PROBLEMA DE MAXIMIZAÇÃO DA UTILIDADE - PMU: 1

$$\max [\beta(X-\delta_X)^\rho + (1-\beta)(Y-\delta_Y)^\rho]^{1/\rho} \quad \text{S. A} \quad P_X \cdot X + P_Y \cdot Y = W$$

X E $Y \geq 0$

LES-CES (LISTA III)

MONTANDO A LAGRANGEANA:

$$\mathcal{L}(X,Y,\lambda) = [\beta(X-\delta_X)^\rho + (1-\beta)(Y-\delta_Y)^\rho]^{1/\rho} + \lambda(W - P_X \cdot X - P_Y \cdot Y)$$

CONDIÇÕES DE PRIMEIRA ORDEM - CPO:

$$\frac{\partial \mathcal{L}}{\partial X} = \frac{1}{\rho}[\beta(X-\delta_X)^\rho + (1-\beta)(Y-\delta_Y)^\rho]^{1/\rho - 1} \cdot [\beta(X-\delta_X)^\rho + (1-\beta)(Y-\delta_Y)^\rho]' - \lambda \cdot P_X = 0$$

ENTÃO,

$$\frac{1}{\rho}[\beta(X-\delta_X)^\rho + (1-\beta)(Y-\delta_Y)^\rho]^{1/\rho - 1} \cdot (\rho \cdot \beta(X-\delta_X)^{\rho-1}) - \lambda \cdot P_X = 0$$

$$\lambda = \frac{\beta(X-\delta_X)^{\rho-1}[\beta(X-\delta_X)^\rho + (1-\beta)(Y-\delta_Y)^\rho]^{1/\rho - 1}}{P_X} \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial Y} = \frac{1}{\rho}[\beta(X-\delta_X)^\rho + (1-\beta)(Y-\delta_Y)^\rho]^{1/\rho - 1} \cdot [\beta(X-\delta_X)^\rho + (1-\beta)(Y-\delta_Y)^\rho]' - \lambda \cdot P_Y = 0$$

ENTÃO,

$$\frac{1}{\rho}[\beta(X-\delta_X)^\rho + (1-\beta)(Y-\delta_Y)^\rho]^{\frac{1}{\rho} - 1} \cdot (\rho(1-\beta)(Y-\delta_Y)^{\rho-1}) - \lambda \cdot P_Y = 0$$

$$\lambda = \frac{(1-\beta)(Y-\delta_Y)^{\rho-1}[\beta(X-\delta_X)^\rho + (1-\beta)(Y-\delta_Y)^\rho]^{\frac{1}{\rho} - 1}}{P_Y} \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = W - P_X \cdot X - P_Y \cdot Y = 0 \therefore W = P_X \cdot X + P_Y \cdot Y \quad (3)$$

IGUALANDO (1) E (2):

$$\frac{\beta(X-\delta_X)^{\rho-1}\cancel{[\beta(X-\delta_X)^\rho + (1-\beta)(Y-\delta_Y)^\rho]^{1/\rho - 1}}}{P_X} = \frac{(1-\beta)(Y-\delta_Y)^{\rho-1}\cancel{[\beta(X-\delta_X)^\rho + (1-\beta)(Y-\delta_Y)^\rho]^{\frac{1}{\rho}-1}}}{P_Y}$$

$$(X-\delta_X)^{\rho-1} = \frac{1-\beta}{\beta} \cdot \frac{P_X}{P_Y} \cdot (Y-\delta_Y)^{\rho-1}$$

$$x - \gamma_x = \left(\frac{P_x}{P_Y} \cdot \frac{1-\beta}{\beta} \right)^{\frac{1}{\rho-1}} \cdot (Y - \gamma_Y)$$

$$x - \gamma_x = \left(\frac{P_Y}{P_x} \cdot \frac{\beta}{1-\beta} \right)^{\frac{1}{1-\rho}} \cdot (Y - \gamma_Y) \qquad \sigma = \frac{1}{1-\rho}, \quad \frac{\beta}{1-\beta} = A \text{ E } \frac{P_Y}{P_x} = P$$

$$x - \gamma_x = (P \cdot A)^{\sigma} \cdot (Y - \gamma_Y)$$

$$x = P^{\sigma} \cdot A^{\sigma} \cdot (Y - \gamma_Y) + \gamma_x \qquad (4)$$

SUBSTITUINDO (4) EM (3):

$$W = P_x \left[P^{\sigma} \cdot A^{\sigma} \cdot (Y - \gamma_Y) + \gamma_x \right] + P_Y \cdot Y$$

$$= P_x \cdot P^{\sigma} A^{\sigma} \cdot (Y - \gamma_Y) + P_x \cdot \gamma_x + P_Y \cdot Y$$

$$= P_x \cdot P^{\sigma} \cdot A^{\sigma} \cdot Y - P_x \cdot P^{\sigma} \cdot A^{\sigma} \cdot \gamma_Y + P_x \cdot \gamma_x + P_Y \cdot Y$$

