0$$

$$(1-\lambda)\cdot(-3-\lambda)-(-5)\cdot(1)=0$$

$$-3-\lambda+3\lambda+\lambda^2+5=0$$

$$\lambda^2+2\lambda+2=0$$

FÓRMULA DE BHASKARA:

$$\Delta=b^2-4ac$$
$$=4-8$$
$$=-4$$

$$\lambda=\frac{-b\pm\sqrt{\Delta}}{2a}$$
$$=\frac{-2\pm 2i}{2}$$

$$\lambda_1=-1+i$$

$$\lambda_2=-1-i$$

$$\alpha\pm\beta i$$

AUTOVETORES:

PARA $\lambda_1=-1+i$

$$[A-(-1+i)I]V_1=0$$

$$\begin{bmatrix} 2-i & -5 \\ 1 & -2-i \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2-i & -5 \\ 1 & -2-i \end{bmatrix} \times\left(-\frac{1}{2-i}\right) + \sim \begin{bmatrix} 2-i & -5 \\ 0 & 0 \end{bmatrix}$$

$$-5\left(\frac{-1}{2-i}\right)+(-2-i)=0$$

$$\frac{5}{2-i}+(-2-i)=0$$

$$5+(-2-i)\cdot(2-i)=0$$

$$5+(-4+\cancel{2i}-\cancel{2i}+i^2)=0$$

$$5+(-4-1)=0$$

$$0=0$$

$(2-i)x_1 - 5x_2 = 0$

$5x_2 = (2-i)x_1$

$x_2 = \dfrac{2-i}{5} x_1$

$x_1 = x_1$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{2-i}{5} \end{pmatrix} x_1$$

FAZENDO $x_1 = 1$ TEM-SE:

$$V_1 = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{2-i}{5} \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 2/5 \end{pmatrix} - \begin{pmatrix} 0 \\ 1/5 \end{pmatrix} i$$

PARA $\lambda_2 = -1-i$

UMA VEZ QUE AS RAÍZES SÃO NÚMEROS CONJUGADOS COMPLEXOS, ENTÃO, V_2 SERÁ DADO POR:

$$V_2 = \begin{pmatrix} 1 \\ 2/5 \end{pmatrix} + \begin{pmatrix} 0 \\ 1/5 \end{pmatrix} i$$

$V = u \pm w \qquad u = \begin{pmatrix} 1 \\ 2/5 \end{pmatrix} \quad w = \begin{pmatrix} 0 \\ 1/5 \end{pmatrix}$

$$x(t) = e^{\alpha t}\left[(c_1 u - c_2 w)\cos \beta t - (c_2 u + c_1 w)\operatorname{sen} \beta t\right]$$

$$= e^{-t}\left[\left(c_1\begin{pmatrix} 1 \\ 2/5 \end{pmatrix} - c_2\begin{pmatrix} 0 \\ 1/5 \end{pmatrix}\right)\cos t - \left(c_2\begin{pmatrix} 1 \\ 2/5 \end{pmatrix} + c_1\begin{pmatrix} 0 \\ 1/5 \end{pmatrix}\right)\operatorname{sen} t\right]$$

$$\boxed{\begin{aligned} x_1(t) &= e^{-t}\left[c_1 \cos t - c_2 \operatorname{sen} t\right] \\ x_2(t) &= e^{-t}\left[(0{,}4c_1 - 0{,}2c_2)\cos t - (0{,}4c_2 + 0{,}2c_1)\operatorname{sen} t\right] \end{aligned}}$$ SOLUÇÃO GERAL

SOLUÇÃO DEFINIDA:

$x_1(0) = e^{(0)}(c_1 \cos(0) - c_2 \overset{0}{\cancel{\operatorname{sen}(0)}})$

$1 = c_1 \therefore c_1 = 1$

$x_2(0) = e^0[(0{,}4c_1 - 0{,}2c_2)\cos(0) - (0{,}4c_2 + 0{,}2c_1)\overset{0}{\cancel{\operatorname{sen}(0)}}]$ 54.A

$1 = 0{,}4c_1 - 0{,}2c_2 \therefore 0{,}4c_1 - 0{,}2c_2 = 1$

UMA VEZ QUE $c_1 = 1$ TEM-SE:

