OBSERVAÇÕES MATEMÁTICA I

2013/1º

EQUAÇÕES DIFERENCIAIS ORDINÁRIAS DE PRIMEIRA ORDEM - EDO

$$\boxed{1\_\ Y' + aY = b}$$

COEFICIENTES CONSTANTES

OBS: EDO LINEAR

SOLUÇÃO:

SE $b \neq 0$ SOLUÇÃO HOMOGÊNEA

SOLUÇÃO PARTICULAR

SOLUÇÃO HOMOGÊNEA:

$b = 0$

$Y' + aY = b$

$Y' = -aY$

$\frac{dY}{dt} = -aY$

$\frac{dY}{Y} = -a\,dt$

$\int \frac{dY}{Y} = \int -a\,dt$

$\ln Y = -at + c$

$e^{\ln Y} = e^{-at+c}$

$Y = e^{-at} \cdot e^{c}$

$Y = e^{-at} \cdot A$

$$\boxed{Y = A \cdot e^{-at}}$$

SOLUÇÃO PARTICULAR:

$Y' = 0$

$aY = b$

$$\boxed{Y = \frac{b}{a}}$$

SOLUÇÃO GERAL:

$Y = Y_H + Y_P$

$$\boxed{Y = A \cdot e^{-at} + \frac{b}{a}}$$

1.A

$\boxed{2 - Y' + a(t)Y = b(t)}$

COEFICIENTES VARIANTES

OBS: EDO LINEAR

APLICA-SE O MÉTODO DO FATOR INTEGRANTE.

MATEMÁTICA:

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EXERCÍCIO 6:

(6.A) $\frac{dY}{dx} + 5Y = 10 \quad = Y' + 5Y = 10$

SOLUÇÃO HOMOGÊNEA ($b=0$):

$Y' + 5Y = 0$

$Y_H = Ae^{-aX}$

$= Ae^{-5X}$

SOLUÇÃO PARTICULAR ($Y'=0$)

$5Y = 10$

$Y_P = 2$

SOLUÇÃO GERAL:

$Y = Y_H + Y_P$

$\boxed{= Ae^{-5X} + 2}$

(6.B) $\frac{dY}{dx} + 2tY = t \quad = Y' + (2t)Y = (t)$

FATOR INTEGRANTE

$\mu = e^{\int 2t\,dt}$

$= e^{2\int t\,dt}$

$\mu = e^{t^2}$

$e^{t^2} \cdot \frac{dY}{dx} + e^{t^2} 2tY = e^{t^2} t$

$\frac{d(\mu \cdot Y)}{dt} = e^{t^2} t$

$d(\mu \cdot Y) = e^{t^2} t\,dt$

$d(e^{t^2} \cdot Y) = e^{t^2} t\,dt$

$\int d(e^{t^2} \cdot Y) = \int e^{t^2} t\,dt$

$e^{t^2} \cdot Y = \int e^{u} \cdot t \cdot \frac{du}{2t}$

$e^{t^2} \cdot Y = \frac{1}{2} \int e^{u}\,du$

$e^{t^2} \cdot Y = \frac{1}{2} e^{u} + c$

$e^{t^2} \cdot Y = \frac{1}{2} e^{t^2} + c$

$\boxed{Y = \frac{1}{2} + c \cdot e^{-t^2}}$

MÉTODO DA SUBSTITUIÇÃO:

$u = t^2 \therefore \frac{du}{dt} = 2t \therefore dt = \frac{du}{2t}$

6.c $y' + y = te^{-t} + 1$

FATOR INTEGRANTE:

$\mu = e^{\int dt}$

$= e^{t}$

$e^{t} \cdot y' + e^{t} y = e^{t}(te^{-t} + 1)$

$\frac{d(e^{t} \cdot y)}{dt} = e^{t}(te^{-t} + 1)$

$d(e^{t} \cdot y) = e^{t}(te^{-t} + 1)\, dt$

$\int d(e^{t} \cdot y) = \int e^{t}(te^{-t} + 1)\, dt$

$e^{t} \cdot y = \int (t + e^{t})\, dt$

$e^{t} \cdot y = \int t\, dt + \int e^{t}\, dt$

$e^{t} \cdot y = \frac{t^2}{2} + e^{t} + c$

$\boxed{y = \frac{t^2 e^{-t}}{2} + c \cdot e^{-t} + 1}$

6.d $y' - 2y = 3e^{t}$

FATOR INTEGRANTE:

$\mu = e^{\int -2\, dt}$

$= e^{-2t}$

$e^{-2t} \cdot y' - e^{-2t} \cdot 2 \cdot y = e^{-2t} \cdot 3e^{t}$

$\frac{d(\mu \cdot y)}{dt} = 3 \cdot e^{-t}$

$d(e^{-2t} \cdot y) = 3e^{-t}\, dt$

$\int d(e^{-2t} \cdot y) = \int 3e^{-t}\, dt$

$e^{-2t} \cdot y = 3 \int e^{-t}\, dt$

$e^{-2t} \cdot y = -3e^{-t} + c$

$\boxed{y = -3e^{t} + c \cdot e^{2t}}$

(6.E) $y' - \frac{1}{t}y = 3\cos 2t \quad t > 0$ (2.9)

FATOR INTEGRANTE:

$\mu = e^{\int -\frac{1}{t}\,dt} \Rightarrow \mu = e^{\ln t^{-1}}$

$= e^{-\ln t} \qquad = t^{-1}$

$t^{-1}y' - t^{-2}y = 3t^{-1}\cos 2t$

$\frac{d(\mu \cdot y)}{dt} = 3t^{-1}\cos 2t$

$d\mu \cdot y = 3t^{-1}\cos 2t\,dt$

$\int d\,t^{-1} \cdot y = \int 3t^{-1}\cos 2t\,dt$

$t^{-1} \cdot y = 3\int t^{-1}\cos 2t\,dt$

MÉTODO DA INTEGRAÇÃO POR PARTES:

$u = \cos 2t \qquad dv = t^{-1}dt$

$\frac{du}{dt} = -2\,\text{sen}\,2t \qquad \int dv = \int \frac{1}{t}\,dt$

$v = \ln t$

$du = -2\,\text{sen}\,2t\,dt$

$\int t^{-1}\cos 2t\,dt = u \cdot v - \int v \cdot du$

$= \cos 2t \cdot \ln t - \int \ln t \cdot (-2\,\text{sen}\,2t\,dt)$

$= \ln t \cdot \cos 2t + 2\int \ln t \cdot \text{sen}\,2t\,dt$

MÉTODO DA INTEGRAÇÃO POR PARTES:

$u = 2t \qquad \int \text{sen}\,u\,\frac{du}{2}$

$dt = \frac{du}{2} \qquad \frac{1}{2}\int \text{sen}\,u\,du$

$u = \ln t \Rightarrow du = \frac{dt}{t}$

$\frac{du}{dt} = \frac{1}{t}$

$dv = \text{sen}\,2t\,dt$

$\int dv = \int \text{sen}\,2t\,dt$

$v = -\frac{\cos 2t}{2}$

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$$\int \ln t \cdot \sin 2t\, dt = u \cdot v - \int v\, du$$

$$= \ln t \cdot -\frac{\cos 2t}{2} - \int -\frac{\cos 2t}{2} \frac{dt}{t}$$

$$= -\frac{1}{2} \ln t \cdot \cos 2t + \frac{1}{2} \int t^{-1} \cos 2t\, dt$$

$$\int t^{-1} \cos 2t\, dt = -\frac{1}{2} \ln t \cdot \cos 2t + \frac{1}{2} \int t^{-1} \cos 2t\, dt$$

FAZENDO $\int t^{-1} \cos 2t\, dt = I$ TEM-SE:

$$I = -\frac{1}{2} \ln t \cdot \cos 2t + \frac{1}{2} I$$

$$I - \frac{1}{2} I = -\frac{1}{2} \ln t \cdot \cos 2t$$

$$\frac{1}{2} I = -\frac{1}{2} \ln t \cdot \cos 2t$$

$$I = -\ln t \cdot \cos 2t$$

$$\int t^{-1} \cos 2t\, dt = -\ln t \cdot \cos 2t + c$$

LOGO,

$$t^{-1} \cdot Y = -3 \ln t \cdot \cos 2t + c$$

$$\boxed{Y = -3t \ln t \cdot \cos 2t + c \cdot t}$$

6.F $2Y' + Y = 3t^2$ $\quad Y' + \frac{1}{2} Y = \frac{3}{2} t^2$

FATOR INTEGRANTE:

$$\mu = e^{\int \frac{1}{2} dt}$$

$$= e^{\frac{1}{2} \int dt}$$

$$= e^{\frac{1}{2} t}$$

3.A

$$e^{\frac{1}{2}t}\cdot y' + \frac{1}{2}e^{\frac{1}{2}t}\cdot y = \frac{3t^2}{2}e^{\frac{1}{2}t}$$

$$\frac{d(\mu\cdot y)}{dt} = \frac{3t^2}{2}\cdot e^{\frac{1}{2}t}$$

$$d(\mu\cdot y) = \frac{3t^2}{2}\cdot e^{\frac{1}{2}t}\cdot dt$$

$$d(e^{\frac{1}{2}t}\cdot y) = \frac{3t^2}{2}e^{\frac{1}{2}t}dt$$

$$\int d(e^{\frac{1}{2}t}\cdot y) = \int \frac{3t^2}{2}\cdot e^{\frac{1}{2}t}dt$$

$$e^{\frac{1}{2}t}\cdot y = \frac{3}{2}\int t^2\cdot e^{\frac{1}{2}t}dt$$

MÉTODO DA INTEGRAÇÃO POR PARTES:

$u = t^2$

$\frac{du}{dt} = 2t$

$du = 2t\,dt$

$dv = e^{\frac{1}{2}t}dt$

$\int dv = \int e^{\frac{1}{2}t}dt$

$v = \int e^{u}\,2du$

$v = 2\int e^{u}du$

$= 2e^{\frac{1}{2}t}$

$u = \frac{1}{2}t$

$\frac{du}{dt} = \frac{1}{2}$

$dt = 2du$

$$\int t^2\cdot e^{\frac{1}{2}t}dt = u\cdot v - \int v\cdot du$$

$$= t^2\cdot 2e^{\frac{1}{2}t} - \int 2e^{\frac{1}{2}t}\cdot 2t\,dt$$

$$= 2t^2e^{\frac{1}{2}t} - 4\int e^{\frac{1}{2}t}t\,dt$$

MÉTODO DA INTEGRAÇÃO POR PARTES:

