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PROBLEMA DE MAXIMIZAÇÃO DA UTILIDADE - PMU:

$$\max_{x, y} [\beta(x - x_x)^{\rho} + (1 - \beta)(y - y_y)^{\rho}]^{1/\rho} \quad \text{s.t.} \quad p_x \cdot x + p_y \cdot y = w$$

$$x, y \geq 0$$

LES-CES (LISTA III)

MONITANDO A LAGRANGIANA:

$$\mathcal{L}(x, y, \lambda) = [\beta(x - x_x)^{\rho} + (1 - \beta)(y - y_y)^{\rho}]^{1/\rho} + \lambda(w - p_x \cdot x - p_y \cdot y)$$

CONDIÇÕES DE PRIMEIRA DERIVADA:

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{1}{\rho} [\beta(x - x_x)^{\rho} + (1 - \beta)(y - y_y)^{\rho}]^{1/\rho - 1} [\beta \rho (x - x_x)^{\rho - 1}] - \lambda \cdot p_x = 0$$

ENTÃO,

$$\frac{1}{\rho} [\beta(x - x_x)^{\rho} + (1 - \beta)(y - y_y)^{\rho}]^{1/\rho - 1} (\rho \cdot \beta (x - x_x)^{\rho - 1}) - \lambda \cdot p_x = 0$$

$$\lambda = \frac{\beta (x - x_x)^{\rho - 1} [\beta(x - x_x)^{\rho} + (1 - \beta)(y - y_y)^{\rho}]^{1/\rho - 1}}{p_x} \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial y} = \frac{1}{\rho} [\beta(x - x_x)^{\rho} + (1 - \beta)(y - y_y)^{\rho}]^{1/\rho - 1} [(1 - \beta) \rho (y - y_y)^{\rho - 1}] - \lambda \cdot p_y = 0$$

ENTÃO,

$$\frac{1}{\rho} [\beta(x - x_x)^{\rho} + (1 - \beta)(y - y_y)^{\rho}]^{1/\rho - 1} ((1 - \beta) \rho (y - y_y)^{\rho - 1}) - \lambda \cdot p_y = 0$$

$$\lambda = \frac{(1 - \beta)(y - y_y)^{\rho - 1} [\beta(x - x_x)^{\rho} + (1 - \beta)(y - y_y)^{\rho}]^{1/\rho - 1}}{p_y} \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = w - p_x \cdot x - p_y \cdot y = 0 \quad \therefore \quad w = p_x \cdot x + p_y \cdot y \quad (3)$$

IGUALANDO (1) E (2):

$$\frac{\beta (x - x_x)^{\rho - 1} [\beta(x - x_x)^{\rho} + (1 - \beta)(y - y_y)^{\rho}]^{1/\rho - 1}}{p_x} = \frac{(1 - \beta)(y - y_y)^{\rho - 1} [\beta(x - x_x)^{\rho} + (1 - \beta)(y - y_y)^{\rho}]^{1/\rho - 1}}{p_y}$$

$$(x - x_x)^{\rho - 1} = \frac{1 - \beta}{\beta} \cdot \frac{p_x}{p_y} \cdot (y - y_y)^{\rho - 1}$$

$$x - \delta_x = \left(\frac{p_x}{p_y} \cdot \frac{1-\beta}{\beta} \right)^{\frac{1}{1-\alpha}} (y - \delta_y)$$

$$x - \delta_x = \left(\frac{p_y}{p_x} \cdot \frac{\beta}{1-\beta} \right)^{\frac{1}{1-\alpha}} (y - \delta_y)$$

$$\phi = \frac{1}{1-\alpha}, \quad \frac{\beta}{1-\beta} = A \Leftrightarrow \frac{p_y}{p_x} = P$$

$$x - \delta_x = (P \cdot A)^{\phi} (y - \delta_y)$$

$$x = P^{\phi} \cdot A^{\phi} \cdot (y - \delta_y) + \delta_x$$

(4)

SUBSTITUINDO (4) EM (3):

$$W = p_x [P^{\phi} \cdot A^{\phi} \cdot (y - \delta_y) + \delta_x] + p_y \cdot y$$

$$= p_x \cdot P^{\phi} \cdot A^{\phi} \cdot (y - \delta_y) + p_x \cdot \delta_x + p_y \cdot y$$

$$= p_x \cdot P^{\phi} \cdot A^{\phi} \cdot y - p_x \cdot P^{\phi} \cdot A^{\phi} \cdot \delta_y + p_x \cdot \delta_x + p_y \cdot y$$

$$= [p_x \cdot P^{\phi} \cdot A^{\phi} + p_y] y - p_x \cdot P^{\phi} \cdot A^{\phi} \cdot \delta_y + p_x \cdot \delta_x$$

$$= [p_x \cdot P^{\phi} \cdot A^{\phi} + p_y] y - p_x \cdot P^{\phi} \cdot A^{\phi} \cdot \delta_y - \delta_y p_y + \delta_y p_y + p_x \cdot \delta_x$$

$$= [p_x \cdot P^{\phi} \cdot A^{\phi} + p_y] y - [p_x \cdot P^{\phi} \cdot A^{\phi} + p_y] \delta_y + \delta_y p_y + p_x \cdot \delta_x$$

$$= [p_x \cdot P^{\phi} \cdot A^{\phi} + p_y] (y - \delta_y) + p_y \cdot \delta_y + p_x \cdot \delta_x$$

$$y - \delta_y = \frac{W - p_y \cdot \delta_y - p_x \cdot \delta_x}{p_x \cdot P^{\phi} \cdot A^{\phi} + p_y}$$

(4.1)

$$y = \frac{W - p_y \cdot \delta_y - p_x \cdot \delta_x}{p_x \cdot P^{\phi} \cdot A^{\phi} + p_y} + \delta_y$$

(5)