$$= \left[P_x \cdot P^{\sigma} \cdot A^{\sigma} + P_Y \right] Y - P_x \cdot P^{\sigma} A^{\sigma} \cdot \gamma_Y + P_x \cdot \gamma_x$$

$$= \left[P_x \cdot P^{\sigma} \cdot A^{\sigma} + P_Y \right] Y - P_x \cdot P^{\sigma} A^{\sigma} \gamma_Y - \gamma_Y P_Y + \gamma_Y \cdot P_Y + P_x \cdot \gamma_x$$

$$= \left[P_x \cdot P^{\sigma} \cdot A^{\sigma} + P_Y \right] Y - \left[P_x \cdot P^{\sigma} \cdot A^{\sigma} + P_Y \right] \gamma_Y + \gamma_Y \cdot P_Y + P_x \cdot \gamma_x$$

$$= \left[P_x \cdot P^{\sigma} \cdot A^{\sigma} + P_Y \right] (Y - \gamma_Y) + P_Y \cdot \gamma_Y + P_x \cdot \gamma_x$$

$$Y - \gamma_Y = \frac{W - P_Y \cdot \gamma_Y - P_x \cdot \gamma_x}{P_x \cdot P^{\sigma} \cdot A^{\sigma} + P_Y} \qquad (4.A)$$

$$\boxed{Y = \frac{W - P_Y \cdot \gamma_Y - P_x \cdot \gamma_x}{P_x \cdot P^{\sigma} \cdot A^{\sigma} + P_Y} + \gamma_Y} \qquad (5)$$

SUBSTITUINDO (4.A) EM (4):

$$x = P^{\sigma} \cdot A^{\sigma} \cdot \frac{W - P_Y \cdot \gamma_Y - P_x \cdot \gamma_x}{P_x \cdot P^{\sigma} \cdot A^{\sigma} + P_Y} + \gamma_x$$

$$X = \frac{w - P_Y \cdot \delta_Y - P_X \cdot \delta_X}{(P^{-\sigma} \cdot A^{-\sigma})(P_X \cdot P^{\sigma} \cdot A^{\sigma} + P_Y)} + \delta_X$$

$$\boxed{= \frac{w - P_Y \cdot \delta_Y - P_X \cdot \delta_X}{P_X + P_Y \cdot P^{-\sigma} \cdot A^{-\sigma}} + \delta_X}$$

ENCONTRANDO A FUNÇÃO UTILIDADE INDIRETA - $v(P,w)$:

$$v(P,w) = \left\{ \beta \left(\frac{w - P_Y \cdot \delta_Y - P_X \cdot \delta_X}{P_X + P_Y \cdot P^{-\sigma} \cdot A^{-\sigma}} + \cancel{\delta_X} - \cancel{\delta_X} \right)^{\rho} + (1-\beta) \left(\frac{w - P_Y \cdot \delta_Y - P_X \cdot \delta_X}{P_X \cdot P^{\sigma} \cdot A^{\sigma} + P_Y} + \cancel{\delta_Y} - \cancel{\delta_Y} \right)^{\rho} \right\}^{1/\rho}$$

$$= \left\{ \beta (w - P_Y \cdot \delta_Y - P_X \cdot \delta_X)^{\rho} \left(\frac{1}{P_X + P_Y \cdot P^{-\sigma} \cdot A^{-\sigma}} \right)^{\rho} + (1-\beta)(w - P_Y \cdot \delta_Y - P_X \cdot \delta_X)^{\rho} \left(\frac{1}{P_X \cdot P^{\sigma} \cdot A^{\sigma} + P_Y} \right)^{\rho} \right\}^{1/\rho}$$

$$= (w - P_X \cdot \delta_Y - P_X \cdot \delta_X) \left\{ \beta \left(\frac{1}{(P_X \cdot P^{\sigma} \cdot A^{\sigma} + P_Y)\frac{1}{P^{\sigma} A^{\sigma}}} \right)^{\rho} + (1-\beta) \left(\frac{1}{P_X \cdot P^{\sigma} \cdot A^{\sigma} + P_Y} \right)^{\rho} \right\}^{1/\rho}$$

$$= (w - P_X \cdot \delta_X - P_Y \cdot \delta_Y) \left\{ \beta \left(\frac{P^{\sigma} \cdot A^{\sigma}}{P_X \cdot P^{\sigma} \cdot A^{\sigma} + P_Y} \right)^{\rho} + (1-\beta) \left(\frac{1}{P_X P^{\sigma} \cdot A^{\sigma} + P_Y} \right)^{\rho} \right\}^{1/\rho}$$