$u = t$

$\frac{du}{dt} = 1$

$du = dt$

$dv = e^{\frac{1}{2}t}\cdot dt$

$\int dv = \int e^{\frac{1}{2}t}dt$

$v = 2e^{\frac{1}{2}t}$

$u = \frac{1}{2}t$

$\frac{du}{dt} = \frac{1}{2}$

$dt = 2du$

$\int e^{u}\,2du$

$2\int e^{u}du$

$2e^{u}$

$$\int e^{1/2t} \cdot t\,dt = u \cdot v - \int v \cdot du$$
$$= t \cdot 2 \cdot e^{1/2t} - \int 2e^{1/2t}\,dt$$
$$= 2te^{1/2t} - 2\int e^{1/2t}\,dt$$
$$= 2te^{1/2t} - 2(2e^{1/2t})$$
$$= 2te^{1/2t} - 4e^{1/2t}$$

$u = \frac{1}{2}t$ $\quad \int e^{u}2\,du$

$\frac{du}{dt} = \frac{1}{2}$ $\quad 2\int e^{u}du$

$dt = 2du$ $\quad 2e^{u}$

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$$\int t^2 \cdot e^{1/2t}\,dt = 2t^2 e^{1/2t} - 4(2te^{1/2t} - 4e^{1/2t})$$
$$= 2t^2 \cdot e^{1/2t} - 8te^{1/2t} + 16e^{1/2t} + c$$

$$e^{1/2t} \cdot Y = \frac{3}{2}(2t^2 \cdot e^{1/2t} - 8te^{1/2t} + 16e^{1/2t} + c)$$
$$e^{1/2t} \cdot Y = 3t^2 e^{1/2t} - 12te^{1/2t} + 24e^{1/2t} + 3c$$

$$\boxed{Y = 3t^2 - 12t + 24 + 3ce^{-1/2t}}$$

6.6 $tY' + 2Y = \text{sen}\, t \quad t > 0 \qquad Y' + \frac{2}{t}Y = \frac{1}{t}\,\text{sen}\, t$

FATOR INTEGRANTE:

$$\mu = e^{\int \frac{2}{t}dt}$$
$$= e^{2\int \frac{1}{t}dt}$$
$$= e^{2\ln t}$$

$$\mu = e^{\ln t^2}$$
$$= t^2$$

$$t^2 y' + 2tY = t\,\text{sen}\, t$$
$$\frac{d(\mu \cdot Y)}{dt} = t\,\text{sen}\, t$$
$$d(t^2 \cdot Y) = t\,\text{sen}\, t\,dt$$

$$\int d(t^2 \cdot Y) = \int t\,\text{sen}\, t\,dt$$
$$t^2 \cdot Y = \int t\,\text{sen}\, t\,dt$$

4.A

MÉTODO DA INTEGRAÇÃO POR PARTES:

$u = t$ $\quad$ $dv = \text{sen}\, t\, dt$

$\frac{du}{dt} = 1$ $\quad$ $\int dv = \int \text{sen}\, t\, dt$

$du = dt$ $\quad$ $v = -\cos t$

$$\int t\, \text{sen}\, t\, dt = u \cdot v - \int v \cdot du$$

$$= t(-\cos t) - \int (-\cos t)\, dt$$

$$= -t\cos t + \text{sen}\, t + c$$

$$t^2 \cdot Y = -t \cdot \cos t + \text{sen}\, t + c$$

$$\boxed{Y = -t^{-1} \cos t + t^{-2}\,\text{sen}\, t + t^{-2} c}$$

6.H $\quad tY' - Y = t^2 e^{-t}$ $\qquad$ $Y' - \frac{1}{t} Y = t e^{-t}$

FATOR INTEGRANTE:

$$\mu = e^{\int -\frac{1}{t} dt}$$

$$= e^{(-1)\int \frac{1}{t} dt}$$

$$= e^{(-1)\ln t}$$

$$\mu = e^{\ln t^{-1}}$$

$$= t^{-1}$$

$$t^{-1} Y' - t^{-2} \cdot Y = e^{-t}$$

$$\frac{d(\mu \cdot Y)}{dt} = e^{-t}$$

$$d(\mu \cdot Y) = e^{-t} dt$$

$$d(t^{-1} \cdot Y) = e^{-t} dt$$

$$\int d(t^{-1} \cdot Y) = \int e^{-t} dt$$

$$t^{-1} \cdot Y = \int e^{-t} dt$$

$$t^{-1} \cdot Y = -e^{-t} + c$$

$$\boxed{Y = -e^{-t} \cdot t + c \cdot t}$$

$u = -t$ $\quad$ $\int e^u(-du)$

$\frac{du}{dt} = -1$ $\quad$ $(-1)\int e^u du$

$dt = -du$

(6.I) $y' - \frac{1}{2}y = 2\cos t$ (5)

FATOR INTEGRANTE:

$\mu = e^{\int -\frac{1}{2} dt}$

$= e^{-1/2 \int dt}$

$\mu = e^{-1/2 t}$

$e^{-1/2 t} \cdot y' - \frac{1}{2} e^{-1/2 t} y = 2e^{-1/2 t} \cos t$

$\frac{d(\mu \cdot y)}{dt} = 2e^{-1/2 t} \cos t$

$d(\mu \cdot y) = 2e^{-1/2 t} \cos t \, dt$

$\int d(e^{-1/2 t} \cdot y) = \int 2e^{-1/2 t} \cos t \, dt$

$e^{-1/2 t} y = 2\int e^{-1/2 t} \cos t \, dt$

MÉTODO DA INTEGRAÇÃO POR PARTES:

$u = \cos t$

$\frac{du}{dt} = -\text{sen}\, t$

$du = -\text{sen}\, t \, dt$

$dv = e^{-1/2 t} dt$

$\int dv = \int e^{-1/2 t} dt$

$v = -2e^{-1/2 t}$

$u = -\frac{1}{2}t$

$\frac{du}{dt} = -\frac{1}{2}$

$dt = -2du$

$\int e^{u}(-2du)$

$-2\int e^{u} du$

$-2e^{u}$

$-2e^{-1/2 t}$

$\int e^{-1/2 t} \cos t \, dt = u \cdot v - \int v \, du$

$= \cos t \cdot (-2e^{-1/2 t}) - \int (-2e^{-1/2 t})(-\text{sen}\, t) dt$

$= -2e^{-1/2 t} \cos t - 2\int e^{-1/2 t} \text{sen}\, t \, dt$

MÉTODO DA INTEGRAÇÃO POR PARTES:

$u = \text{sen}\, t$

$\frac{du}{dt} = \cos t$

$du = \cos t \, dt$

$dv = e^{-1/2 t} dt$

$\int dv = \int e^{-1/2 t} dt$

$v = -2e^{-1/2 t}$

$u = -\frac{1}{2}t$

$\frac{du}{dt} = -\frac{1}{2}$

$dt = -2du$

$\int e^{u}(-2du)$

$-2\int e^{u} du$

$-2e^{u}$

$-2e^{-1/2 t}$

5.A

$$\int e^{-1/2 t}\, sen\, t\, dt = u \cdot v - \int v \cdot du$$

$$= sen\, t\,(-2e^{-1/2 t}) - \int (-2e^{-1/2 t}) \cdot \cos t\, dt$$

$$= -2e^{-1/2 t}\, sen\, t + 2\int e^{-1/2 t} \cos t\, dt$$

. FAZENDO, $\int e^{-1/2 t} \cos t\, dt = I$ TEM-SE:

$$I = -2e^{-1/2 t}\cos t - 2(-2e^{-1/2 t}\, sen\, t + 2I)$$

$$I = -2e^{-1/2 t}\cos t + 4e^{-1/2 t}\, sen\, t - 4I$$

$$5I = -2e^{-1/2 t}\cos t + 4e^{-1/2 t}\, sen\, t$$

$$\int e^{-1/2 t}\cos t\, dt = -0{,}4e^{-1/2 t}\cos t + 0{,}8e^{-1/2 t}\, sen\, t + c$$

LOGO,

$$e^{-1/2 t}\, y = 2(-0{,}4e^{-1/2 t}\cos t + 0{,}8e^{-1/2 t}\, sen\, t + c)$$

$$\boxed{y = -0{,}8\cos t + 1{,}6\, sen\, t + k_1 e^{1/2 t}}$$ //

6.J $y' + t^2 y = 3t^2$

FATOR INTEGRANTE:

$$\mu = e^{\int t^2 dt}$$

$$= e^{t^3/3}$$

$$e^{t^3/3} \cdot y' + e^{t^3/3} \cdot t^2 \cdot y = e^{t^3/3} \cdot 3t^2$$

$$\frac{d(\mu \cdot y)}{dt} = 3t^2 \cdot e^{t^3/3}$$

$$d(\mu \cdot y) = 3t^2 e^{t^3/3} dt$$

$$\int d(e^{t^3/3} \cdot y) = \int 3t^2 e^{t^3/3} dt$$

$$e^{t^3/3} \cdot y = 3\int t^2 \cdot e^{t^3/3} dt$$

MÉTODO DA SUBSTITUIÇÃO:

(6)

$u = \frac{t^3}{3}$ $\quad$ $\int t^2 . e^{t^3/3} dt = \int t^2 e^u . \frac{du}{t^2}$

$\frac{du}{dt} = t^2$ $\quad$ $= \int e^u . du$

$dt = \frac{du}{t^2}$ $\quad$ $= e^{t^3/3} + c$

$e^{t^3/3} . Y = 3e^{t^3/3} + c$

$\boxed{Y = c . e^{-t^3/3} + 3}$ //

(6.K) $Y' + Y^2 \operatorname{sen} X = 0$

$Y' = -Y^2 \operatorname{sen} X$

$\frac{dY}{dx} = -Y^2 \operatorname{sen} X$

$dY = -Y^2 \operatorname{sen} x \, dx$

$\frac{dY}{Y^2} = -\operatorname{sen} x \, dX$

$\int \frac{1}{Y^2} dY = \int -\operatorname{sen} x \, dX$

$-\frac{1}{Y} = (-1) \int \operatorname{sen} X \, dX$

$-\frac{1}{Y} = (-1)(-\cos x) + c$

$\boxed{Y = -\frac{1}{\cos X + c}}$ //

(6.L) $\frac{dY}{dx} = \frac{X - e^{-X}}{Y - e^Y}$

$(Y - e^Y) dY = (X - e^{-X}) dX$

$\int (Y - e^Y) dY = \int (X - e^{-X}) dX$

$\int Y dY - \int e^Y dY = \int x \, dx - \int e^{-x} dX$

$\frac{Y^2}{2} - e^Y = \frac{x^2}{2} + e^{-X} + c$

$u = -x$ $\quad$ $\int e^u (-du)$

$\frac{du}{dx} = -1$ $\quad$ $(-1) \int e^u du$

$dx = -du$ $\quad$ $(-1) e^u$

$$\boxed{y^2 - 2e^y = x^2 + 2e^{-x} + 2c}$$

6.A

6.M $xy' = (1-y^2)^{1/2}$

$$y' = \frac{(1-y^2)^{1/2}}{x}$$

$$\frac{dy}{dx} = \frac{(1-y^2)^{1/2}}{x}$$

$$\frac{dy}{(1-y^2)^{1/2}} = \frac{dx}{x}$$

$$\int \frac{1}{\sqrt{1-y^2}}\, dy = \int \frac{1}{x}\, dx$$

$$\text{arcsen } y = \ln x + c$$

$$\boxed{y = \text{sen}(\ln x + c)}$$

6.N $\dfrac{dy}{dx} = \dfrac{x^2 + 3y^2}{2xy}$

$$\frac{dy}{dx} = \frac{1}{2}\frac{x}{y} + \frac{3y}{2x}$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{y}{x}\right)^{-1} + \frac{3}{2}\frac{y}{x}$$