SUBSTITUINDO (4.1) EM (4):

$$x = P^{\phi} \cdot A^{\phi} \cdot \frac{W - p_y \cdot \delta_y - p_x \cdot \delta_x}{p_x \cdot P^{\phi} \cdot A^{\phi} + p_y} + \delta_x$$

(2)

$$X = \frac{w - P_Y \cdot \delta_Y - P_X \cdot \delta_X}{(P^{-\phi} A^{-\phi})(P_X P^{\phi} A^{\phi} + P_Y)} + \delta_X$$

$$X = \frac{w - P_Y \cdot \delta_Y - P_X \cdot \delta_X}{P_X + P_Y P^{-\phi} A^{-\phi}} + \delta_X$$

ENCONTRANDO A FUNÇÃO UTILIDADE INDIVIDUAL $= u(P, w)$:

$$u(P, w) = \left\{ \beta \left(\frac{w - P_Y \cdot \delta_Y - P_X \cdot \delta_X}{P_X + P_Y P^{-\phi} A^{-\phi}} + \delta_X - \delta_X \right)^{\rho} + (1-\beta) \left(\frac{w - P_Y \cdot \delta_Y - P_X \cdot \delta_X}{P_X P^{\phi} A^{\phi} + P_Y} + \delta_Y - \delta_Y \right)^{\rho} \right\}^{\frac{1}{\rho}}$$

$$= \left\{ \beta (w - P_Y \cdot \delta_Y - P_X \cdot \delta_X)^{\rho} \left(\frac{1}{P_X + P_Y P^{-\phi} A^{-\phi}} \right)^{\rho} + (1-\beta) (w - P_Y \cdot \delta_Y - P_X \cdot \delta_X)^{\rho} \left(\frac{1}{P_X P^{\phi} A^{\phi} + P_Y} \right)^{\rho} \right\}^{\frac{1}{\rho}}$$

$$= (w - P_X \cdot \delta_X - P_Y \cdot \delta_Y) \left\{ \beta \left(\frac{1}{(P_X P^{\phi} A^{\phi} + P_Y) \frac{1}{P^{\phi} A^{\phi}}} \right)^{\rho} + (1-\beta) \left(\frac{1}{P_X P^{\phi} A^{\phi} + P_Y} \right)^{\rho} \right\}^{\frac{1}{\rho}}$$

$$= (w - P_X \cdot \delta_X - P_Y \cdot \delta_Y) \left\{ \beta \left(\frac{P^{\phi} A^{\phi}}{P_X P^{\phi} A^{\phi} + P_Y} \right)^{\rho} + (1-\beta) \left(\frac{1}{P_X P^{\phi} A^{\phi} + P_Y} \right)^{\rho} \right\}^{\frac{1}{\rho}}$$

$$= (w - P_X \cdot \delta_X - P_Y \cdot \delta_Y) \left\{ P^{\phi \rho} A^{\phi \rho} \beta \left(\frac{1}{P_X P^{\phi} A^{\phi} + P_Y} \right)^{\rho} + (1-\beta) \left(\frac{1}{P_X P^{\phi} A^{\phi} + P_Y} \right)^{\rho} \right\}^{\frac{1}{\rho}}$$

$$= (w - P_X \cdot \delta_X - P_Y \cdot \delta_Y) \left\{ \left(\frac{1}{P_X P^{\phi} A^{\phi} + P_Y} \right)^{\rho} (P^{\phi \rho} A^{\phi \rho} \beta + 1 - \beta) \right\}^{\frac{1}{\rho}}$$

$$= \frac{w - P_X \cdot \delta_X - P_Y \cdot \delta_Y}{P_X P^{\phi} A^{\phi} + P_Y} (P^{\phi \rho} A^{\phi \rho} \beta + 1 - \beta)^{\frac{1}{\rho}}$$

$(1-\beta)^{\frac{1}{\rho}} (P^{\phi-1} A^{\phi-1} \beta + 1)^{\frac{1}{\rho}}$
 $(1-\beta)^{\frac{1}{\rho}} (P^{\phi-1} A^{\phi-1} A + 1)^{\frac{1}{\rho}}$
 $(1-\beta)^{\frac{1}{\rho}} (P^{\phi-1} A^{\phi} + 1)^{\frac{1}{\rho}}$

$P_Y \left(\frac{P_X P^{\phi} A^{\phi} + 1}{P_Y} \right)^{\frac{1}{\rho}}$
 $P_Y (P^{-1} P^{\phi} A^{\phi} + 1)^{\frac{1}{\rho}}$

$$(p^{\phi p} A^{\phi p} \beta + 1 - \beta)^{1/p} = \left[\frac{(1-\beta)^{1/p} (p^{\phi p} A^{\phi p} \beta + 1)}{1-\beta} \right]^{1/p}$$

$$= (1-\beta)^{1/p} \left(p^{\phi p} A^{\phi p} \frac{\beta}{1-\beta} + 1 \right)^{1/p}$$

$$= (1-\beta)^{1/p} (p^{\phi p} A^{\phi p} A + 1)^{1/p}$$

$$\phi = \frac{1}{1-\rho}$$

$$= (1-\beta)^{1/p} (p^{\phi-1} A^{\phi-1} A + 1)^{1/p}$$

$$\phi - \phi\rho = 1$$

$$= (1-\beta)^{1/p} (p^{\phi-1} A^{\phi} + 1)^{1/p}$$

$$\phi - 1 = \phi\rho$$

$$\mathcal{L}(W) = \frac{W - P_X X - P_Y Y}{P_X p^{\phi} A^{\phi} + P_Y} \cdot (1-\beta)^{1/p} (p^{\phi-1} A^{\phi} + 1)^{1/p}$$