$0{,}4(1) - 0{,}2c_2 = 1 \therefore c_2 = -3$

$$\boxed{\begin{aligned} x_1(t) &= e^{-t}(\cos t + 3\operatorname{sen} t) \\ x_2(t) &= e^{-t}(\cos t + \operatorname{sen} t) \end{aligned}}$$

SOLUÇÃO DEFINIDA

// $\lim_{t\to\infty} x_1(t) = 0 \quad \lim_{t\to\infty} x_2(t) = 0$

21.C $\dot{x}_1 = x_1 - 4x_2$

$\dot{x}_2 = 4x_1 - 7x_2$

$x_1(0) = 3 \quad x_2(0) = 2$

$$\dot{x} = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix} x$$

$$A = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix} \therefore \dot{x} = Ax \therefore \dot{x} - Ax = 0$$

SOLUÇÃO HOMOGÊNEA:

AUTOVALORES:

$|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & -4 \\ 4 & -7-\lambda \end{vmatrix} = 0$$

$(1-\lambda)(-7-\lambda) - (-4)(4) = 0$

$-7 - \lambda + 7\lambda + \lambda^2 + 16 = 0$

$\lambda^2 + 6\lambda + 9 = 0$

FÓRMULA DE BHASKARA:

$\Delta = b^2 - 4ac$

$= 36 - 36$

$= 0$

$\lambda = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$= \dfrac{-6 \pm 0}{2}$ $\nearrow \lambda_1 = -3$, $\searrow \lambda_2 = -3$

AUTOVETORES:

PARA $\lambda_1 = -3$:

$[A-(-3)I]V_1 = 0$

$$\begin{bmatrix} 4 & -4 \\ 4 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -4 \\ 4 & -4 \end{bmatrix} \times (1/4) \sim \begin{bmatrix} 1 & -1 \\ 4 & -4 \end{bmatrix} \times(-4) \sim \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

$x_1 - x_2 = 0$

$x_1 = x_2$

$x_2 = x_2$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} x_2$$

FAZENDO $x_2 = 1$ TEM-SE QUE:

$$V_1 = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

PARA $\lambda_2 = -3$: (VETOR GENERALIZADO)

$[A-(-3)I]V_2 = V_1$

$$\begin{bmatrix} 4 & -4 \\ 4 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 4 & -4 & 1 \\ 4 & -4 & 1 \end{array}\right] \times(1/4) \sim \left[\begin{array}{cc|c} 1 & -1 & 1/4 \\ 4 & -4 & 1 \end{array}\right] \times(-4) \sim \left[\begin{array}{cc|c} 1 & -1 & 1/4 \\ 0 & 0 & 0 \end{array}\right]$$

$x_1 - x_2 = \frac{1}{4}$

$x_1 = \frac{1}{4} + x_2$

$x_2 = x_2$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} x_2 + \begin{pmatrix} 1/4 \\ 0 \end{pmatrix}$$

FAZENDO $x_2 = 0$ TEM-SE QUE:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1/4 \\ 0 \end{pmatrix}$$

$$x(t) = V_1(c_1 + c_2 t)e^{\lambda t} + V_2 c_2 e^{\lambda t}$$

5.5.A

$$x(t)=\begin{pmatrix}1\\1\end{pmatrix}(c_1+c_2t)e^{\lambda t}+\begin{pmatrix}1/4\\0\end{pmatrix}c_2e^{\lambda t}$$

$$\boxed{\begin{aligned}x_1(t)&=(c_1+c_2t)e^{-3t}+\frac{1}{4}c_2e^{-3t}\\x_2(t)&=(c_1+c_2t)e^{-3t}\end{aligned}}$$

SOLUÇÃO GERAL

SOLUÇÃO DEFINIDA:

$$x_1(0)=(c_1+c_2(0))e^{-3(0)}+\frac{1}{4}c_2e^{-3(0)}$$

$$3=c_1+\frac{1}{4}c_2$$

$$c_1+\frac{1}{4}c_2=3 \therefore c_1+0{,}25c_2=3$$

$$c_1+0{,}25c_2=3$$

$$2+0{,}25c_2=3$$

$$0{,}25c_2=1$$

$$c_2=4$$

$$x_2(0)=(c_1+c_2(0))e^{-3(0)}$$

$$2=c_1 \therefore c_1=2$$

$$x_1(t)=(2+4t)e^{-3t}+\frac{1}{4}(4)e^{-3t} \qquad x_2(t)=(2+4t)e^{-3t}$$

$$=(2+4t)e^{-3t}+e^{-3t}$$

$$\boxed{\begin{aligned}x_1(t)&=(2+4t)e^{-3t}+e^{-3t}\\x_2(t)&=(2+4t)e^{-3t}\end{aligned}}$$