$$u = \frac{y}{x} \therefore y = u \cdot x \therefore \frac{dy}{dx} = u \cdot (x)' + x \cdot \frac{du}{dx} \therefore \frac{dy}{dx} = u + x \cdot \frac{du}{dx}$$

$$u + x \cdot \frac{du}{dx} = \frac{1}{2}\left(\frac{y}{x}\right)^{-1} + \frac{3}{2}\frac{y}{x}$$

$$u + x \cdot \frac{du}{dx} = \frac{1}{2}u^{-1} + \frac{3}{2}u$$

$$u + x \cdot \frac{du}{dx} = \frac{2 + 6u^2}{4u}$$

$$2 + 6u^2 = 4u^2 + 4u \cdot x \cdot \frac{du}{dx}$$

$$2 + 2u^2 = 4u \cdot x \cdot \frac{du}{dx}$$

$$2ux\frac{du}{dx} = 1 + u^2$$

$$2u\, du = \frac{1+u^2}{x}\, dx$$

$$\frac{2u}{1+u^2}\, du = \frac{1}{x}\, dx$$

$$\int \frac{2u}{1+u^2}\, du = \int \frac{1}{x}\, dx$$

MÉTODO DA SUBSTITUIÇÃO:

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$v = 1 + u^2$

$\frac{dv}{du} = 2u$

$du = \frac{dv}{2u}$

$$\int \frac{2u}{1+u^2}\, du = \int \frac{2u}{v} \frac{dv}{2u}$$
$$= \int \frac{1}{v}\, dv$$
$$= \ln v$$
$$= \ln(1+u^2)$$

$\ln(1+u^2) = \ln x + c$

$e^{\ln(1+u^2)} = e^{\ln x + c}$

$1 + u^2 = e^{\ln x} \cdot e^{c}$

$1 + \left(\frac{y}{x}\right)^2 = x \cdot c_1$

$\frac{y^2}{x^2} = c_1 \cdot x - 1$

$y^2 = x^2 (c_1 \cdot x - 1)$

$$\boxed{y = x\sqrt{c_1 \cdot x - 1}}$$

6.0 $\frac{dy}{dx} = \frac{4y - 3x}{2x - y}$

$$\frac{dy}{dx} = \frac{4\frac{y}{x} - 3}{2 - \frac{y}{x}}$$

$u = \frac{y}{x} \therefore y = ux \therefore \frac{dy}{dx} = u\,(x)' + x \cdot \frac{du}{dx} \therefore \frac{dy}{dx} = u + x \cdot \frac{du}{dx}$

$$\frac{4\frac{y}{x} - 3}{2 - \frac{y}{x}} = u + x \cdot \frac{du}{dx}$$

$$\frac{4u - 3}{2 - u} = u + x \cdot \frac{du}{dx}$$

$$\frac{4u - 3}{2 - u} - u = x \cdot \frac{du}{dx}$$

$$\frac{4u - 3 - 2u + u^2}{2 - u} = x \cdot \frac{du}{dx}$$

$$\frac{u^2 + 2u - 3}{2 - u} = x \cdot \frac{du}{dx}$$

$$\frac{2 - u}{u^2 + 2u - 3}\, du = \frac{1}{x}\, dx$$

$$\int \frac{2 - u}{u^2 + 2u - 3}\, du = \int \frac{1}{x}\, dx$$

7.A

$$\frac{2-u}{u^2+2u-3} = \frac{2-u}{(u-1)(u+3)} = \frac{A_1}{(u-1)} + \frac{A_2}{(u+3)}$$

$$= \frac{A_1(u+3)}{(u-1)} + \frac{A_2(u-1)}{(u+3)}$$

ENTÃO,

$$2-u = A_1(u+3) + A_2(u-1)$$

$$2-u = A_1 u + A_1 3 + A_2 u - A_2$$

$$2-u = (3A_1 - A_2) + (A_1 + A_2)u$$

$$3A_1 - A_2 = 2$$

$$A_2 = 3A_1 - 2$$

$$= 3\left(\frac{1}{4}\right) - 2$$

$$= -\frac{5}{4}$$

$$A_1 + A_2 = -1$$

$$A_1 = -1 - A_2$$

$$A_1 = -1 - 3A_1 + 2$$

$$4A_1$$

$$4A_1 = 1$$

$$A_1 = \frac{1}{4}$$

$$\int \frac{2-u}{u^2+2u-3}\,du = \int \frac{1}{4(u-1)}\,du + \int -\frac{5}{4(u+3)}\,du$$

$$= \frac{1}{4}\int \frac{1}{u-1}\,du - \frac{5}{4}\int \frac{1}{u+3}\,du$$

$$= \frac{1}{4}\ln(u-1) - \frac{5}{4}\ln(u+3)$$

LOGO,

$$\int \frac{2-u}{u^2+2u-3}\,du = \int \frac{1}{x}\,dx$$

$$\frac{1}{4}\ln(u-1) - \frac{5}{4}\ln(u+3) = \ln x + c$$

$$\ln(u-1)^{1/4} + \ln(u+3)^{-5/4} = \ln x + c$$

$$e^{\ln(u-1)^{1/4}} + e^{\ln(u+3)^{-5/4}} = e^{\ln x + c}$$

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$$(u-1)^{1/4}+(u+3)^{-3/4}=e^{\ln x}\cdot e^{c}$$

$$\boxed{\left(\frac{y}{x}-1\right)^{1/4}+\left(\frac{y}{x}+3\right)^{-3/4}=c_1 x}$$

6.7 $(x^2+3xy+y^2)\,dx=x^2\,dy$

$$\frac{dy}{dx}=\frac{x^2+3xy+y^2}{x^2}$$

$$=\frac{x^2}{x^2}+\frac{3xy}{x^2}+\frac{y^2}{x^2}$$

$$=1+3\frac{y}{x}+\left(\frac{y}{x}\right)^2$$

$$u=\frac{y}{x}\ \therefore\ y=ux\ \therefore\ \frac{dy}{dx}=u(x)'+x\cdot\frac{du}{dx}\ \therefore\ \frac{dy}{dx}=u+x\cdot\frac{du}{dx}$$

$$1+3\frac{y}{x}+\left(\frac{y}{x}\right)^2=u+x\cdot\frac{du}{dx}$$

$$1+3u+u^2=u+x\cdot\frac{du}{dx}$$

$$u^2+2u+1=x\cdot\frac{du}{dx}$$

$$\frac{1}{u^2+2u+1}\,du=\frac{1}{x}\,dx$$

$$\int\frac{1}{u^2+2u+1}\,du=\int\frac{1}{x}\,dx$$

$$\int\frac{1}{(u+1)^2}\,du=\int\frac{1}{x}\,dx$$

8.A

MÉTODO DA INTEGRAÇÃO POR SUBSTITUIÇÃO:

$v = u + 1$

$\frac{dv}{du} = 1$

$du = dv$

$$\int \frac{1}{(u+1)^2}\, du = \int \frac{1}{v^2}\, dv$$

$$= -\frac{1}{v}$$

$$= -\frac{1}{u+1}$$

$$-\frac{1}{u+1} = \ln x + c$$

$$u + 1 = -\frac{1}{\ln x + c}$$

$$u = -\frac{1}{\ln x + c} - 1$$

$$\frac{y}{x} = -\frac{1}{\ln x + c} - 1$$

$$\boxed{y = -\frac{x}{\ln x + c} - x}$$

EXERCÍCIO 8:

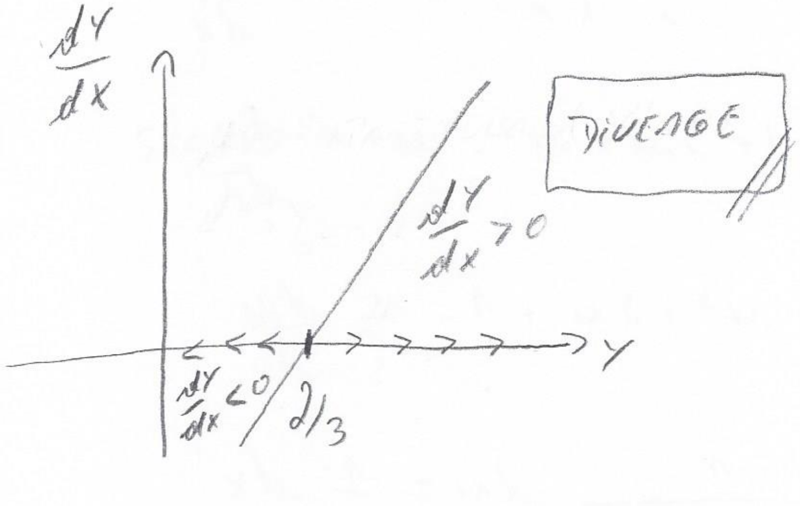
8.A $\frac{dy}{dx} - 3y = -2$

$$\frac{dy}{dx} = 3y - 2$$

PONTO ESTACIONÁRIO:

$$3y - 2 = 0$$

$$y = \frac{2}{3}$$

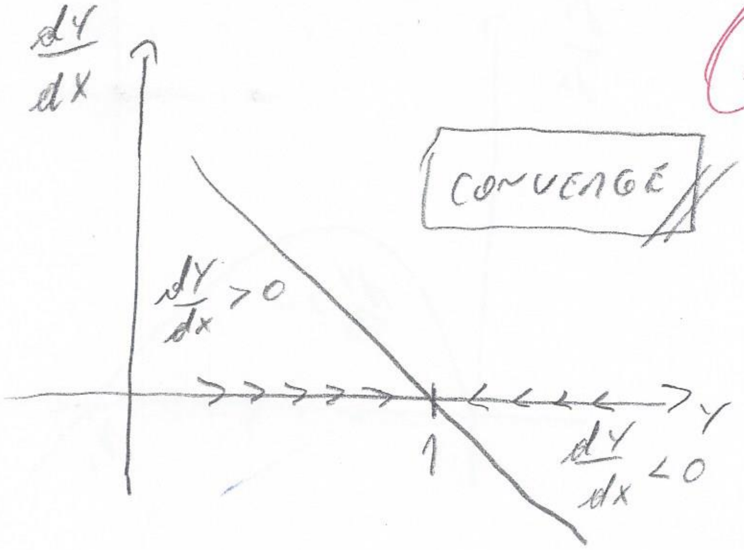


8.B $\frac{dy}{dx} = 1 - y$

PONTO ESTACIONÁRIO:

$1 - Y = 0$

$Y = 1$



9

(8.C) $\dfrac{dY}{dx} = (Y+1)^2 - 12$ $\quad Y > 0$

PONTO ESTACIONÁRIO:

$(Y+1)^2 - 12 = 0$

$Y^2 + 2Y + 1 - 12 = 0$

$Y^2 + 2Y - 11 = 0$

FÓRMULA DE BHASKARA:

$\Delta = b^2 - 4ac$

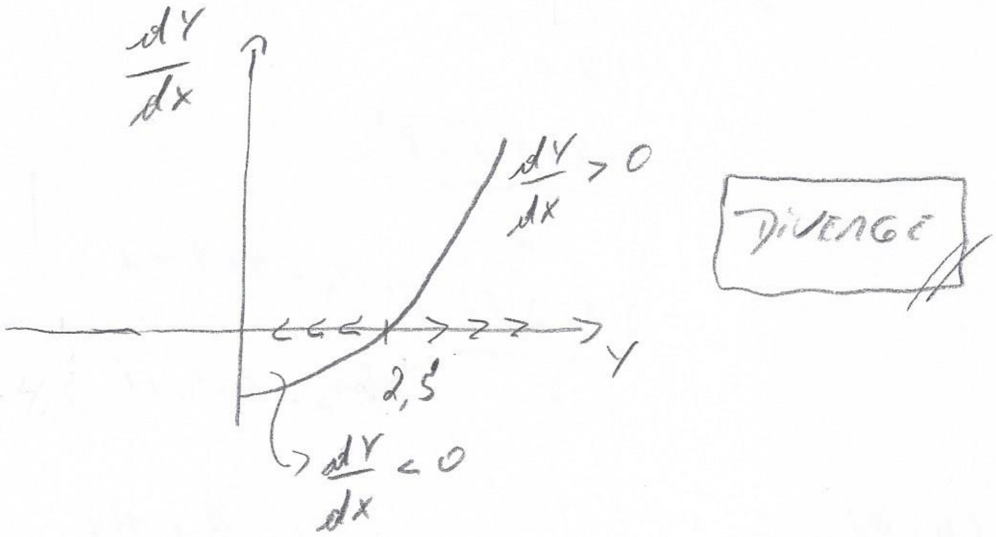
$= 4 + 44$

$= 48$

$Y = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$= \dfrac{-2 \pm \sqrt{48}}{2}$ $\rightarrow Y_1$, $\rightarrow Y_2$

$Y_1 = 2{,}5$ E $\cancel{Y_2 = -4{,}5}$



(8.D) $\dfrac{dY}{dx} = -Y^2 + \dfrac{1}{2}Y$ $\quad Y > 0$

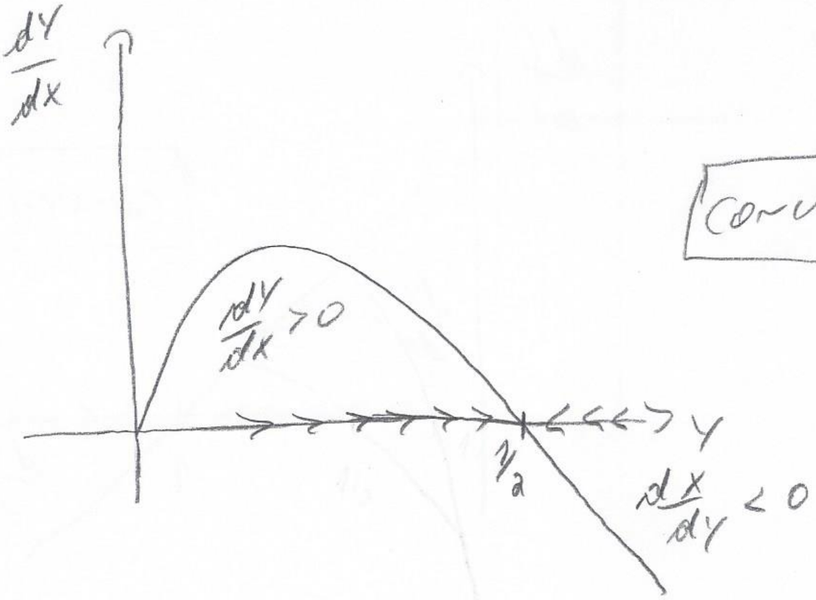
PONTO ESTACIONÁRIO:

$-Y^2 + \dfrac{1}{2}Y = 0$

$Y^2 - \dfrac{1}{2}Y = 0$

$Y\left(Y - \dfrac{1}{2}\right) = 0$

$\cancel{Y_1 = 0}$ E $Y_2 = \dfrac{1}{2}$



9.A

$\boxed{\text{CONVERGE}}$

EXERCÍCIO 12:

12.A $Y'' - 2Y' + 3Y = 2$

RAIZES CARACTERÍSTICAS:

$r^2 - 2r + 3 = 0$

FÓRMULA DE BHASKARA:

$\Delta = b^2 - 4ac$

$= 4 - 12$

$= -8$

$r = \frac{-b \pm \sqrt{\Delta}}{2a}$

$= \frac{-(-2) \pm \sqrt{-8}}{2}$

$= \frac{2 \pm 2{,}8i}{2}$ $\nearrow r_1 = 1 + 1{,}4i$ $\searrow r_2 = 1 - 1{,}4i$

SOLUÇÃO HOMOGÊNEA ($b=0$):

$r = a \pm \alpha i$

$Y_H = e^{ax}\{A_1\cos(\alpha x) + A_2 \operatorname{sen}(\alpha x)\}$

$= e^{x}\{A_1\cos(1{,}4x) + A_2 \operatorname{sen}(1{,}4x)\}$

SOLUÇÃO PARTICULAR ($Y'=0$):

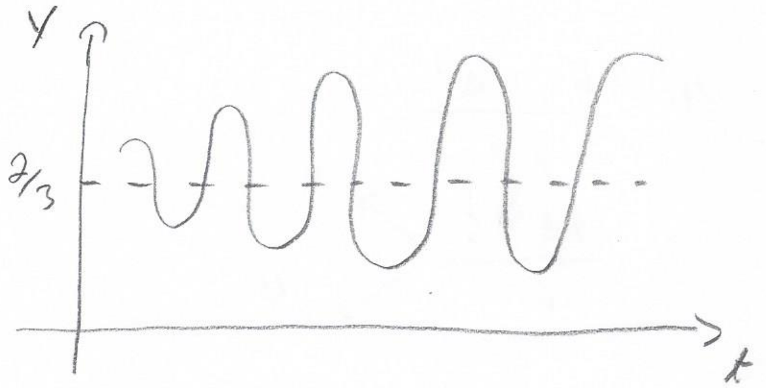
$3Y_p = 2 \therefore Y_p = \frac{2}{3}$

10

SOLUÇÃO GERAL:

$$Y = Y_H + Y_P$$

$$\boxed{= e^{t}\{A_1 \cos(1{,}4t) + A_2 \operatorname{sen}(1{,}4t)\} + \frac{2}{3}}$$



$\boxed{\text{DIVERGE}}$

$$\lim_{t \to \infty} Y = \infty$$

12.3 $Y'' + 3Y = 9$

RAÍZES CARACTERÍSTICAS:

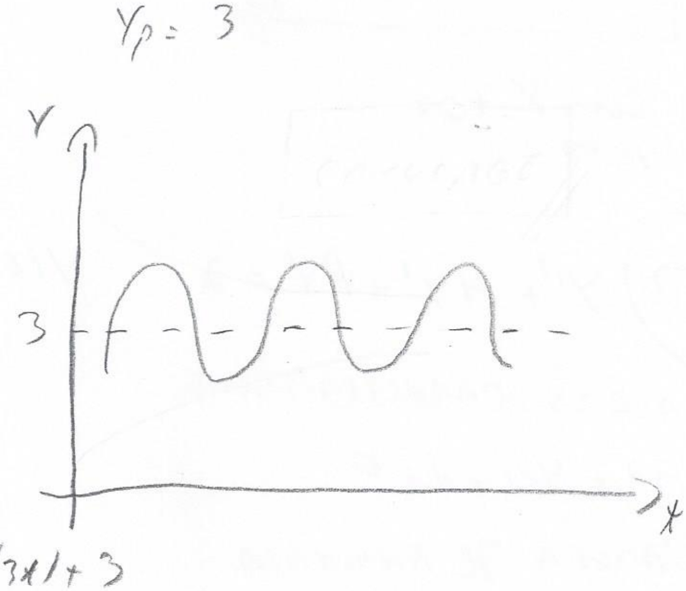
$$r^2 + 3 = 0 \qquad r = \pm 3i$$
$$r^2 = -3$$

SOLUÇÃO HOMOGÊNEA ($b=0$):

$$Y_H = e^{at}\{A_1 \cos(\omega t) + A_2 \operatorname{sen}(\omega t)\}$$
$$= e^{(0)t}\{A_1 \cos(3t) + A_2 \operatorname{sen}(3t)\}$$
$$= A_1 \cos(3t) + A_2 \operatorname{sen}(3t)$$

SOLUÇÃO PARTICULAR ($Y''=0$):

$$3Y_P = 9$$
$$Y_P = 3$$



SOLUÇÃO GERAL:

$$Y = Y_H + Y_P$$

$$\boxed{= A_1 \cos(3t) + A_2 \operatorname{sen}(3t) + 3}$$

$$\lim_{t \to \infty} Y = A_1 \cos(3t) + A_2 \operatorname{sen}(3t) + 3$$

12.C $3Y'' + 9Y' - 12Y = 36$ 10.A

RAÍZES CARACTERÍSTICAS:

$3r^2 + 9r - 12 = 0$

FÓRMULA DE BHASKARA:

$\Delta = b^2 - 4ac$

$= 81 + 144$

$= 225$

$r = \frac{-b \pm \sqrt{\Delta}}{2a}$

$= \frac{-9 \pm 15}{6} \nearrow 1, \searrow -4$

SOLUÇÃO HOMOGÊNEA ($b=0$):

$Y_H = A_1 e^{r_1 t} + A_2 e^{r_2 t}$

$= A_1 e^{t} + A_2 e^{-4t}$

SOLUÇÃO PARTICULAR ($Y' = 0$):