$$= \frac{W - P_X X - P_Y Y \cdot (1-\beta)^{1/p} (p^{\phi-1} A^{\phi} + 1)^{1/p}}{P_Y (P_X p^{\phi} A^{\phi} + 1)}$$

$$= \frac{W - P_X X - P_Y Y \cdot (1-\beta)^{1/p} (p^{\phi-1} A^{\phi} + 1)^{1/p}}{P_Y (p^{\phi-1} P_X A^{\phi} + 1)}$$

$$= \frac{W - P_X X - P_Y Y \cdot (1-\beta)^{1/p} (p^{\phi-1} A^{\phi} + 1)^{1/p}}{P_Y (p^{\phi-1} A^{\phi} + 1)}$$

$$= \frac{(W - P_X X - P_Y Y) \cdot (1-\beta)^{1/p} (p^{\phi-1} A^{\phi} + 1)^{1/p-1}}{P_Y}$$

$$= \frac{(W - P_X X - P_Y Y) \cdot (1-\beta)^{1/p} (p^{\phi-1} A^{\phi} + 1)^{\frac{1-\rho}{p}}}{P_Y}$$

$$= (W - P_X X - P_Y Y) \cdot (1-\beta)^{1/p} (p^{\phi-1} A^{\phi} + 1)^{\frac{1-\rho}{p}} P_Y^{-1}$$

$$= (W - P_X X - P_Y Y) \cdot \left[(1-\beta)^{\frac{1}{p}} p^{\phi-1} A^{\phi} P_Y^{\frac{\rho}{p}} + (1-\beta)^{\frac{1}{p}} P_Y^{\frac{\rho}{p}} \right]^{\frac{1-\rho}{p}}$$

$$= (W - P_X X - P_Y Y) \cdot \left[(1-\beta)^{\frac{1}{p}} p^{\phi-1} A^{\phi} P_Y^{1-\phi} + (1-\beta)^{\frac{1}{p}} P_Y^{1-\phi} \right]^{\frac{1-\rho}{p}}$$

$$= (\cdot) \left[\frac{(1-\beta)^{\frac{1}{p}} p^{\phi-1} A^{\phi} P_Y^{1-\phi}}{P_X^{\phi-1} (1-\beta)^{\frac{1}{p}}} + (1-\beta)^{\frac{1}{p}} P_Y^{1-\phi} \right]^{\frac{1-\rho}{p}}$$

$$w(P, W) = (W - P_x \cdot X - P_y \cdot Y) (\beta^0 \cdot P_x^{1-\beta} + (1-\beta)^0 \cdot P_y^{1-\beta})^{\frac{1-\beta}{\beta}}$$

$$= (W - P_x \cdot X - P_y \cdot Y) \cdot (\beta^0 \cdot P_x^{1-\beta} + (1-\beta)^0 \cdot P_y^{1-\beta})^{\frac{1}{\beta-1}}$$

(3)

$$(X - \delta_x)^{1-\beta} = \frac{1-\beta}{\beta} \cdot \frac{P_x}{P_y} \cdot (Y - \delta_y)^{1-\beta}$$

$$X - \delta_x = \left(\frac{1-\beta}{\beta} \cdot \frac{P_x}{P_y} \right)^{\frac{1}{1-\beta}} \cdot (Y - \delta_y)$$

$$X - \delta_x = \left(\frac{\beta}{1-\beta} \cdot \frac{P_y}{P_x} \right)^{\frac{1}{1-\beta}} \cdot (Y - \delta_y)$$

$$b = \frac{1}{1-\beta}, \quad A = \frac{\beta}{1-\beta} \quad \text{and} \quad P = \frac{P_y}{P_x}$$

$$X - \delta_x = A^b \cdot P^b \cdot (Y - \delta_y)$$

$$\cancel{X - \delta_x = A^b \cdot P^b \cdot Y - A^b \cdot P^b \cdot \delta_y}$$

$$X = A^b \cdot P^b \cdot (Y - \delta_y) + \delta_x$$

(4)

SUBSTITUTING (4) IN (3):

$$W = P_x [A^b \cdot P^b \cdot (Y - \delta_y) + \delta_x] + P_y \cdot Y$$

$$W = P_x \cdot A^b \cdot P^b \cdot (Y - \delta_y) + P_x \cdot \delta_x + P_y \cdot Y$$

$$W = P_x \cdot A^b \cdot P^b \cdot Y - P_x \cdot A^b \cdot P^b \cdot \delta_y + P_x \cdot \delta_x + P_y \cdot Y$$

$$W = [P_x \cdot A^b \cdot P^b + P_y] \cdot Y - P_x \cdot A^b \cdot P^b \cdot \delta_y + P_x \cdot \delta_x$$

$$W = [P_x \cdot A^b \cdot P^b + P_y] \cdot Y - P_x \cdot A^b \cdot P^b \cdot \delta_y - P_y \cdot \delta_y + P_y \cdot \delta_y + P_x \cdot \delta_x$$

$$W = [P_x \cdot A^b \cdot P^b + P_y] Y - [P_x \cdot A^b \cdot P^b + P_y] \delta_y + P_y \cdot \delta_y + P_x \cdot \delta_x$$

$$W = (P_x \cdot A^b \cdot P^b + P_y) \cdot (Y - \delta_y) + P_y \cdot \delta_y + P_x \cdot \delta_x$$

$$Y - \delta_y = \frac{W - P_x \cdot \delta_x - P_y \cdot \delta_y}{P_x \cdot A^b \cdot P^b + P_y}$$

$$P_y = \frac{W - P_x \cdot \delta_x - P_y \cdot \delta_y}{P_x \cdot A^b \cdot P^b + P_y} + \delta_y$$

(3)