$$= (w - P_X \cdot \delta_X - P_Y \cdot \delta_Y) \left\{ P^{\sigma\rho} \cdot A^{\sigma\rho} \cdot \beta \left(\frac{1}{P_X \cdot P^{\sigma} \cdot A^{\sigma} + P_Y} \right)^{\rho} + (1-\beta) \left(\frac{1}{P_X \cdot P^{\sigma} \cdot A^{\sigma} + P_Y} \right)^{\rho} \right\}^{1/\rho}$$

$$= (w - P_X \cdot \delta_X - P_Y \cdot \delta_Y) \left\{ \left(\frac{1}{P_X \cdot P^{\sigma} \cdot A^{\sigma} + P_Y} \right)^{\rho} (P^{\sigma\rho} \cdot A^{\sigma\rho} \cdot \beta + 1 - \beta) \right\}^{1/\rho}$$

$$= \frac{w - P_X \cdot \delta_X - P_Y \cdot \delta_X}{P_X \cdot P^{\sigma} \cdot A^{\sigma} + P_Y} (P^{\sigma\rho} \cdot A^{\sigma\rho} \cdot \beta + 1 - \beta)^{1/\rho}$$

$(1-\beta)^{1/\rho} \left(P^{\sigma-1} A^{\sigma-1} \frac{\beta}{1-\beta} + 1 \right)^{1/\rho}$

$(1-\beta)^{1/\rho} \left(P^{\sigma-1} A^{\sigma-1} \cdot A + 1 \right)^{1/\rho}$

$(1-\beta)^{1/\rho} \left(P^{\sigma-1} A^{\sigma} + 1 \right)^{1/\rho}$

$P_y \left(\frac{P_x}{P_y} P^{\sigma} A^{\sigma} + 1 \right)$

$P_y (P^{-1} P^{\sigma} A^{\sigma} + 1)$

$$(p^{\sigma\rho} A^{\sigma\rho} \beta + 1 - \beta)^{1/\rho} = \left[(1-\beta)\left(p^{\sigma\rho} A^{\sigma\rho} \frac{\beta}{1-\beta} + 1\right)\right]^{1/\rho}$$

$$= (1-\beta)^{1/\rho}\left(p^{\sigma\rho} A^{\sigma\rho} \frac{\beta}{1-\beta} + 1\right)^{1/\rho}$$

$$= (1-\beta)^{1/\rho}\left(p^{\sigma\rho} A^{\sigma\rho} A + 1\right)^{1/\rho} \qquad \sigma = \frac{1}{1-\rho}$$

$$= (1-\beta)^{1/\rho}\left(p^{\sigma-1} A^{\sigma-1} A + 1\right)^{1/\rho} \qquad \sigma - \sigma\rho = 1$$

$$= (1-\beta)^{1/\rho}\left(p^{\sigma-1} A^{\sigma} + 1\right)^{1/\rho} \qquad \sigma - 1 = \sigma\rho$$

$$x(p,w) = \frac{W - P_X X - P_Y Y}{P_X p^{\sigma} A^{\sigma} + P_Y} \cdot (1-\beta)^{1/\rho}\left(p^{\sigma-1} A^{\sigma} + 1\right)^{1/\rho}$$

$$= \frac{W - P_X X - P_Y Y}{P_Y\left(\frac{P_X}{P_Y} p^{\sigma} A^{\sigma} + 1\right)} \cdot (1-\beta)^{1/\rho}\left(p^{\sigma-1} A^{\sigma} + 1\right)^{1/\rho}$$

$$= \frac{W - P_X X - P_Y Y}{P_Y\left(p^{-1} p^{\sigma} A^{\sigma} + 1\right)} (1-\beta)^{1/\rho}\left(p^{\sigma-1} A^{\sigma} + 1\right)^{1/\rho}$$

$$= \frac{W - P_X X - P_Y Y}{P_Y\left(p^{\sigma-1} A^{\sigma} + 1\right)} (1-\beta)^{1/\rho}\left(p^{\sigma-1} A^{\sigma} + 1\right)^{1/\rho}$$

$$= \frac{(W - P_X X - P_Y Y)(1-\beta)^{1/\rho}\left(p^{\sigma-1} A^{\sigma} + 1\right)^{1/\rho - 1}}{P_Y}$$

$$= \frac{(W - P_X X - P_Y Y)(1-\beta)^{1/\rho}\left(p^{\sigma-1} A^{\sigma} + 1\right)^{\frac{1-\rho}{\rho}}}{P_Y}$$

$$= (W - P_X X - P_Y Y)(1-\beta)^{1/\rho}\left(p^{\sigma-1} A^{\sigma} + 1\right)^{\frac{1-\rho}{\rho}} P_Y^{-1}$$

$$= (W - P_X X - P_Y Y)\left[(1-\beta)^{\frac{1}{1-\rho}} p^{\sigma-1} A^{\sigma} P_Y^{\frac{\rho}{\rho-1}} + (1-\beta)^{\frac{1}{1-\rho}} P_Y^{\frac{\rho}{\rho-1}}\right]^{\frac{1-\rho}{\rho}}$$