$-12 Y_p = 36$

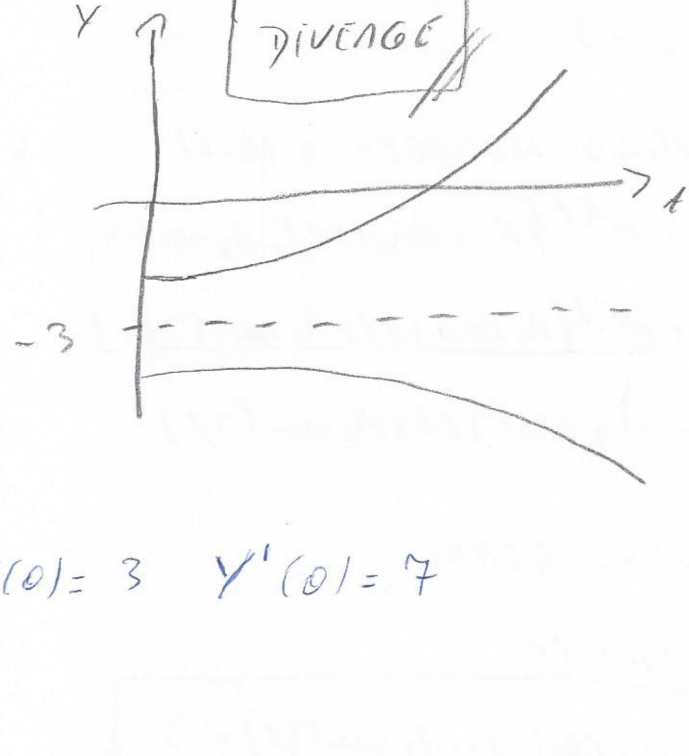
$Y_p = -3$

SOLUÇÃO GERAL:

$Y = Y_H + Y_P$

$\boxed{Y = A_1 e^{t} + A_2 e^{-4t} - 3}$

$\lim_{t \to \infty} Y = \pm\infty$



12.D $Y'' + 4Y' + 8Y = 2$ $Y(0) = 3$ $Y'(0) = 7$

RAÍZES CARACTERÍSTICAS:

$r^2 + 4r + 8 = 0$

FÓRMULA DE BHASKARA:

$\Delta = b^2 - 4ac$

$= 16 - 32 \rightarrow \Delta = -16$

$r = \frac{-b \pm \sqrt{\Delta}}{2a}$

$= \frac{-4 \pm 4i}{2} \nearrow r_1 = -2 + 2i, \searrow r_2 = -2 - 2i$

SOLUÇÃO HOMOGÊNEA $(b=0)$: $\quad r = a \pm \omega i$

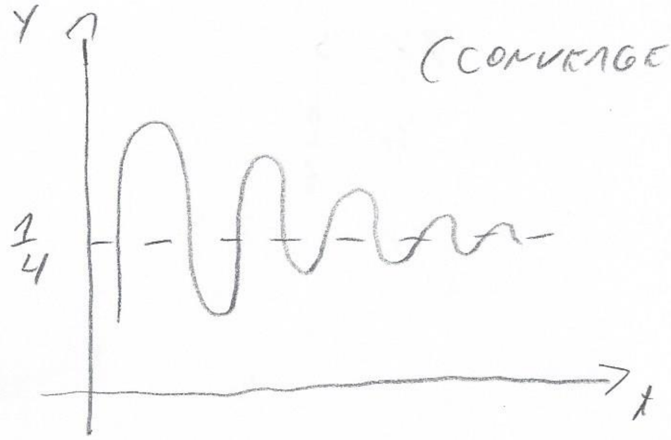
$$Y_H = e^{at}\{A_1 \cos(\omega t) + A_2 \operatorname{sen}(\omega t)\}$$
$$= e^{-2t}\{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\}$$

SOLUÇÃO PARTICULAR $(Y'=0)$:

$$8Y_p = 2$$
$$Y_p = \frac{1}{4}$$

SOLUÇÃO GERAL:

$$Y = Y_H + Y_P$$
$$\boxed{= e^{-2t}\{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\} + \frac{1}{4}}$$



(CONVERGE)

$$Y(0) = e^{-2(0)}\{A_1 \cos(2(0)) + A_2 \operatorname{sen}(2(0))\} + \frac{1}{4}$$
$$= A_1 + \frac{1}{4}$$

$$A_1 + \frac{1}{4} = 3$$
$$A_1 = 3 - \frac{1}{4} \quad \to \quad A_1 = \frac{11}{4}$$

$$Y' = e^{-2t}\{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\}' + \{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\}(e^{-2t})'$$
$$= e^{-2t}\{-8A_1 \operatorname{sen}(2t) + 8A_2 \cos(2t)\} + \{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\}(-2e^{-2t})$$
$$= e^{-2(0)}\{-8A_1 \operatorname{sen}(2(0)) + 8A_2 \cos(2(0))\} + \{A_1 \cos(2(0)) + A_2 \operatorname{sen}(2(0))\}(-2e^{-2(0)})$$
$$= 2A_2 - 2A_1$$

$$2A_2 - 2A_1 = 7$$
$$2A_2 - 2\left(\frac{11}{4}\right) = 7$$
$$2A_2 - \frac{11}{2} = 7 \quad \to \quad 2A_2 = 7 + \frac{11}{2}$$
$$A_2 = \frac{14 + 11}{4}$$
$$= \frac{25}{4}$$

$$\boxed{Y = e^{-2t}\left\{\frac{11}{4}\cos(2t) + \frac{23}{4}\operatorname{sen}(2t)\right\} + \frac{1}{4}}$$

$\lim_{t \to \infty} Y = \frac{1}{4}$

11.A

SOLUÇÃO DEFINIDA

12.E $\quad Y'' + 9Y = 3 \qquad Y(0) = 1 \qquad Y'(0) = 3$

$r^2 + 9 = 0 \rightarrow r = \pm 3i \qquad \lambda \pm \mu i$

$r^2 = -9$

SOLUÇÃO HOMOGÊNEA (b=0)

$$Y_H = e^{\lambda t}\{A_1 \cos(\mu t) + A_2 \operatorname{sen}(\mu t)\}$$

$$= e^{(0)t}\{A_1 \cos(3t) + A_2 \operatorname{sen}(3t)\}$$

$$= \{A_1 \cos(3t) + A_2 \operatorname{sen}(3t)\}$$

SOLUÇÃO PARTICULAR ($Y''=0$):

$$9Y_p = 3$$

$$Y_p = \frac{1}{3}$$

SOLUÇÃO GERAL:

$$Y = Y_H + Y_P$$

$$\boxed{= \{A_1 \cos(3t) + A_2 \operatorname{sen}(3t)\} + \frac{1}{3}}$$

$$Y' = \{-A_1 \operatorname{sen}(3t) + A_2 \cos(3t)\}$$

$$Y(0) = \{A_1 \cos(3(0)) + A_2 \operatorname{sen}(3(0))\} + \frac{1}{3}$$

$$= A_1 + \frac{1}{3}$$

$$1 = A_1 + \frac{1}{3}$$

$$A_1 = 1 - \frac{1}{3}$$

$$= \frac{2}{3}$$

$$Y'(0) = \{-A_1 \operatorname{sen}(3(0)) + A_2 \cos(3(0))\}$$

$$= A_2$$

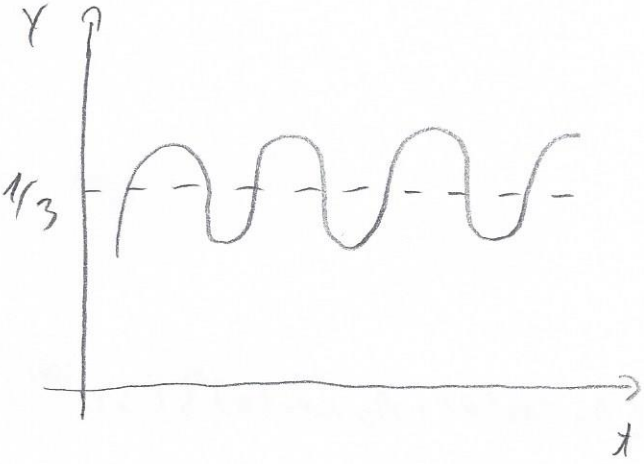
$$A_2 = 3$$

$$\boxed{Y = \left\{ \frac{2}{3} \cos(3t) + 3 \operatorname{sen}(3t) \right\} + \frac{1}{3}}$$

SOLUÇÃO DEFINIDA

12

$$\lim_{t \to \infty} Y = \left\{ \frac{2}{3} \cos(3t) + 3 \operatorname{sen}(3t) \right\} + \frac{1}{3}$$



12.F $2Y'' - 12Y' + 20Y = 40$ $\quad Y(0) = 4 \quad Y'(0) = 5$

$Y'' - 6Y' + 10Y = 20$ RAÍZES CARACTERÍSTICAS: $r^2 - 6r + 10 = 0$

$\alpha \pm \beta i$

FÓRMULA DE BHASKARA:

$$\Delta = b^2 - 4ac$$
$$= 36 - 40$$
$$= -4$$

$$r = \frac{-b \pm \sqrt{\Delta}}{2a}$$
$$= \frac{6 \pm 2i}{2} \quad r_1 = 3 + i, \quad r_2 = 3 - i$$

SOLUÇÃO HOMOGÊNEA ($b = 0$):

$$Y_H = e^{\alpha t} \{ A_1 \cos(\beta t) + A_2 \operatorname{sen}(\beta t) \}$$
$$= e^{3t} \{ A_1 \cos(t) + A_2 \operatorname{sen}(t) \}$$

SOLUÇÃO PARTICULAR ($Y' = 0$):

$$10 Y_p = 20$$
$$Y_p = 2$$

SOLUÇÃO GERAL:

$$Y = Y_H + Y_p$$
$$\boxed{= e^{3t} \{ A_1 \cos(t) + A_2 \operatorname{sen}(t) \} + 2}$$

(12.A)

$$Y' = e^{3t}\{A_1\cos(t) + A_2\,\text{sen}(t)\}' + \{A_1\cos(t) + A_2\,\text{sen}(t)\}(e^{3t})'$$
$$= e^{3t}\{-A_1\,\text{sen}(t) + A_2\cos(t)\} + \{A_1\cos(t) + A_2\,\text{sen}(t)\}(3e^{3t})$$

$$Y(0) = e^{3(0)}\{A_1\cos(0) + A_2\,\text{sen}(0)\} + 2$$
$$= A_1 + 2$$

$$4 = A_1 + 2$$

$$A_1 = 2$$

$$Y'(0) = e^{3(0)}\{-A_1\,\text{sen}(0) + A_2\cos(0)\} + \{A_1\cos(0) + A_2\,\text{sen}(0)\}(3e^{3(0)})$$
$$= A_2 + 3A_1$$