SUBSTITUINDO (3) EM (4):

$$X = A^{\frac{1}{\sigma}} \cdot P^{\frac{\sigma}{1-\sigma}} \left(\frac{W - P_Y \cdot Y - P_X \cdot X}{P_X \cdot A^{\frac{1}{\sigma}} P^{\frac{\sigma}{1-\sigma}} + P_Y} + X_Y - X_X \right) + X_X$$

$$= \frac{W - P_Y \cdot Y - P_X \cdot X}{A^{\frac{1}{\sigma}} \cdot P^{\frac{\sigma}{1-\sigma}} (P_X \cdot A^{\frac{1}{\sigma}} P^{\frac{\sigma}{1-\sigma}} + P_Y)} + X_X$$

$$\boxed{= \frac{W - P_Y \cdot Y - P_X \cdot X}{P_X + P_Y \cdot A^{-\frac{1}{\sigma}} \cdot P^{-\frac{\sigma}{1-\sigma}}} + X_X}$$

ENCONTRANDO A FUNÇÃO UTILIDADE INDIVIDUAL - $u(P, W)$:

$$u(P, W) = \left\{ \beta \left(\frac{W - P_Y \cdot Y - P_X \cdot X}{P_X + P_Y \cdot A^{-\frac{1}{\sigma}} \cdot P^{-\frac{\sigma}{1-\sigma}}} + X_X - X_X \right)^{\rho} + (1-\beta) \left(\frac{W - P_Y \cdot Y - P_X \cdot X}{P_X \cdot A^{\frac{1}{\sigma}} P^{\frac{\sigma}{1-\sigma}} + P_Y} + X_X - X_X \right)^{\rho} \right\}^{\frac{1}{1-\rho}}$$

$$= \left\{ \beta \cdot (W - P_Y \cdot Y - P_X \cdot X)^{\rho} \left(\frac{1}{P_X + P_Y \cdot A^{-\frac{1}{\sigma}} \cdot P^{-\frac{\sigma}{1-\sigma}}} \right)^{\rho} + (1-\beta) \cdot (W - P_Y \cdot Y - P_X \cdot X)^{\rho} \left(\frac{1}{P_X \cdot A^{\frac{1}{\sigma}} P^{\frac{\sigma}{1-\sigma}} + P_Y} \right)^{\rho} \right\}^{\frac{1}{1-\rho}}$$

$$= (W - P_Y \cdot Y - P_X \cdot X) \left\{ \beta \left(\frac{1}{(A^{\frac{1}{\sigma}} P^{\frac{\sigma}{1-\sigma}} P_X + P_Y) \left(\frac{1}{A^{\frac{1}{\sigma}} P^{\frac{\sigma}{1-\sigma}}} \right)} \right)^{\rho} + (1-\beta) \left(\frac{1}{P_X \cdot A^{\frac{1}{\sigma}} P^{\frac{\sigma}{1-\sigma}} + P_Y} \right)^{\rho} \right\}^{\frac{1}{1-\rho}}$$

$$= (\cdot) \left\{ \beta \left(\frac{A^{\frac{1}{\sigma}} P^{\frac{\sigma}{1-\sigma}}}{P_X \cdot A^{\frac{1}{\sigma}} P^{\frac{\sigma}{1-\sigma}} + P_Y} \right)^{\rho} + (1-\beta) \left(\frac{1}{P_X \cdot A^{\frac{1}{\sigma}} P^{\frac{\sigma}{1-\sigma}} + P_Y} \right)^{\rho} \right\}^{\frac{1}{1-\rho}}$$

$$= (\cdot) \left\{ \beta \cdot A^{\frac{\rho}{\sigma}} P^{\frac{\rho \sigma}{1-\sigma}} \left(\frac{1}{P_X \cdot A^{\frac{1}{\sigma}} P^{\frac{\sigma}{1-\sigma}} + P_Y} \right)^{\rho} + (1-\beta) \left(\frac{1}{P_X \cdot A^{\frac{1}{\sigma}} P^{\frac{\sigma}{1-\sigma}} + P_Y} \right)^{\rho} \right\}^{\frac{1}{1-\rho}}$$

$$= (\cdot) \left\{ \left(\frac{1}{P_X \cdot A^{\frac{1}{\sigma}} P^{\frac{\sigma}{1-\sigma}} + P_Y} \right)^{\rho} (\beta \cdot A^{\frac{\rho}{\sigma}} P^{\frac{\rho \sigma}{1-\sigma}} + 1 - \beta) \right\}^{\frac{1}{1-\rho}}$$

(4)

$$u(p, w) = \frac{w - p_y \cdot x_y - p_x \cdot x_x}{p_x \cdot A^{\frac{1}{\sigma}} \cdot p^{\frac{\sigma-1}{\sigma}} + p_y} (\beta \cdot A^{\frac{1}{\sigma}} \cdot p^{\frac{\sigma-1}{\sigma}} + 1 - \beta)^{\frac{1}{1-\sigma}}$$

$$= (-) (1 - \beta)^{\frac{1}{1-\sigma}} \left(\frac{\beta}{1 - \beta} \cdot A^{\frac{1}{\sigma}} \cdot p^{\frac{\sigma-1}{\sigma}} + 1 \right)^{\frac{1}{1-\sigma}}$$

$$= (-) (1 - \beta)^{\frac{1}{1-\sigma}} (A \cdot A^{\frac{1}{\sigma}} \cdot p^{\frac{\sigma-1}{\sigma}} + 1)^{\frac{1}{1-\sigma}}$$

$$= (-) (1 - \beta)^{\frac{1}{1-\sigma}} (A \cdot A^{\frac{1}{\sigma} - 1} \cdot p^{\frac{\sigma-1}{\sigma}} + 1)^{\frac{1}{1-\sigma}}$$

$$= (-) (1 - \beta)^{\frac{1}{1-\sigma}} (A^{\frac{1}{\sigma}} \cdot p^{\frac{\sigma-1}{\sigma}} + 1)^{\frac{1}{1-\sigma}}$$