$$= (W - P_X X - P_Y Y)\left[(1-\beta)^{\sigma} p^{\sigma-1} A^{\sigma} P_Y^{1-\sigma} + (1-\beta)^{\sigma} P_Y^{1-\sigma}\right]^{\frac{1-\rho}{\rho}}$$

$$= (\cdot)\left[\cancel{(1-\beta)^{\sigma}} \cdot \frac{P_Y^{\sigma-1}}{P_X^{\sigma-1}} \cdot \frac{A^{\sigma}}{\cancel{(1-\beta)^{\sigma}}} \cdot P_Y^{1-\sigma} + (1-\beta)^{\sigma} P_Y^{1-\sigma}\right]^{\frac{1-\rho}{\rho}}$$

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$$v(P,W) = (W - P_X \cdot X - P_Y \cdot Y)\left(\beta^{\sigma} \cdot P_X^{1-\sigma} + (1-\beta)^{\sigma} \cdot P_Y^{1-\sigma}\right)^{\frac{1-\rho}{\rho}}$$

$$= (W - P_X \cdot X - P_Y \cdot Y) \cdot \left(\beta^{\sigma} \cdot P_X^{1-\sigma} + (1-\beta)^{\sigma} \cdot P_Y^{1-\sigma}\right)^{\frac{1}{\sigma-1}}$$

$$(X-\gamma_X)^{\rho-1} = \frac{1-\beta}{\beta} \cdot \frac{P_X}{P_Y} \cdot (Y-\gamma_Y)^{\rho-1}$$

$$X-\gamma_X = \left(\frac{1-\beta}{\beta} \cdot \frac{P_X}{P_Y}\right)^{1/\rho-1} \cdot (Y-\gamma_Y)$$

$$X-\gamma_X = \left(\frac{\beta}{1-\beta} \cdot \frac{P_Y}{P_X}\right)^{\frac{1}{1-\rho}} \cdot (Y-\gamma_Y) \qquad \sigma = \frac{1}{1-\rho}, \; A = \frac{\beta}{1-\beta} \text{ E } P = \frac{P_Y}{P_X}$$

$$X-\gamma_X = A^{\sigma} \cdot P^{\sigma} \cdot (Y-\gamma_Y)$$

$$\cancel{X-\gamma_X = A^{\sigma} \cdot P^{\sigma} \cdot Y - A^{\sigma} \cdot P^{\sigma} \cdot \gamma_Y}$$

$$X = A^{\sigma} \cdot P^{\sigma} \cdot (Y-\gamma_Y) + \gamma_X \qquad (4)$$

SUBSTITUINDO (4) EM (3):

$$W = P_X \left[A^{\sigma} \cdot P^{\sigma} \cdot (Y-\gamma_Y) + \gamma_X\right] + P_Y \cdot Y$$

$$W = P_X \cdot A^{\sigma} \cdot P^{\sigma} (Y-\gamma_Y) + P_X \cdot \gamma_X + P_Y \cdot Y$$

$$W = P_X \cdot A^{\sigma} \cdot P^{\sigma} \cdot Y - P_X \cdot A^{\sigma} \cdot P^{\sigma} \cdot \gamma_Y + P_X \cdot \gamma_X + P_Y \cdot Y$$

$$W = \left[P_X \cdot A^{\sigma} \cdot P^{\sigma} + P_Y\right] Y - P_X \cdot A^{\sigma} \cdot P^{\sigma} \cdot \gamma_Y + P_X \cdot \gamma_X$$

$$W = \left[P_X \cdot A^{\sigma} \cdot P^{\sigma} + P_Y\right] \cdot Y - P_X \cdot A^{\sigma} \cdot P^{\sigma} \cdot \gamma_Y - P_Y \cdot \gamma_Y + P_Y \cdot \gamma_Y + P_X \cdot \gamma_X$$

$$W = \left[P_X \cdot A^{\sigma} \cdot P^{\sigma} + P_Y\right] Y - \left[P_X \cdot A^{\sigma} \cdot P^{\sigma} + P_Y\right] \gamma_Y + P_Y \cdot \gamma_Y + P_X \cdot \gamma_X$$

$$W = (P_X \cdot A^{\sigma} \cdot P^{\sigma} + P_Y) \cdot (Y-\gamma_Y) + P_Y \cdot \gamma_Y + P_X \cdot \gamma_X$$

$$Y-\gamma_Y = \frac{W - P_Y \cdot \gamma_Y - P_X \cdot \gamma_X}{P_X \cdot A^{\sigma} \cdot P^{\sigma} + P_Y}$$

$$\boxed{Y = \frac{W - P_Y \cdot \gamma_Y - P_X \cdot \gamma_X}{P_X \cdot A^{\sigma} \cdot P^{\sigma} + P_Y} + \gamma_Y} \qquad (5)$$