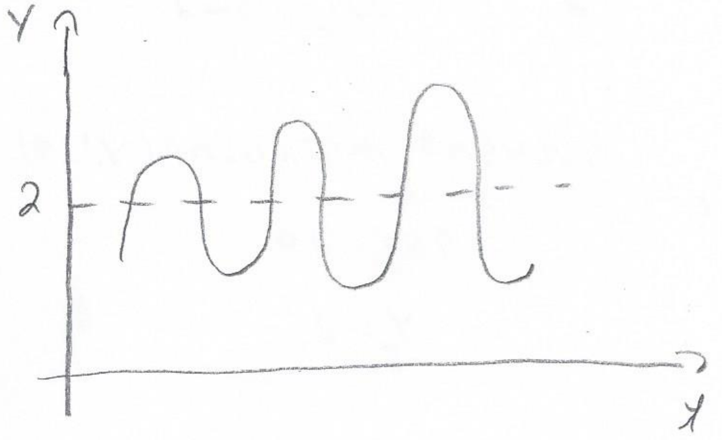
$$5 = A_2 + 3(2)$$

$$A_2 = -1$$

$$\boxed{Y = e^{3t}\{2\cos(t) - \text{sen}(t)\} + 2}$$

SOLUÇÃO DEFINIDA

$$\lim_{t\to\infty} Y = \infty$$



(12.6) $Y'' + Y' - 2Y = 0 \qquad Y(0) = 1 \qquad Y'(0) = 1$

RAÍZES CARACTERÍSTICAS:

$$r^2 + r - 2r = 0$$

13

FÓRMULA DE BHASKARA:

$$\Delta = b^2 - 4ac = 1 + 8 = 9$$

$$r = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-1 \pm 3}{2} \rightarrow r_1 = 1, \quad r_2 = -2$$

SOLUÇÃO HOMOGÊNEA ($b=0$):

$$Y_H = A_1 e^{r_1 t} + A_2 e^{r_2 t} = A_1 e^{t} + A_2 e^{-2t}$$

SOLUÇÃO PARTICULAR ($Y' = 0$):

$$-2Y_p = 0$$

$$Y_p = 0$$

SOLUÇÃO GERAL:

$$Y = Y_H + Y_P = A_1 e^{t} + A_2 e^{-2t} + 0 = \boxed{A_1 e^{t} + A_2 e^{-2t}}$$

$$Y' = A_1 e^{t} - 2A_2 e^{-2t}$$

$$Y(0) = A_1 e^{0} + A_2 e^{-2(0)}$$

$$1 = A_1 + A_2$$

$$A_1 + A_2 = 1$$

$$Y'(0) = A_1 e^{0} - 2A_2 e^{-2(0)}$$

$$1 = A_1 - 2A_2$$

$$A_1 - 2A_2 = 1$$

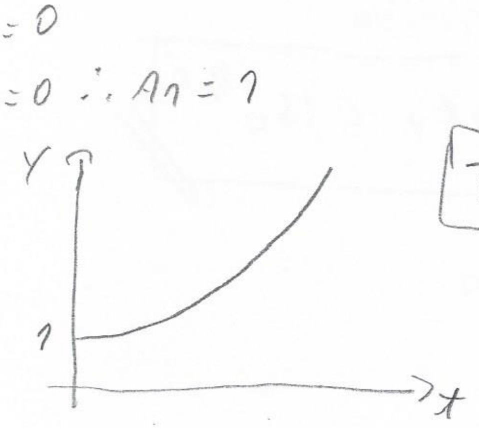
$$\begin{cases} A_1 + A_2 = 1 \\ A_1 - 2A_2 = 1 \quad \times(-1) \end{cases} \sim \begin{cases} \cancel{A_1} + A_2 = 1 \\ -\cancel{A_1} + 2A_2 = -1 \end{cases}$$

$$3A_2 = 0$$

$$A_2 = 0 \therefore A_1 = 1$$

SOLUÇÃO DEFINIDA:

$$\boxed{Y = e^{t}} \qquad \lim_{t \to \infty} Y = \infty$$



$\boxed{\text{DIVERGE}}$

12.H $4Y'' - Y = 0 \quad Y(-2) = 1 \quad Y'(-2) = -1$

13.A

$$Y'' - \frac{1}{4}Y = 0$$

RAÍZES CARACTERÍSTICAS:

$$r^2 - \frac{1}{4} = 0$$

$$r = \pm \frac{1}{2}$$

SOLUÇÃO GERAL:

$$Y = A_1 e^{r_1 t} + A_2 e^{r_2 t}$$

$$\boxed{= A_1 e^{1/2 t} + A_2 e^{-1/2 t}}$$

$$Y' = \frac{1}{2} A_1 e^{1/2 t} - \frac{1}{2} A_2 e^{-1/2 t}$$

$$Y(-2) = A_1 e^{1/2(-2)} + A_2 e^{-1/2(-2)}$$

$$1 = A_1 e^{-1} + A_2 e^{1}$$

$$A_1 e^{-1} + A_2 e^{1} = 1$$

$$0,4 A_1 + 2,7 A_2 = 1$$

$$Y'(-2) = \frac{1}{2} A_1 e^{1/2(-2)} - \frac{1}{2} A_2 e^{-1/2(-2)}$$

$$-1 = \frac{1}{2} A_1 e^{-1} - \frac{1}{2} A_2 e$$

$$0,2 A_1 - 1,4 A_2 = -1$$

$$\begin{cases} 0,4 A_1 + 2,7 A_2 = 1 \\ 0,2 A_1 - 1,4 A_2 = -1 \end{cases} \quad x(-2) \sim \begin{cases} \cancel{0,4 A_1} + 2,7 A_2 = 1 \\ \cancel{-0,4 A_1} + 2,8 A_2 = 2 \end{cases}$$

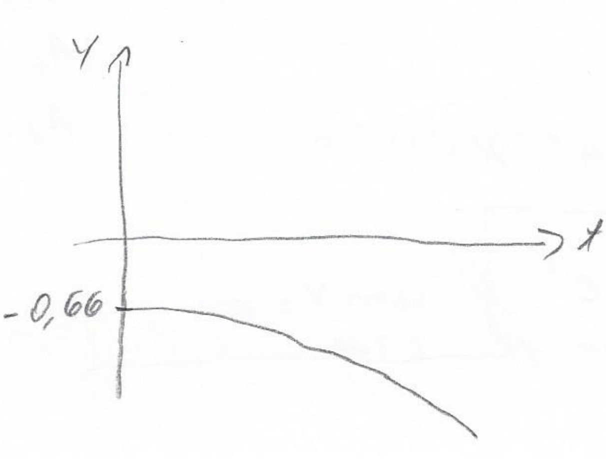
$$5,5 A_2 = 3$$

$$A_2 = 0,55 \therefore A_1 = -1,21$$

SOLUÇÃO DEFINIDA:

$$\boxed{Y = -1,21 e^{1/2 t} + 0,55 e^{-1/2 t}}$$

$$\lim_{t \to \infty} Y = -\infty$$



12.I $2Y'' + Y' - 4Y = 6 \quad Y(0) = 4 \quad Y'(0) = 0$ 14

$Y'' + \frac{1}{2}Y' - 2Y = 3$

RAIZES CARACTERÍSTICAS:

$r^2 + \frac{1}{2}r - 2 = 0$

FÓRMULA DE BHASKARA:

$\Delta = b^2 - 4ac$

$= \frac{1}{4} + 8$

$= \frac{33}{4}$

$r = \frac{-b \pm \sqrt{\Delta}}{2a}$

$= \frac{-0,5 \pm 2,9}{2}$ $\rightarrow r_1 = 1,2$; $\rightarrow r_2 = -1,7$

SOLUÇÃO HOMOGÊNEA ($b=0$):

$Y_H = A_1 e^{r_1 t} + A_2 e^{r_2 t}$

$= A_1 e^{1,2t} + A_2 e^{-1,7t}$

SOLUÇÃO PARTICULAR ($Y'=0$):

$-2Y_p = 3$

$Y_p = -1,5$

SOLUÇÃO GERAL:

$Y = Y_H + Y_p$

$\boxed{= A_1 e^{1,2t} + A_2 e^{-1,7t} - 1,5}$

$Y' = 1,2A_1 e^{1,2t} - 1,7A_2 e^{-1,7t}$

$Y'(0) = 1,2A_1 e^{1,2(0)} - 1,7A_2 e^{-1,7(0)}$

$0 = 1,2A_1 - 1,7A_2$

$1,2A_1 - 1,7A_2 = 0$

$1,2A_1 = 1,7A_2$

$A_1 = 1,4A_2$

$Y(0) = A_1 e^{1,2(0)} + A_2 e^{-1,7(0)} - 1,5$

$4 = A_1 + A_2 - 1,5$

$A_1 + A_2 = 5,5$

$1,4A_2 + A_2 = 5,5$

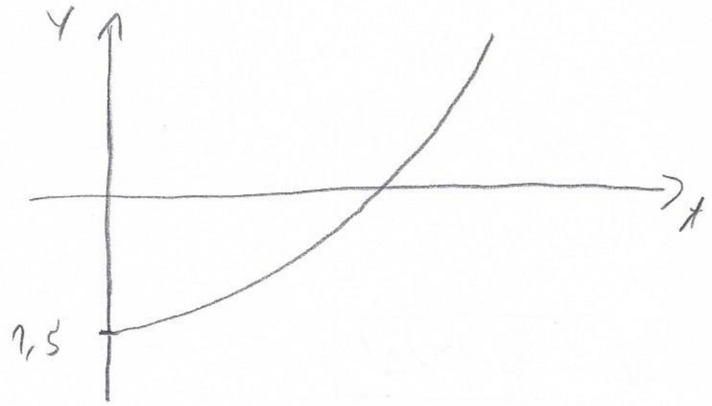
$A_2 = 2,3 \therefore A_1 = 3,2$

14.A

SOLUÇÃO DEFINIDA:

$$\boxed{Y = 3,2e^{1,2t} + 2,3e^{-2,7t} - 1,5}$$

$$\lim_{t \to \infty} Y = \infty$$



$\boxed{\text{DIVERGE}}$

12.J $Y'' - 2Y' + 5Y = 0$ $\quad Y\left(\frac{\pi}{2}\right) = 0 \quad Y'\left(\frac{\pi}{2}\right) = 2$

RAIZES CARACTERÍSTICAS:

$r^2 - 2r + 5 = 0$

FÓRMULA DE BHASKARA:

$\lambda \pm \mu i$

$\Delta = b^2 - 4ac$

$= 4 - 20$

$= -16$

$r = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$= \dfrac{2 \pm 4i}{2}$ $\nearrow r_1 = 1 + 2i$ $\searrow r_2 = 1 - 2i$

SOLUÇÃO GERAL:

$Y = e^{\lambda t}\{A_1 \cos(\mu t) + A_2 \operatorname{sen}(\mu t)\}$

$$\boxed{= e^{t}\{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\}}$$

$Y' = e^{t}\{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\}' + \{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\} \cdot (e^{t})'$

$= e^{t}\{-2A_1 \operatorname{sen}(2t) + 2A_2 \cos(2t)\} + \{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\}(e^{t})$