SOLUÇÃO DEFINIDA

$$\lim_{t\to\infty}x_1(t)=0 \qquad \lim_{t\to\infty}x_2(t)=0$$

21.7

$$\dot{x}_1=3x_1+9x_2$$
$$\dot{x}_2=-x_1-3x_2$$

$$x_1(0)=2 \quad x_2(0)=4$$

LISTA VIII MATEMÁTICA I

2013/1º

$$\dot{X} = \begin{pmatrix} 3 & 9 \\ -1 & -3 \end{pmatrix} X$$

$$A = \begin{pmatrix} 3 & 9 \\ -1 & -3 \end{pmatrix} \therefore \dot{X} = AX \therefore \dot{X} - AX = 0$$

SOLUÇÃO HOMOGÊNEA:

AUTOVALORES:

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 3-\lambda & 9 \\ -1 & -3-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)\cdot(-3-\lambda) - (9)\cdot(-1) = 0$$

$$-\cancel{9} - \cancel{3\lambda} + \cancel{3\lambda} + \lambda^2 + \cancel{9} = 0$$

$$\lambda^2 = 0 \therefore \lambda_1 = \lambda_2 = 0$$

AUTOVETORES:

PARA $\lambda_1 = 0$:

$$[A - (0)I]V_1 = 0$$

$$\begin{bmatrix} 3 & 9 \\ -1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 9 \\ -1 & -3 \end{bmatrix} \times (1/3) \sim \begin{bmatrix} 1 & 3 \\ -1 & -3 \end{bmatrix} \times (1) + \sim \begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix}$$

$$x_1 + 3x_2 = 0$$

$$x_1 = -3x_2$$

$$x_2 = x_2$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} x_2$$

FAZENDO $x_2 = 1$ TEM-SE:

$$V_1 = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

PARA $\lambda_2 = 0$: (VETOR GENERALIZADO) 56.A

$[A-(0)I]V_1 = V_2$

$$\begin{bmatrix} 3 & 9 \\ -1 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 3 & 9 & -3 \\ -1 & -3 & 1 \end{array}\right] \times (1/3) \sim \left[\begin{array}{cc|c} 1 & 3 & -1 \\ -1 & -3 & 1 \end{array}\right] \times(1) \sim \left[\begin{array}{cc|c} 1 & 3 & -1 \\ 0 & 0 & 0 \end{array}\right]$$

$x_1 + 3x_2 = -1$

$x_1 = -1 - 3x_2$

$x_2 = x_2$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} x_2 + \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

FAZENDO $x_2 = 0$ TEM-SE:

$$V_2 = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$x(t) = V_1(c_1 + c_2 t)e^{\lambda t} + V_2 c_2 e^{\lambda t}$$

$$x(t) = \begin{pmatrix} -3 \\ 1 \end{pmatrix}(c_1 + c_2 t)e^{(0)t} + \begin{pmatrix} -1 \\ 0 \end{pmatrix} c_2 e^{(0)t}$$

$$\boxed{\begin{aligned} x_1(t) &= -3(c_1 + c_2 t) - c_2 \\ x_2(t) &= (c_1 + c_2 t) \end{aligned}}$$

SOLUÇÃO GERAL //

SOLUÇÃO DEFINIDA:

$x_1(0) = -3(c_1 + c_2(0)) - c_2$

$2 = -3c_1 - c_2 \therefore 3c_1 + c_2 = -2$

$x_2(0) = c_1 + c_2(0)$

$4 = c_1 \therefore c_1 = 4$

$3c_1 + c_2 = -2$

$3(4) + c_2 = -2$

$c_2 = -14$

$x_1(t) = -3(4 - 14t) + 14$

$= -12 + 42t + 14$

$= 42t + 2$

$x_2(t) = 4 - 14t$

LISTA VIII MATEMÁTICA I

2013/1º

$$\boxed{\begin{aligned} x_1(t) &= 42t + 2 \\ x_2(t) &= 4 - 14t \end{aligned}}$$

SOLUÇÃO DEFINIDA

$\lim\limits_{t \to \infty} x_1(t) = \infty$ $\lim\limits_{t \to \infty} x_2(t) = -\infty$