SUBSTITUINDO (3) EM (4):

$$x = A^{\sigma}\cdot p^{\sigma}\left(\frac{w - P_y\cdot\gamma_y - P_x\cdot\gamma_x}{P_x\cdot A^{\sigma}\cdot p^{\sigma} + P_y} + \cancel{\gamma_y} - \cancel{\gamma_y}\right) + \gamma_x$$

$$= \frac{w - P_y\cdot\gamma_y - P_x\cdot\gamma_x}{A^{-\sigma}\cdot p^{-\sigma}\left(P_x\cdot A^{\sigma}\cdot p^{\sigma} + P_y\right)} + \gamma_x$$

$$\boxed{= \frac{w - P_y\cdot\gamma_y - P_x\cdot\gamma_x}{P_x + P_y\cdot A^{-\sigma}\cdot p^{-\sigma}} + \gamma_x}$$

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ENCONTRANDO A FUNÇÃO UTILIDADE INDIRETA - $v(p,w)$:

$$v(p,w) = \left\{\beta\left(\frac{w - P_y\cdot\gamma_y - P_x\cdot\gamma_x}{P_x + P_y\cdot A^{-\sigma}\cdot p^{-\sigma}} + \cancel{\gamma_x} - \cancel{\gamma_x}\right)^{\rho} + (1-\beta)\left(\frac{w - P_y\cdot\gamma_y - P_x\cdot\gamma_x}{P_x\cdot A^{\sigma}\cdot p^{\sigma} + P_y} + \cancel{\gamma_x} - \cancel{\gamma_x}\right)^{\rho}\right\}^{1/\rho}$$

$$= \left\{\beta\cdot(w - P_y\cdot\gamma_y - P_x\cdot\gamma_x)^{\rho}\cdot\left(\frac{1}{P_x + P_y\cdot A^{-\sigma}\cdot p^{-\sigma}}\right)^{\rho} + (1-\beta)\cdot(w - P_y\cdot\gamma_y - P_x\cdot\gamma_x)^{\rho}\cdot\left(\frac{1}{P_x\cdot A^{\sigma}\cdot p^{\sigma} + P_y}\right)^{\rho}\right\}^{1/\rho}$$

$$= (w - P_y\cdot\gamma_y - P_x\cdot\gamma_x)\left\{\beta\left(\frac{1}{(A^{\sigma}\cdot p^{\sigma}\cdot P_x + P_y)\left(\frac{1}{A^{\sigma}\cdot p^{\sigma}}\right)}\right)^{\rho} + (1-\beta)\left(\frac{1}{P_x\cdot A^{\sigma}\cdot p^{\sigma} + P_y}\right)^{\rho}\right\}^{1/\rho}$$

$$= (\cdot)\left\{\beta\left(\frac{A^{\sigma}\cdot p^{\sigma}}{P_x\cdot A^{\sigma}\cdot p^{\sigma} + P_y}\right)^{\rho} + (1-\beta)\left(\frac{1}{P_x\cdot A^{\sigma}\cdot p^{\sigma} + P_y}\right)^{\rho}\right\}^{1/\rho}$$

$$= (\cdot)\left\{\beta\cdot A^{\sigma\rho}\cdot p^{\sigma\rho}\left(\frac{1}{P_x\cdot A^{\sigma}\cdot p^{\sigma} + P_y}\right)^{\rho} + (1-\beta)\left(\frac{1}{P_x\cdot A^{\sigma}\cdot p^{\sigma} + P_y}\right)^{\rho}\right\}^{1/\rho}$$

$$= (\cdot)\left\{\left(\frac{1}{P_x\cdot A^{\sigma}\cdot p^{\sigma} + P_y}\right)^{\rho}\left(\beta\cdot A^{\sigma\rho}\cdot p^{\sigma\rho} + 1 - \beta\right)\right\}^{1/\rho}$$

$$x(p,w) = \frac{w - p_y \cdot \gamma_y - p_x \cdot \gamma_x}{p_x \cdot A^\sigma \cdot p^\sigma + p_y} \left(\beta \cdot A^{\sigma\rho} \cdot p^{\sigma\rho} + 1 - \beta\right)^{1/\rho}$$

$$= (\cdot)\,(1-\beta)^{1/\rho}\left(\frac{\beta}{1-\beta} \cdot A^{\sigma\rho} \cdot p^{\sigma\rho} + 1\right)^{1/\rho}$$

$$= (\cdot)\,(1-\beta)^{1/\rho}\left(A \cdot A^{\sigma\rho} \cdot p^{\sigma\rho} + 1\right)^{1/\rho}$$

$$= (\cdot)\,(1-\beta)^{1/\rho}\left(A \cdot A^{\sigma-1} \cdot p^{\sigma\rho} + 1\right)^{1/\rho}$$

$$= (\cdot)\,(1-\beta)^{1/\rho}\left(A^\sigma \cdot p^{\sigma\rho} + 1\right)^{1/\rho}$$