$$= \frac{w - p_y \cdot x_y - p_x \cdot x_x}{p_y \left(\frac{p_x}{p_y} \cdot A^{\frac{1}{\sigma}} \cdot p^{\frac{\sigma-1}{\sigma}} + 1 \right)} (1 - \beta)^{\frac{1}{1-\sigma}} (A^{\frac{1}{\sigma}} \cdot p^{\frac{\sigma-1}{\sigma}} + 1)^{\frac{1}{1-\sigma}}$$

$$= \frac{w - p_y \cdot x_y - p_x \cdot x_x}{p_y (p^{-\frac{1}{\sigma}} \cdot A^{\frac{1}{\sigma}} \cdot p^{\frac{\sigma-1}{\sigma}} + 1)} (1 - \beta)^{\frac{1}{1-\sigma}} (A^{\frac{1}{\sigma}} \cdot p^{\frac{\sigma-1}{\sigma}} + 1)^{\frac{1}{1-\sigma}}$$

$$= \frac{w - p_y \cdot x_y - p_x \cdot x_x}{p_y (A^{\frac{1}{\sigma}} \cdot p^{\frac{\sigma-1}{\sigma}} + 1)} (1 - \beta)^{\frac{1}{1-\sigma}} (A^{\frac{1}{\sigma}} \cdot p^{\frac{\sigma-1}{\sigma}} + 1)^{\frac{1-\sigma}{\sigma}} \cdot p_y^{-1}$$

$$= (w - p_y \cdot x_y - p_x \cdot x_x) \cdot (1 - \beta)^{\frac{1}{1-\sigma}} (A^{\frac{1}{\sigma}} \cdot p^{\frac{\sigma-1}{\sigma}} + 1)^{\frac{1-\sigma}{\sigma}} \cdot p_y^{-1}$$
$$= (w - p_y \cdot x_y - p_x \cdot x_x) \cdot \left[(1 - \beta)^{\frac{1}{1-\sigma}} A^{\frac{1}{\sigma}} \cdot p^{\frac{\sigma-1}{\sigma}} + (1 - \beta)^{\frac{1}{1-\sigma}} \right]^{\frac{1-\sigma}{\sigma}} \cdot p_y^{-1}$$

$$= (-) \cdot \left[(1 - \beta)^{\frac{1}{\sigma}} \cdot A^{\frac{1}{\sigma}} \cdot p^{\frac{\sigma-1}{\sigma}} \cdot p_y^{\frac{1-\sigma}{\sigma}} + (1 - \beta)^{\frac{1}{\sigma}} \cdot p_y^{\frac{1-\sigma}{\sigma}} \right]^{\frac{1-\sigma}{\sigma}}$$

$$= (-) \cdot \left[\frac{(1 - \beta)^{\frac{1}{\sigma}} \cdot p^{\frac{\sigma-1}{\sigma}} \cdot p_y^{\frac{1-\sigma}{\sigma}}}{(1 - \beta)^{\frac{1}{\sigma}} \cdot p_x^{\frac{\sigma-1}{\sigma}}} \cdot p_y^{\frac{1-\sigma}{\sigma}} + (1 - \beta)^{\frac{1}{\sigma}} \cdot p_y^{\frac{1-\sigma}{\sigma}} \right]^{\frac{1-\sigma}{\sigma}}$$

$$= (-) \left[\beta^{\frac{1}{\sigma}} \cdot p_x^{\frac{\sigma-1}{\sigma}} + (1 - \beta)^{\frac{1}{\sigma}} \cdot p_y^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1-\sigma}{\sigma}}$$

$$= (w - p_y \cdot x_y - p_x \cdot x_x) \cdot \left[\beta^{\frac{1}{\sigma}} \cdot p_x^{\frac{\sigma-1}{\sigma}} + (1 - \beta)^{\frac{1}{\sigma}} \cdot p_y^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1-\sigma}{\sigma}}$$

$$(Y - \delta_Y)^{1-\beta} = \frac{1-\beta}{\beta} \cdot \frac{P_X}{P_Y} \cdot (Y - \delta_Y)^{1-\beta}$$

LES-CES (LISA III)

$$Y - \delta_Y = \left(\frac{1-\beta}{\beta} \cdot \frac{P_X}{P_Y} \right)^{\frac{1}{1-\beta}} \cdot (Y - \delta_Y)$$

$$Y - \delta_Y = \left(\frac{\beta}{1-\beta} \cdot \frac{P_Y}{P_X} \right)^{\frac{1}{1-\beta}} \cdot (Y - \delta_Y)$$

$$\theta = \frac{1}{1-\beta}, \quad A = \frac{\beta}{1-\beta} \quad \text{E} \quad P = \frac{P_Y}{P_X}$$

$$Y - \delta_Y = A^\theta \cdot P^\theta \cdot (Y - \delta_Y)$$

$$Y = A^\theta \cdot P^\theta \cdot (Y - \delta_Y) + \delta_Y$$

(4)

SUBSTITUINDO (4) EM (3):

$$W = P_X \cdot [A^\theta \cdot P^\theta \cdot (Y - \delta_Y) + \delta_Y] + P_Y \cdot Y$$

$$W = P_X \cdot A^\theta \cdot P^\theta \cdot (Y - \delta_Y) + P_X \cdot \delta_Y + P_Y \cdot Y$$

$$W = P_X \cdot A^\theta \cdot P^\theta \cdot Y - P_X \cdot A^\theta \cdot P^\theta \cdot \delta_Y + P_X \cdot \delta_Y + P_Y \cdot Y$$

$$W = (P_X \cdot A^\theta \cdot P^\theta + P_Y) \cdot Y - P_X \cdot A^\theta \cdot P^\theta \cdot \delta_Y + P_X \cdot \delta_Y$$