EXERCÍCIO 22:

$\dot{x}_1 = a x_1 + b x_2$

$\dot{x}_2 = c x_1 - d x_2$

DERIVANDO $\dot{x}_1$ EM RELAÇÃO A t:

$\ddot{x}_1 = a \dot{x}_1 + b \dot{x}_2$

$= a \dot{x}_1 + b(c x_1 - d x_2)$

$= a \dot{x}_1 + bc x_1 - bd x_2$

$= a \dot{x}_1 + bc x_1 - d(\dot{x}_1 - a x_1)$

$= a \dot{x}_1 + bc x_1 - d \dot{x}_1 + da x_1$

$\ddot{x}_1 = (a - d)\dot{x}_1 + (ad + bc) x_1$

$\ddot{x}_1 - (a - d)\dot{x}_1 - (ad + bc) x_1 = 0$

$\ddot{x}_1 - tr(A)\dot{x}_1 - |A| x_1 = 0$

SABE-SE DA ÁLGEBRA LINEAR QUE:

$tr(A) = \lambda_1 + \lambda_2$ E $|A| = \lambda_1 \cdot \lambda_2$

COMO PARA HAVER ESTABILIDADE $tr(A) < 0$ E $|A| > 0$. LOGO,

$(a - d) < 0$ E $(ad + bc) > 0$ ENTÃO, A QUESTÃO É FALSA.

EXERCÍCIO 23:

23.A $\dot{x}_1 = -3x_1$

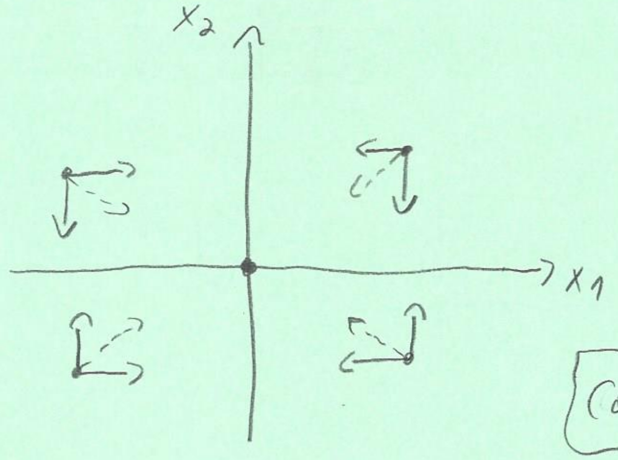
$\dot{x}_2 = -2x_2$

(57.1)

PONTO ESTACIONÁRIO:

$\dot{x}_1 = 0 \therefore x_1 = 0$

$\dot{x}_2 = 0 \therefore x_2 = 0$



$\boxed{\text{CONVERGE}}$

$\dot{x}_1 > 0 \therefore x_1 < 0$ $\qquad$ $\dot{x}_2 > 0 \therefore x_2 < 0$

$\dot{x}_1 < 0 \therefore x_1 > 0$ $\qquad$ $\dot{x}_2 < 0 \therefore x_2 > 0$

$$A = \begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix} \quad tr(A) = -5 < 0 \quad |A| = 6 > 0$$

GLOBALMENTE ESTÁVEL

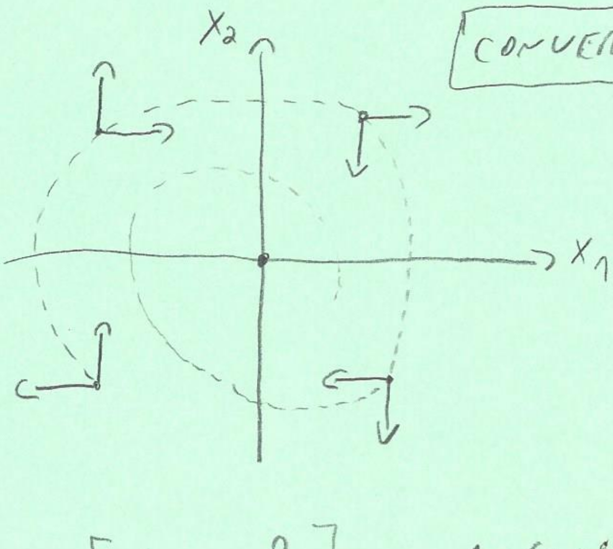
(23.3) $\dot{x}_1 = 2x_2$

$\dot{x}_2 = -4x_1$

PONTO ESTACIONÁRIO:

$\dot{x}_1 = 0 \therefore x_2 = 0$

$\dot{x}_2 = 0 \therefore x_1 = 0$



$\boxed{\text{CONVERGE}}$

$x_2 > 0 \therefore \dot{x}_1 > 0$

$x_2 < 0 \therefore \dot{x}_1 < 0$

$x_1 > 0 \therefore \dot{x}_2 < 0$

$x_1 < 0 \therefore \dot{x}_2 > 0$

$$A = \begin{bmatrix} 0 & 2 \\ -4 & 0 \end{bmatrix} \quad tr(A) = 0 \quad |A| = 8 > 0$$

GLOBALMENTE ESTÁVEL

LISTA VIII MATEMÁTICA I

2023/1.º

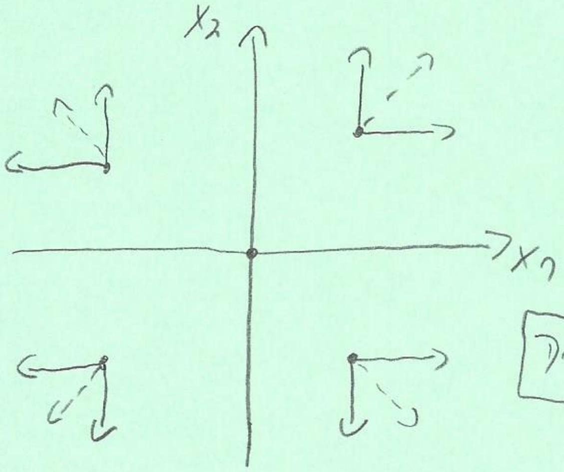
23.C $\dot{x}_1 = 2x_1$

$\dot{x}_2 = x_2$

PONTO ESTACIONÁRIO:

$\dot{x}_1 = 0 \therefore x_1 = 0$

$\dot{x}_2 = 0 \therefore x_2 = 0$



$\boxed{\text{DIVERGE}}$

$\dot{x}_1 > 0 \therefore x_1 > 0$

$\dot{x}_1 < 0 \therefore x_1 < 0$

$\dot{x}_2 > 0 \therefore x_2 > 0$

$\dot{x}_2 < 0 \therefore x_2 < 0$

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

$tr(A) = 3 > 0$

$|A| = 2 > 0$

GLOBALMENTE INSTÁVEL

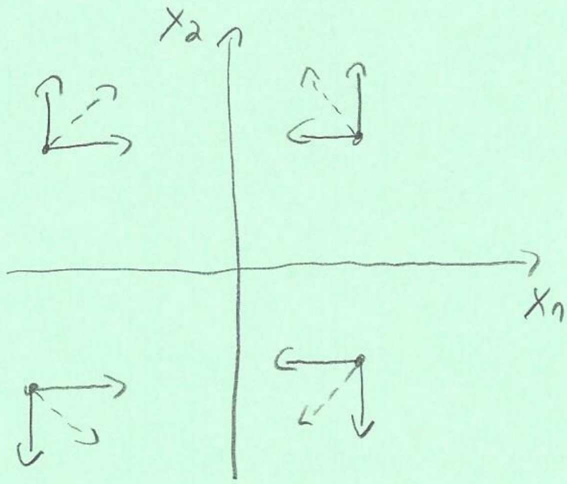
23.D $\dot{x}_1 = -x_1$

$\dot{x}_2 = 2x_2$

PONTO ESTACIONÁRIO:

$\dot{x}_1 = 0 \therefore x_1 = 0$

$\dot{x}_2 = 0 \therefore x_2 = 0$



$\dot{x}_1 > 0 \therefore x_1 < 0$

$\dot{x}_1 < 0 \therefore x_1 > 0$

$\dot{x}_2 > 0 \therefore x_2 > 0$

$\dot{x}_2 < 0 \therefore x_2 < 0$

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$$

$tr(A) = 1 > 0$

$|A| = -2 < 0$

PONTO DE SELA, POIS $|A| < 0$.