$$= \frac{w - p_y \cdot \gamma_y - p_x \cdot \gamma_x}{p_y\left(\frac{p_x}{p_y} \cdot A^\sigma \cdot p^\sigma + 1\right)} \cdot (1-\beta)^{1/\rho} \cdot \left(A^\sigma \cdot p^{\sigma\rho} + 1\right)^{1/\rho}$$

$$= \frac{w - p_y \cdot \gamma_y - p_x \cdot \gamma_x}{p_y\left(p^{-1} \cdot A^\sigma \cdot p^\sigma + 1\right)} \cdot (1-\beta)^{1/\rho} \cdot \left(A^\sigma \cdot p^{\sigma-1} + 1\right)^{1/\rho}$$

$$= \frac{w - p_y \cdot \gamma_y - p_x \cdot \gamma_x}{p_y\left(A^\sigma \cdot p^{\sigma-1} + 1\right)} \cdot (1-\beta)^{1/\rho} \left(A^\sigma \cdot p^{\sigma-1} + 1\right)^{1/\rho}$$

$$= (w - p_y \cdot \gamma_y - p_x \cdot \gamma_x) \cdot (1-\beta)^{1/\rho} \left(A^\sigma \cdot p^{\sigma-1} + 1\right)^{\frac{1-\rho}{\rho}} \cdot p_y^{-1}$$

$$= (w - p_y \cdot \gamma_y - p_x \cdot \gamma_x) \cdot \left[(1-\beta)^{\frac{1}{1-\rho}} A^\sigma \cdot p^{\sigma-1} + (1-\beta)^{\frac{1}{1-\rho}}\right]^{\frac{1-\rho}{\rho}} \cdot p_y^{-1}$$

$$= (\cdot) \cdot \left[(1-\beta)^\sigma \cdot A^\sigma \cdot p^{\sigma-1} \cdot p_y^{\frac{\rho}{\rho-1}} + (1-\beta)^\sigma \cdot p_y^{\frac{\rho}{\rho-1}}\right]^{\frac{1-\rho}{\rho}}$$

$$= (\cdot) \cdot \left[\cancel{(1-\beta)^\sigma} \cdot \frac{\beta^\sigma}{\cancel{(1-\beta)^\sigma}} \cdot \frac{\cancel{p_y^{\sigma-1}}}{p_x^{\sigma-1}} \cdot \cancel{p_y^{1-\sigma}} + (1-\beta)^\sigma \cdot p_y^{1-\sigma}\right]^{\frac{1-\rho}{\rho}}$$

$$= (\cdot)\left[\beta^\sigma \cdot p_x^{1-\sigma} + (1-\beta)^\sigma \cdot p_y^{1-\sigma}\right]^{\frac{1-\rho}{\rho}}$$

$$= (w - p_y \cdot \gamma_y - p_x \cdot \gamma_x) \cdot \left[\beta^\sigma \cdot p_x^{1-\sigma} + (1-\beta)^\sigma \cdot p_y^{1-\sigma}\right]^{\frac{1}{\sigma-1}}$$

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LES-CES (LISTA III)

$$(X-\gamma_X)^{\rho-1} = \frac{1-\beta}{\beta}\cdot\frac{P_X}{P_Y}\cdot(Y-\gamma_Y)^{\rho-1}$$

$$X-\gamma_X = \left(\frac{1-\beta}{\beta}\cdot\frac{P_X}{P_Y}\right)^{\frac{1}{\rho-1}}\cdot(Y-\gamma_Y)$$

$$X-\gamma_X = \left(\frac{\beta}{1-\beta}\cdot\frac{P_Y}{P_X}\right)^{\frac{1}{1-\rho}}\cdot(Y-\gamma_Y) \qquad \sigma = \frac{1}{1-\rho},\ A = \frac{\beta}{1-\beta}\ \text{E}\ p = \frac{P_Y}{P_X}$$

$$X-\gamma_X = A^{\sigma}\cdot p^{\sigma}\cdot(Y-\gamma_Y)$$

$$X = A^{\sigma}\cdot p^{\sigma}\cdot(Y-\gamma_Y)+\gamma_X \qquad (4)$$

SUBSTITUINDO (4) EM (3):

$$W = P_X\cdot[A^{\sigma}\cdot p^{\sigma}\cdot(Y-\gamma_Y)+\gamma_X]+P_Y\cdot Y$$

$$W = P_X\cdot A^{\sigma}\cdot p^{\sigma}\cdot(Y-\gamma_Y)+P_X\cdot\gamma_X+P_Y\cdot Y$$

$$W = P_X\cdot A^{\sigma}\cdot p^{\sigma}\cdot Y - P_X\cdot A^{\sigma}\cdot p^{\sigma}\cdot\gamma_Y + P_X\cdot\gamma_X+P_Y\cdot Y$$