$$W = (P_X \cdot A^\theta \cdot P^\theta + P_Y) \cdot Y - P_X \cdot A^\theta \cdot P^\theta \cdot \delta_Y - P_Y \cdot \delta_Y + P_Y \cdot \delta_Y + P_X \cdot \delta_Y$$

$$W = (P_X \cdot A^\theta \cdot P^\theta + P_Y) \cdot Y - (P_X \cdot A^\theta \cdot P^\theta + P_Y) \cdot \delta_Y + P_Y \cdot \delta_Y + P_X \cdot \delta_Y$$

$$W = (P_X \cdot A^\theta \cdot P^\theta + P_Y) \cdot (Y - \delta_Y) + P_Y \cdot \delta_Y + P_X \cdot \delta_Y$$

$$Y - \delta_Y = \frac{W - P_X \cdot \delta_Y - P_Y \cdot \delta_Y}{P_X \cdot A^\theta \cdot P^\theta + P_Y}$$

$$Y = \frac{W - P_X \cdot \delta_Y - P_Y \cdot \delta_Y}{P_X \cdot A^\theta \cdot P^\theta + P_Y} + \delta_Y$$

(5)

SUBSTITUINDO (5) EM (4):

$$Y = A^\theta \cdot P^\theta \cdot \left(\frac{W - P_X \cdot \delta_Y - P_Y \cdot \delta_Y}{P_X \cdot A^\theta \cdot P^\theta + P_Y} + \delta_Y \right) + \delta_Y$$

$$= \frac{W - P_X \cdot \delta_Y - P_Y \cdot \delta_Y}{A^\theta \cdot P^\theta \cdot (P_X \cdot A^\theta \cdot P^\theta + P_Y)} + \delta_Y$$

$$X = \frac{w - P_x \cdot x_x - P_y \cdot x_y}{P_x + P_y \cdot A^{-\phi} \cdot P^{-\phi}} + x_x$$

(6)

ENCONTRAR A FUNÇÃO UTILIDADE INDIRETA - $v(P, w)$:

$$v(P, w) = \left\{ \beta \left(\frac{w - P_x \cdot x_x - P_y \cdot x_y}{P_x + P_y \cdot A^{-\phi} \cdot P^{-\phi}} + x_x - x_x \right)^{\rho} + (1-\beta) \left(\frac{w - P_x \cdot x_x - P_y \cdot x_y}{P_x \cdot A^{\phi} \cdot P^{\phi} + P_y} + x_y - x_y \right)^{\rho} \right\}^{1/\rho}$$

$$= \left\{ \beta \left(\frac{w - P_x \cdot x_x - P_y \cdot x_y}{(P_x \cdot A^{\phi} \cdot P^{\phi} + P_y) \left(\frac{A^{\phi} \cdot P^{\phi}}{A^{\phi} \cdot P^{\phi}} \right)} \right)^{\rho} + (1-\beta) \left(\frac{w - P_x \cdot x_x - P_y \cdot x_y}{P_x \cdot A^{\phi} \cdot P^{\phi} + P_y} \right)^{\rho} \right\}^{1/\rho}$$

$$= \left\{ (w - P_x \cdot x_x - P_y \cdot x_y)^{\rho} \left(\frac{A^{\phi} \cdot P^{\phi}}{P_x \cdot A^{\phi} \cdot P^{\phi} + P_y} \right)^{\rho} \beta + (1-\beta) \left(\frac{1}{P_x \cdot A^{\phi} \cdot P^{\phi} + P_y} \right)^{\rho} \right\}^{1/\rho}$$

$$= (w - P_x \cdot x_x - P_y \cdot x_y) \left\{ \left(\frac{1}{P_x \cdot A^{\phi} \cdot P^{\phi} + P_y} \right)^{\rho} [\beta (A^{\phi} \cdot P^{\phi})^{\rho} + (1-\beta)] \right\}^{1/\rho}$$

$$= \frac{w - P_x \cdot x_x - P_y \cdot x_y}{P_x \cdot A^{\phi} \cdot P^{\phi} + P_y} (A^{\phi} \cdot P^{\phi} \cdot \beta + 1 - \beta)^{1/\rho}$$

$$= \frac{w - P_x \cdot x_x - P_y \cdot x_y}{P_x \cdot A^{\phi} \cdot P^{\phi} + P_y} \cdot (1-\beta)^{1/\rho} \cdot \left(A^{\phi} \cdot P^{\phi} \cdot \frac{\beta}{1-\beta} + 1 \right)^{1/\rho} \quad \begin{aligned} \phi &= \frac{1}{1-\rho} \\ \phi - \phi\rho &= 1 \\ \phi\rho &= \phi - 1 \end{aligned}$$

$$= (\cdot) (1-\beta)^{1/\rho} (A^{\phi-1} \cdot P^{\phi-1} \cdot A + 1)^{1/\rho}$$

$$= (\cdot) (1-\beta)^{1/\rho} (A^{\phi} \cdot P^{\phi-1} + 1)^{1/\rho}$$

$$= \frac{w - P_x \cdot x_x - P_y \cdot x_y}{P_y \left(\frac{P_x \cdot A^{\phi} \cdot P^{\phi}}{P_y} + 1 \right)} (1-\beta)^{1/\rho} (A^{\phi} \cdot P^{\phi-1} + 1)^{1/\rho}$$

$$= \frac{w - P_x \cdot x_x - P_y \cdot x_y}{(P^{-1} \cdot A^{\phi} \cdot P^{\phi} + 1)} \cdot (1-\beta)^{1/\rho} \cdot (A^{\phi} \cdot P^{\phi-1} + 1)^{1/\rho} \cdot P_y^{-1}$$

$$= \frac{w - P_x \cdot x_x - P_y \cdot x_y}{A^{\phi} \cdot P^{\phi-1} + 1} \cdot (1-\beta)^{1/\rho} \cdot (A^{\phi} \cdot P^{\phi-1} + 1)^{1/\rho} \cdot P_y^{-1}$$