15

$$Y\left(\frac{\pi}{2}\right) = e^{\frac{\pi}{2}}\{A_1 \cos(\pi) + A_2 \operatorname{sen}(\pi)\}$$

$$0 = e^{\pi/2}\{A_1(-1) + A_2(0)\}$$

$$0 = e^{\pi/2}(-A_1)$$

$$A_1 = 0$$

$$Y'\left(\frac{\pi}{2}\right) = e^{\pi/2}\{-2A_1 \operatorname{sen}(\pi) + 2A_2 \cos(\pi)\} + \{A_1 \cos(\pi) + A_2 \operatorname{sen}(\pi)\}e^{\pi/2}$$

$$2 = e^{\pi/2}\{0 - 2A_2\} + \{-A_1 + 0\}e^{\pi/2}$$

$$2 = -9{,}6A_2 - 4{,}8A_1$$

$$2{,}4A_1 + 4{,}8A_2 = -1$$

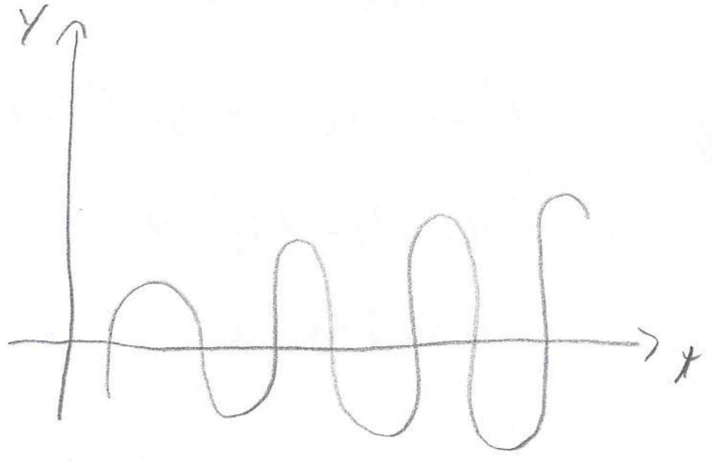
$$2{,}4(0) + 4{,}8A_2 = -1$$

$$A_2 = -0{,}2$$

SOLUÇÃO DEFINIDA:

$$\boxed{Y = e^{x}\{-0{,}2 \operatorname{sen}(2x)\}}$$

$$\lim_{x \to \infty} Y = \infty$$



$\boxed{\text{DIVERGE}}$

13.A

EXERCÍCIO 13:

13.A $Y'' + Y' - 2Y = 2t \qquad Y(0) = 0 \qquad Y'(0) = 1$

RAÍZES CARACTERÍSTICAS:

$r^2 + r - 2 = 0$

FÓRMULA DE BHASKARA:

$\Delta = b^2 - 4ac$

$= 1 + 8$

$= 9$

$r = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$= \dfrac{-1 \pm 3}{2}$ → $r_1 = 1$, → $r_2 = -2$

SOLUÇÃO HOMOGÊNEA:

$Y_H = A_1 e^{r_1 t} + A_2 e^{r_2 t}$

$= A_1 e^{t} + A_2 e^{-2t}$

SOLUÇÃO PARTICULAR:

$Y_p = A_1 t + A_2 \quad Y_p' = A_1 \quad Y_p'' = 0$

$0 + A_1 - 2(A_1 t + A_2) = 2t$

$A_1 - 2A_1 t - 2A_2 = 2t$

$A_1 - 2A_1 t = 2A_2 + 2t$

$A_1 = -1 \qquad \therefore Y_p = -t - \frac{1}{2}$

$A_2 = \frac{1}{2} A_1 \therefore A_2 = -\frac{1}{2}$

SOLUÇÃO GERAL:

$Y = Y_H + Y_p$

$\boxed{= A_1 e^{t} + A_2 e^{-2t} - t - \frac{1}{2}}$

$Y(0) = A_1 e^{0} + A_2 e^{-2(0)} - 0 - \frac{1}{2}$

$0 = A_1 + A_2 - \frac{1}{2}$

$A_1 + A_2 = \frac{1}{2}$

$Y' = A_1 e^{t} - 2A_2 e^{2t} - 1$

$Y'(0) = A_1 e^{0} - 2A_2 e^{-2(0)} - 1$

$1 = A_1 - 2A_2 - 1$

$A_1 - 2A_2 = 2$

16

$$\begin{cases} A_1 - 2A_2 = 2 \\ A_1 + A_2 = 1/2 \quad \times(-1) \end{cases} \sim \begin{cases} A_1 - 2A_2 = 2 \\ -A_1 - A_2 = -\frac{1}{2} \end{cases}$$

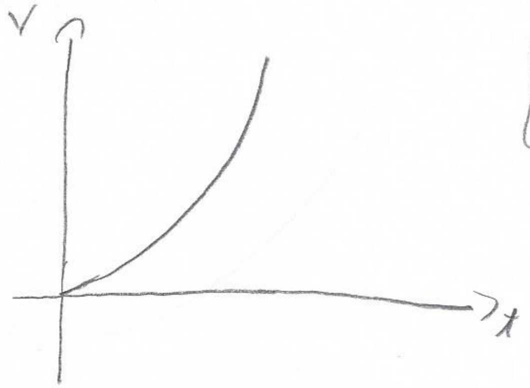
$$-3A_2 = \frac{3}{2}$$

$$A_2 = -\frac{1}{2} \therefore A_1 = 1$$

SOLUÇÃO DEFINIDA:

$$\boxed{Y = e^{t} - \frac{1}{2}e^{-2t} - t - \frac{1}{2}}$$

$$\lim_{t \to \infty} Y = \infty$$



$\boxed{\text{DIVERGE}}$

13.B $Y'' + Y' + 2Y = e^{t}$ $\quad Y(0) = 1 \quad Y'(0) = 0$

RAÍZES CARACTERÍSTICAS:

$$r^2 + r + 2 = 0$$

FÓRMULA DE BHASKARA:

$$\Delta = b^2 - 4ac \qquad r = \frac{-b \pm \sqrt{\Delta}}{2a} \qquad \lambda \pm \mu i$$

$$= 1 - 8$$

$$= -7 \qquad = \frac{-1 \pm 2,6i}{2} \to r_1 = -0,5 + 1,3i, \; r_2 = -0,5 - 1,3i$$

SOLUÇÃO HOMOGÊNEA ($b=0$):

$$Y_H = e^{\lambda t}\{A_1 \cos(\mu t) + A_2 \operatorname{sen}(\mu t)\}$$

$$= e^{-0,5t}\{A_1 \cos(1,3t) + A_2 \operatorname{sen}(1,3t)\}$$

SOLUÇÃO PARTICULAR:

$$Y_P = Ae^{t} \quad Y'_P = Ae^{t} \quad Y''_P = Ae^{t}$$

$$Ae^{t} + Ae^{t} + 2Ae^{t} = e^{t}$$

$$(A + A + 2A)e^{t} = e^{t}$$

$$A = 1/4 \therefore Y_P = \frac{1}{4}e^{t}$$

16.A

SOLUÇÃO GERAL:

$Y = Y_H + Y_P$

$$\boxed{= e^{-0,5t}\{A_1 \cos(1,3t) + A_2 \operatorname{sen}(1,3t)\} + \frac{1}{4} e^t}$$

$$Y' = e^{-0,5t}\{A_1 \cos(1,3t) + A_2 \operatorname{sen}(1,3t)\}' + \{A_1 \cos(1,3t) + A_2 \operatorname{sen}(1,3t)\}(e^{-0,5t})' + \frac{1}{4}e^t$$

$$= e^{-0,5t}\{-1,3A_1 \operatorname{sen}(1,3t) + 1,3A_2 \cos(1,3t)\} + \{A_1 \cos(1,3t) + A_2 \operatorname{sen}(1,3t)\}\left(-\frac{1}{2}e^{-0,5t}\right) + \frac{1}{4}e^t$$

$$Y'(0) = e^{-0,5(0)}\{-1,3A_1 \overbrace{\operatorname{sen}(1,3(0))}^{0} + 1,3A_2 \cos(1,3(0))\} + \{A_1 \cos(1,3(0)) + A_2 \overbrace{\operatorname{sen}(1,3(0))}^{0}\}\left(-\frac{1}{2}e^{-0,5(0)}\right) + \frac{1}{4}e^0$$

$$0 = 1,3A_2 - \frac{1}{2}A_1 + \frac{1}{4}$$

$$\frac{1}{2}A_1 - 1,3A_2 = \frac{1}{4}$$

$$Y(0) = e^{-0,5(0)}\{A_1 \cos(1,3(0)) + A_2 \overbrace{\operatorname{sen}(1,3(0))}^{0}\} + \frac{1}{4}e^0$$

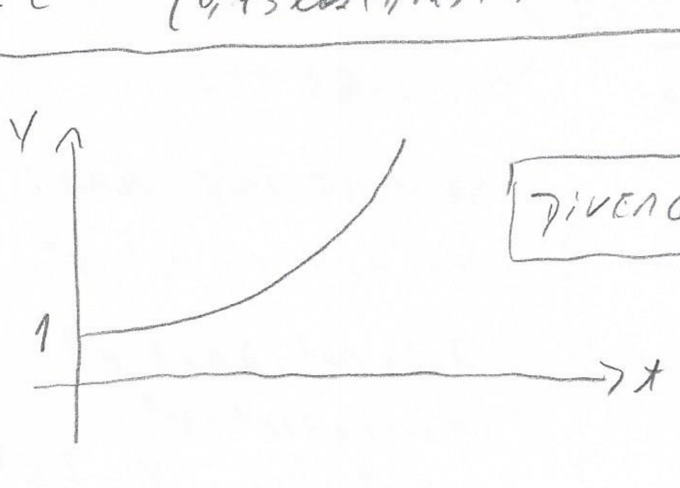
$$1 = A_1 + \frac{1}{4}$$

$$A_1 = \frac{3}{4} \therefore A_2 = 0,1$$

SOLUÇÃO DEFINIDA:

$$\boxed{Y = e^{-0,5t}\{0,75 \cos(1,3t) + 0,1 \operatorname{sen}(1,3t)\} + 0,25e^t}$$

$$\lim_{t \to \infty} Y = \infty$$



$\boxed{\text{DIVERGE}}$

13.C $y'' + 4y = t^2 + 3e^t$ $\quad y(0) = 0 \quad y'(0) = 2$ (17)

RAIZES CARACTERÍSTICAS:

$r^2 + 4 = 0$ $\qquad \lambda \pm \mu i$

$r = \pm 2i$

SOLUÇÃO HOMOGÊNEA ($b=0$):

$y_H = e^{\lambda t}\{A_1 \cos(\mu t) + A_2 \operatorname{sen}(\mu t)\}$

$= e^{(0)t}\{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\}$

$= \{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\}$

SOLUÇÃO PARTICULAR:

$y_p = At^2 + Be^t - \frac{1}{2}A \qquad y'_p = 2At + Be^t \qquad y''_p = 2A + Be^t$

$2A + Be^t + 4\left(At^2 + Be^t - \frac{1}{2}A\right) = t^2 + 3e^t$

$\cancel{2A} + Be^t + 4At^2 + 4Be^t - \cancel{2A} = t^2 + 3e^t$

$4At^2 + 5Be^t = t^2 + 3e^t$

$4A = 1 \qquad 5B = 3 \qquad \therefore \; y_p = \frac{1}{4}t^2 + \frac{3}{5}e^t - \frac{1}{8}$

$A = \frac{1}{4} \qquad B = \frac{3}{5}$

SOLUÇÃO GERAL:

$y = y_H + y_p$

$\boxed{= \{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\} + \frac{1}{4}t^2 + \frac{3}{5}e^t - \frac{1}{8}}$ //

$y' = \{-2A_1 \operatorname{sen}(2t) + 2A_2 \cos(2t)\} + \frac{1}{2}t + \frac{3}{5}e^t$

$y(0) = \{-2A_1 \cancel{\operatorname{sen}}^{\,0}(2(0)) + 2A_2 \cos(2(0))\} + \frac{1}{2}(0) + \frac{3}{5}e^0$

17.A

$$2 = 2A_2 + \frac{3}{5} \qquad Y(0) = \{$$

$$2A_2 = \frac{7}{5}$$

$$A_2 = 0{,}7$$

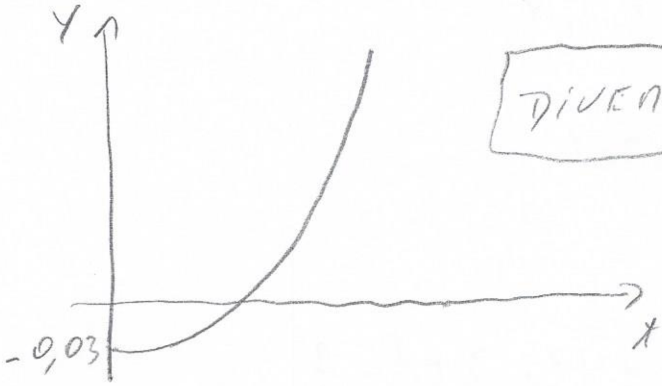
$$Y(0) = \{A_1 \cos(2(0)) + A_2 \cancel{\operatorname{sen}(2(0))}^{\,0}\} + \frac{1}{4}(0)^2 + \frac{3}{5}e^{(0)} - \frac{1}{8}$$

$$0 = A_1 + \frac{3}{5} - \frac{1}{8}$$

$$A_1 = -0{,}5$$

SOLUÇÃO DEFINIDA:

$$\boxed{Y = \{-0{,}5\cos(2t) + 0{,}7\operatorname{sen}(2t)\} + \frac{1}{4}t^2 + \frac{3}{5}e^t - \frac{1}{8}} \qquad \lim_{t\to\infty} Y = \infty$$



DIVERGE

18.D $Y'' - 2Y' + Y = te^t + 4 \qquad Y(0) = 1 \qquad Y'(0) = 1$

RAIZES CARACTERÍSTICAS:

$$m^2 - 2m + 1 = 0$$

FÓRMULA DE BHASKARA:

$$\Delta = b^2 - 4ac \qquad m = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$= 4 - 4$$

$$= 0 \qquad = \frac{2 \pm 0}{2} \to m_1 = 1, \; m_2 = 1$$

SOLUÇÃO HOMOGÊNEA (b=0):

$$Y_H = A_1 e^{r_1 t} + A_2 e^{r_2 t}$$

$$= A_1 e^{2t} + A_2 e^{2t}$$

SOLUÇÃO PARTICULAR:

$Y_p = A t e^{t} + B \quad Y'_p = A t e^{t} + A e^{t} \quad Y''_p = A t e^{t} + 2 A e^{t}$

$A t e^{t} + 2 A e^{t} - 2 A t e^{t} - 2 A e^{t} + A t e^{t} + B$

13.E $y'' - 2y' - 3y = 3xe^{2x}$ $y(0) = 1$ $y'(0) = 0$

18.A

RAÍZES CARACTERÍSTICAS:

$r^2 - 2r - 3 = 0$

FÓRMULA DE BHASKARA:

$\Delta = b^2 - 4ac$

$= 4 + 12$

$= 16$

$r = \frac{-b \pm \sqrt{\Delta}}{2a}$

$= \frac{2 \pm 4}{2}$ $\nearrow r_1 = 3$, $\searrow r_2 = -1$

SOLUÇÃO HOMOGÊNEA ($b = 0$):

$y_H = A_1 e^{r_1 x} + A_2 e^{r_2 x}$

$= A_1 e^{3x} + A_2 e^{-x}$

SOLUÇÃO PARTICULAR:

$y_p = Axe^{2x}$ $y_p' = 2Ae^{2x}$

13.F $y'' + 4y = 3 \operatorname{sen} 2t \qquad y(0) = 2 \qquad y'(0) = -1$ 19

RAÍZES CARACTERÍSTICAS:

$r^2 + 4 = 0 \qquad \lambda \pm \mu i$

$r = \pm 2i$

SOLUÇÃO HOMOGÊNEA ($b=0$):

$$y_H = e^{\lambda t}\{A_1 \cos(\mu t) + A_2 \operatorname{sen}(\mu t)\}$$
$$= e^{(0)t}\{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\}$$
$$= \{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\}$$

SOLUÇÃO PARTICULAR:

$$y_p = A t \cos 2t$$
$$y'_p = -2At \operatorname{sen} 2t + A \cos 2t$$
$$y''_p = -4At \cos 2t - 2A \operatorname{sen} 2t - 2A \operatorname{sen} 2t$$
$$= -4At \cos 2t - 4A \operatorname{sen} 2t$$
$$-4At \cancel{\cos 2t} - 4A \operatorname{sen} 2t + 4t \cancel{\cos 2t} = 3 \operatorname{sen} 2t$$
$$-4A \cancel{\operatorname{sen} 2t} = 3 \cancel{\operatorname{sen} 2t}$$
$$A = -\frac{3}{4} \therefore y_p = -\frac{3}{4} t \cos 2t$$

SOLUÇÃO GERAL:

$$y = y_H + y_p$$
$$\boxed{= \{A_1 \cos(2t) + A_2 \operatorname{sen}(2t)\} - \frac{3}{4} t \cos(2t)}$$

$$y' = \{-2A_1 \operatorname{sen}(2t) + 2A_2 \cos(2t)\} + \frac{3}{2} t \operatorname{sen}(2t) - \frac{3}{4} \cos(2t)$$

$$y'(0) = \{-2A_1 \cancelto{0}{\operatorname{sen}(2(0))} + 2A_2 \cos(2(0))\} + \frac{3}{2}(0) \cancelto{0}{\operatorname{sen}(2(0))} - \frac{3}{4} \cos(2(0))$$

$$-1 = 2A_2 - \frac{3}{4}$$

$$2A_2 = \frac{1}{4}$$

$$A_2 = -0{,}125$$

$$Y(0) = \{A_1 \cos(2(0)) + A_2 \cancel{sen}(2(0))^{0}\} - \frac{3}{4}(0)\cos(2(0))$$

19.A

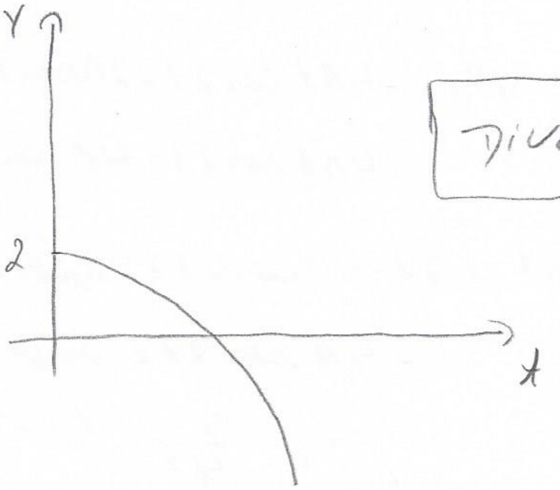
$2 = A_1$

$A_1 = 2$

SOLUÇÃO DEFINIDA:

$$\boxed{Y = \{2\cos(2t) - 0,125\,sen(2t)\} - \frac{3}{4}\,t\cos(2t)}$$

$\lim_{t \to \infty} Y = -\infty$



$\boxed{\text{DIVERGE}}$

13.G $\quad Y'' + 2Y' + 5Y = 4e^{t}\cos 2t \qquad Y(0) = 1 \quad Y'(0) = 1$

RAIZES CARACTERÍSTICAS:

$r^2 + 2r + 5 = 0$

FÓRMULA DE BHASKARA:

$\Delta = b^2 - 4ac \qquad r = \dfrac{-b \pm \sqrt{\Delta}}{2a}$ $\qquad \alpha \pm \beta i$

$= 4 - 20$

$= -16 \qquad = \dfrac{-2 \pm 4i}{2}$ $\quad \nearrow r_1 = -1 + 2i$, $\quad \searrow r_2 = -1 - 2i$

SOLUÇÃO HOMOGÊNEA (b=0):

$Y_H = e^{\alpha t}\{A_1 \cos(\beta t) + A_2\, sen(\beta t)\}$

$= e^{-t}\{A_1 \cos(2t) + A_2\, sen(2t)\}$

SOLUÇÃO PARTICULAR:

$Y_p = Ae^x \cos 2x \quad Y'_p = -2Ae^x \operatorname{sen} 2x + Ae^x \cos 2x$

$Y''_p = -4Ae^x \cos 2x - 2Ae^x \operatorname{sen} 2x - 2Ae^x \operatorname{sen} 2x + Ae^x \cos 2x$

$= -3Ae^x \cos 2x - 4Ae^x \operatorname{sen} 2x$

$Y_p = Ae^x \operatorname{sen} 2x \quad Y'_p = 2Ae^x \cos 2x + Ae^x \operatorname{sen} 2x$

$-3Ae^x \cos 2x - 4Ae^x \operatorname{sen} 2x - 4Ae^x \operatorname{sen} 2x + 2Ae^x \cos 2x + 5e^x \cos 2x$

$2Ae^x \cos 2x + Ae^x \operatorname{sen} 2x$

$Y''_p = -4Ae^x \operatorname{sen} 2x + 2Ae^x \cancel{\cos} 2x + 2A\cancel{e^x} \cos 2x + Ae^x \operatorname{sen} 2x$

$= -3Ae^x \operatorname{sen} 2x + 4Ae^x \cos 2x$

$-3Ae^x \operatorname{sen} 2x + 4Ae^x \cos 2x + 2Ae^x \operatorname{sen} 2x + 5Ae^x \operatorname{sen} 2x$