$$W = (P_X\cdot A^{\sigma}\cdot p^{\sigma}+P_Y)\cdot Y - P_X\cdot A^{\sigma}\cdot p^{\sigma}\cdot\gamma_Y + P_X\cdot\gamma_X$$

$$W = (P_X\cdot A^{\sigma}\cdot p^{\sigma}+P_Y)\cdot Y - P_X\cdot A^{\sigma}\cdot p^{\sigma}\cdot\gamma_Y - P_Y\cdot\gamma_Y + P_Y\cdot\gamma_Y + P_X\cdot\gamma_X$$

$$W = (P_X\cdot A^{\sigma}\cdot p^{\sigma}+P_Y)\cdot Y - (P_X\cdot A^{\sigma}\cdot p^{\sigma}+P_Y)\cdot\gamma_Y + P_Y\cdot\gamma_Y + P_X\cdot\gamma_X$$

$$W = (P_X\cdot A^{\sigma}\cdot p^{\sigma}+P_Y)\cdot(Y-\gamma_Y) + P_Y\cdot\gamma_Y + P_X + \gamma_X$$

$$Y-\gamma_Y = \frac{W-P_X\cdot\gamma_X-P_Y\cdot\gamma_Y}{P_X\cdot A^{\sigma}\cdot p^{\sigma}+P_Y}$$

$$\boxed{Y = \frac{W-P_X\cdot\gamma_X-P_Y\cdot\gamma_Y}{P_X\cdot A^{\sigma}\cdot p^{\sigma}+P_Y}+\gamma_Y} \qquad (5)$$

SUBSTITUINDO (5) EM (4):

$$X = A^{\sigma}\cdot p^{\sigma}\cdot\left(\frac{W-P_X\cdot\gamma_X-P_Y\cdot\gamma_Y}{P_X\cdot A^{\sigma}p^{\sigma}+P_Y}+\cancel{\gamma_Y}-\cancel{\gamma_Y}\right)+\gamma_X$$

$$= \frac{W-P_X\cdot\gamma_X-P_Y\cdot\gamma_Y}{A^{-\sigma}\cdot p^{-\sigma}(P_X\cdot A^{\sigma}\cdot p^{\sigma}+P_Y)}+\gamma_X$$

$$\boxed{X = \frac{w - P_X\cdot\gamma_X - P_Y\cdot\gamma_Y}{P_X + P_Y\cdot A^{-\sigma}\cdot p^{-\sigma}} + \gamma_X}$$

ENCONTRANDO A FUNÇÃO UTILIDADE INDIRETA - $v(P,w)$:

$$v(P,w) = \left\{\beta\left(\frac{w - P_X\cdot\gamma_X - P_Y\cdot\gamma_Y}{P_X + P_Y\cdot A^{-\sigma}\cdot p^{-\sigma}} + \cancel{\gamma_X} - \cancel{\gamma_X}\right)^{\rho} + (1-\beta)\left(\frac{w - P_X\cdot\gamma_X - P_Y\cdot\gamma_Y}{P_X\cdot A^{\sigma}\cdot p^{\sigma} + P_Y} + \cancel{\gamma_Y} - \cancel{\gamma_Y}\right)^{\rho}\right\}^{1/\rho}$$

$$= \left\{\beta\left(\frac{w - P_X\cdot\gamma_X - P_Y\cdot\gamma_Y}{(P_X\cdot A^{\sigma}\cdot p^{\sigma} + P_Y)\left(\frac{1}{A^{\sigma}\cdot p^{\sigma}}\right)}\right)^{\rho} + (1-\beta)\left(\frac{w - P_X\cdot\gamma_X - P_Y\cdot\gamma_Y}{P_X A^{\sigma}\cdot p^{\sigma} + P_Y}\right)^{\rho}\right\}^{1/\rho}$$

$$= \left\{(w - P_X\cdot\gamma_X - P_Y\cdot\gamma_Y)^{\rho}\left(\frac{A^{\sigma}\cdot p^{\sigma}}{P_X\cdot A^{\sigma}\cdot p^{\sigma} + P_Y}\right)^{\rho}\cdot\beta + (1-\beta)\left(\frac{1}{P_X\cdot A^{\sigma}\cdot p^{\sigma} + P_Y}\right)^{\rho}\right\}^{1/\rho}$$

$$= (w - P_X\cdot\gamma_X - P_Y\cdot\gamma_Y)\left\{\left(\frac{1}{P_X\cdot A^{\sigma}\cdot p^{\sigma} + P_Y}\right)^{\rho}\left[(A^{\sigma}\cdot p^{\sigma})^{\rho}\cdot\beta + (1-\beta)\right]\right\}^{1/\rho}$$

$$= \frac{w - P_X\cdot\gamma_X - P_Y\cdot\gamma_Y}{P_X\cdot A^{\sigma}\cdot p^{\sigma} + P_Y}\left(A^{\sigma\rho}\cdot p^{\sigma\rho}\cdot\beta + 1 - \beta\right)^{1/\rho}$$

$$= \frac{w - P_X\cdot\gamma_X - P_Y\cdot\gamma_Y}{P_X\cdot A^{\sigma}\cdot p^{\sigma} + P_Y}\cdot(1-\beta)^{1/\rho}\cdot\left(A^{\sigma\rho}\cdot p^{\sigma\rho}\cdot\frac{\beta}{1-\beta} + 1\right)^{1/\rho}$$