(2)

$$\begin{aligned}
\mathcal{L}(P, W) &= (W - P_X \cdot \delta_X - P_Y \cdot \delta_Y) (1-\beta)^{\frac{1}{1-\rho}} \cdot (A^\phi \cdot P^{\phi-1} + 1)^{\frac{1-\rho}{\phi}} \cdot P_Y^{-1} \\
&= (W - P_X \cdot \delta_X - P_Y \cdot \delta_Y) \left[(1-\beta)^{\frac{1}{1-\rho}} \cdot A^\phi \cdot P^{\phi-1} \cdot P_Y^{\frac{\rho}{1-\rho}} + (1-\beta)^{\frac{1}{1-\rho}} \cdot P_Y^{\frac{\rho}{1-\rho}} \right]^{\frac{1-\rho}{\rho}} \\
&= (-) \left[\cancel{(1-\beta)^\phi} \cdot \frac{\beta^\phi}{\cancel{(1-\beta)^\phi}} \cdot \frac{P_X^{\phi-1}}{P_X^{\phi-1}} \cdot \cancel{P_Y^{\phi-1}} + (1-\beta)^\phi \cdot P_Y^{\phi-1} \right]^{\frac{1-\rho}{\rho}} \\
&= (-) \cdot \left[\beta^\phi \cdot P_X^{\phi-1} + (1-\beta)^\phi \cdot P_Y^{\phi-1} \right]^{\frac{1-\rho}{\rho}} \\
&= (W - P_X \cdot \delta_X - P_Y \cdot \delta_Y) (\beta^\phi \cdot P_X^{\phi-1} + (1-\beta)^\phi \cdot P_Y^{\phi-1})^{\frac{1}{\phi-1}}
\end{aligned}$$

$$|X - \delta_X|^{\rho-1} = \frac{1-\beta}{\beta} \cdot \frac{P_X}{P_Y} \cdot (Y - \delta_Y)^{\rho-1}$$

$$X - \delta_X = \left(\frac{1-\beta}{\beta} \cdot \frac{P_X}{P_Y} \right)^{\frac{1}{\rho-1}} \cdot (Y - \delta_Y)$$

$$X - \delta_X = \left(\frac{\beta}{1-\beta} \cdot \frac{P_Y}{P_X} \right)^{\frac{1}{1-\rho}} \cdot (Y - \delta_Y)$$

$$\phi = \frac{1}{1-\rho}, \quad A = \frac{\beta}{1-\beta} \quad \text{e} \quad P = \frac{P_Y}{P_X}$$

$$X - \delta_X = A^\phi \cdot P^\phi \cdot (Y - \delta_Y)$$

$$X = A^\phi \cdot P^\phi \cdot (Y - \delta_Y) + \delta_X \tag{4}$$

SUBSTITUINDO (4) EM (3):

$$W = P_X \cdot [A^\phi \cdot P^\phi \cdot (Y - \delta_Y) + \delta_X] + P_Y \cdot Y$$

$$W = P_X \cdot A^\phi \cdot P^\phi \cdot (Y - \delta_Y) + P_X \cdot \delta_X + P_Y \cdot Y$$

$$W = P_X \cdot A^\phi \cdot P^\phi \cdot Y - P_X \cdot A^\phi \cdot P^\phi \cdot \delta_Y + P_X \cdot \delta_X + P_Y \cdot Y$$

$$W = (P_X \cdot A^\phi \cdot P^\phi + P_Y) Y - P_X \cdot A^\phi \cdot P^\phi \cdot \delta_Y + P_X \cdot \delta_X$$

$$W = (P_X \cdot A^\phi \cdot P^\phi + P_Y) Y - P_X \cdot A^\phi \cdot P^\phi \cdot \delta_Y - P_Y \cdot \delta_Y + P_Y \cdot \delta_Y + P_X \cdot \delta_X$$

$$W = (P_X \cdot A^\phi \cdot P^\phi + P_Y) Y - (P_X \cdot A^\phi \cdot P^\phi + P_Y) \delta_Y + P_Y \cdot \delta_Y + P_X \cdot \delta_X$$

$$W = (P_X \cdot A^\phi \cdot P^\phi + P_Y) \cdot (Y - \delta_Y) + P_Y \cdot \delta_Y + P_X \cdot \delta_X$$

$$Y - \delta_Y = \frac{W - P_X \cdot \delta_X - P_Y \cdot \delta_Y}{P_X \cdot A^q \cdot P^q + P_Y}$$

$$Y = \frac{W - P_X \cdot \delta_X - P_Y \cdot \delta_Y}{P_X \cdot A^q \cdot P^q + P_Y} + \delta_Y$$

(5)

SUBSTITUINDO (5) EM (4):

$$X = A^q \cdot P^q \cdot \left(\frac{W - P_X \cdot \delta_X - P_Y \cdot \delta_Y}{P_X \cdot A^q \cdot P^q + P_Y} + \delta_X - \delta_X \right) + \delta_X$$

$$= \frac{W - P_X \cdot \delta_X - P_Y \cdot \delta_Y}{A^q \cdot P^q \cdot (P_X \cdot A^q \cdot P^q + P_Y)} + \delta_X$$

$$= \frac{W - P_X \cdot \delta_X - P_Y \cdot \delta_Y}{P_X + P_Y \cdot A^{-q} \cdot P^{-q}} + \delta_X$$

(6)

ENCONTRANDO A FUNÇÃO DE UTILIDADE INFINITA - $u(P, W)$:

$$u(P, W) = \left\{ \beta \left[\frac{W - P_X \cdot \delta_X - P_Y \cdot \delta_Y}{P_X + P_Y \cdot A^{-q} \cdot P^{-q}} + \delta_X - \delta_X \right]^p + (1 - \beta) \left(\frac{W - P_X \cdot \delta_X - P_Y \cdot \delta_Y}{P_X \cdot A^q \cdot P^q + P_Y} + \delta_X - \delta_X \right)^p \right\}^{1/p}$$

$$= \left\{ (W - P_X \cdot \delta_X - P_Y \cdot \delta_Y)^p \left[\beta \left(\frac{1}{P_X + P_Y \cdot A^{-q} \cdot P^{-q}} \right)^p + (1 - \beta) \left(\frac{1}{P_X \cdot A^q \cdot P^q + P_Y} \right)^p \right] \right\}^{1/p}$$

$$= (W - P_X \cdot \delta_X - P_Y \cdot \delta_Y) \left\{ \beta \left[\frac{1}{(P_X \cdot A^q \cdot P^q + P_Y) \cdot A^{-q} \cdot P^{-q}} \right]^p + (1 - \beta) \left(\frac{1}{P_X \cdot A^q \cdot P^q + P_Y} \right)^p \right\}^{1/p}$$

$$= (W - P_X \cdot \delta_X - P_Y \cdot \delta_Y) \left\{ \left(\frac{1}{P_X \cdot A^q \cdot P^q + P_Y} \right)^p \cdot [\beta \cdot (A^q \cdot P^q)^p + (1 - \beta)] \right\}^{1/p}$$

$$= \frac{W - P_X \cdot \delta_X - P_Y \cdot \delta_Y}{P_X \cdot A^q \cdot P^q + P_Y} \cdot (\beta \cdot A^q \cdot P^q + 1 - \beta)^{1/p}$$

$$q = \frac{1}{1 - p}$$

$$q - qp = 1$$

$$qp = q - 1$$

(3)

$$\pi(P, w) = (\cdot) \cdot (1-\beta)^{1/\rho} \cdot \left(\frac{\beta}{1-\beta} \cdot A^{\sigma-1} \cdot P^{\sigma-1} + 1 \right)^{1/\rho}$$

$$= (\cdot) \cdot (1-\beta)^{1/\rho} \cdot (A \cdot A^{\sigma-1} \cdot P^{\sigma-1} + 1)^{1/\rho}$$

$$= (\cdot) \cdot (1-\beta)^{1/\rho} \cdot (A^{\sigma} \cdot P^{\sigma-1} + 1)^{1/\rho}$$

$$= \frac{w - P_x \cdot x - P_y \cdot y}{P_y \left(\frac{P_x \cdot A^{\sigma} \cdot P^{\sigma-1} + 1}{P_y} \right)} \cdot (1-\beta)^{1/\rho} \cdot (A^{\sigma} \cdot P^{\sigma-1} + 1)^{1/\rho}$$

$$= \frac{w - P_x \cdot x - P_y \cdot y}{P_y (P^{-1} \cdot A^{\sigma} \cdot P^{\sigma} + 1)} \cdot (1-\beta)^{1/\rho} \cdot (A^{\sigma} \cdot P^{\sigma-1} + 1)^{1/\rho}$$

$$= (w - P_x \cdot x - P_y \cdot y) \cdot (1-\beta)^{1/\rho} \cdot (A^{\sigma} \cdot P^{\sigma-1} + 1)^{\frac{1-\rho}{\rho}} \cdot P_y^{-1}$$

$$= (w - P_x \cdot x - P_y \cdot y) \cdot \left[(1-\beta)^{\frac{1}{\rho}} \cdot A^{\sigma} \cdot P^{\sigma-1} \cdot P_y^{\frac{\rho}{\rho-1}} + (1-\beta)^{\frac{1}{\rho}} \cdot P_y^{\frac{\rho}{\rho-1}} \right]^{\frac{\rho-1}{\rho}}$$

$$= (\cdot) \cdot \left[\cancel{(1-\beta)^{\frac{1}{\rho}}} \cdot \frac{\beta^{\sigma}}{(A \cdot P)^{\sigma}} \cdot \frac{P^{\sigma-1}}{P_x^{\sigma-1}} \cdot P_y^{\frac{\rho}{\rho-1}} + (1-\beta)^{\frac{1}{\rho}} \cdot P_y^{\frac{\rho}{\rho-1}} \right]^{\frac{\rho-1}{\rho}}$$

$$= (\cdot) \cdot \left(\beta^{\sigma} \cdot P_x^{1-\sigma} + (1-\beta)^{\sigma} \cdot P_y^{1-\sigma} \right)^{\frac{\rho-1}{\rho}}$$

$$\boxed{\mu = (w - P_x \cdot x - P_y \cdot y) \cdot \left[\beta^{\sigma} \cdot P_x^{1-\sigma} + (1-\beta)^{\sigma} \cdot P_y^{1-\sigma} \right]^{\frac{1}{\sigma-1}}}$$

É CONTRAINDO A FUNÇÃO DISPENSA DA QUALIDADE COM A FUNÇÃO UTILIDADE INDIVIDUAL:

$$\bar{\mu} = (w - P_x \cdot x - P_y \cdot y) \cdot \left[\beta^{\sigma} \cdot P_x^{1-\sigma} + (1-\beta)^{\sigma} \cdot P_y^{1-\sigma} \right]^{\frac{1}{\sigma-1}}$$

$$w - P_x \cdot x - P_y \cdot y = \bar{\mu} \cdot \left[\beta^{\sigma} \cdot P_x^{1-\sigma} + (1-\beta)^{\sigma} \cdot P_y^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

$$\boxed{\pi(P, \mu) = \bar{\mu} \cdot \left[\beta^{\sigma} \cdot P_x^{1-\sigma} + (1-\beta)^{\sigma} \cdot P_y^{1-\sigma} \right]^{\frac{1}{1-\sigma}} + P_x \cdot x + P_y \cdot y}$$