$\sigma = \frac{1}{1-\rho}$

$\sigma - \sigma\rho = 1$

$\sigma\rho = \sigma - 1$

$$= (\cdot)\,(1-\beta)^{1/\rho}\left(A^{\sigma-1}\cdot p^{\sigma-1}\cdot A + 1\right)^{1/\rho}$$

$$= (\cdot)\,(1-\beta)^{1/\rho}\left(A^{\sigma}\cdot p^{\sigma-1} + 1\right)^{1/\rho}$$

$$= \frac{w - P_X\cdot\gamma_X - P_Y\cdot\gamma_Y}{P_Y\left(\frac{P_X}{P_Y}\cdot A^{\sigma}\cdot p^{\sigma} + 1\right)}\,(1-\beta)^{1/\rho}\cdot\left(A^{\sigma}\cdot p^{\sigma-1} + 1\right)^{1/\rho}$$

$$= \frac{w - P_X\cdot\gamma_X - P_Y\cdot\gamma_Y}{(p^{-1}\cdot A^{\sigma}\cdot p^{\sigma} + 1)}\cdot(1-\beta)^{1/\rho}\cdot\left(A^{\sigma}\cdot p^{\sigma-1} + 1\right)^{1/\rho}\cdot P_Y^{-1}$$

$$= \frac{w - P_X\cdot\gamma_X - P_Y\cdot\gamma_Y}{A^{\sigma}\cdot p^{\sigma-1} + 1}\cdot(1-\beta)^{1/\rho}\cdot\left(A^{\sigma}\cdot p^{\sigma-1} + 1\right)^{1/\rho}\cdot P_Y^{-1}$$

2

$$\mathcal{V}(P,W) = (W - P_X\cdot\gamma_X - P_Y\cdot\gamma_Y)(1-\beta)^{1/\rho}\cdot\left(A^{\sigma}\cdot P^{\sigma-1}+1\right)^{\frac{1-\rho}{\rho}}\cdot P_Y^{-1}$$

$$= (W - P_X\cdot\gamma_X - P_Y\cdot\gamma_Y)\left[(1-\beta)^{\frac{1}{1-\rho}}\cdot A^{\sigma}\cdot P^{\sigma-1}\cdot P_Y^{\frac{\rho}{\rho-1}} + (1-\beta)^{\frac{1}{1-\rho}}\cdot P_Y^{\frac{\rho}{\rho-1}}\right]^{\frac{1-\rho}{\rho}}$$

$$= (\cdot)\left[\cancel{(1-\beta)^{\sigma}}\cdot\frac{\beta^{\sigma}}{\cancel{(1-\beta)^{\sigma}}}\cdot\frac{\cancel{P_Y^{\sigma-1}}}{P_X^{\sigma-1}}\cdot\cancel{P_Y^{1-\sigma}} + (1-\beta)^{\sigma}\cdot P_Y^{1-\sigma}\right]^{\frac{1-\rho}{\rho}}$$

$$= (\cdot)\cdot\left[\beta^{\sigma}\cdot P_X^{1-\sigma} + (1-\beta)^{\sigma}\cdot P_Y^{1-\sigma}\right]^{\frac{1-\rho}{\rho}}$$

$$= (W - P_X\cdot\gamma_X - P_Y\cdot\gamma_Y)\left(\beta^{\sigma}\cdot P_X^{1-\sigma} + (1-\beta)^{\sigma}\cdot P_Y^{1-\sigma}\right)^{\frac{1}{\sigma-1}}$$

$$(X-\gamma_X)^{\rho-1} = \frac{1-\beta}{\beta}\cdot\frac{P_X}{P_Y}\cdot(Y-\gamma_Y)^{\rho-1}$$

$$X-\gamma_X = \left(\frac{1-\beta}{\beta}\cdot\frac{P_X}{P_Y}\right)^{\frac{1}{\rho-1}}\cdot(Y-\gamma_Y)$$

$$X-\gamma_X = \left(\frac{\beta}{1-\beta}\cdot\frac{P_Y}{P_X}\right)^{\frac{1}{1-\rho}}\cdot(Y-\gamma_Y) \qquad \sigma = \frac{1}{1-\rho},\ A = \frac{\beta}{1-\beta}\ \text{E}\ P = \frac{P_Y}{P_X}$$

$$X-\gamma_X = A^{\sigma}\cdot P^{\sigma}\cdot(Y-\gamma_Y)$$

$$X = A^{\sigma}\cdot P^{\sigma}\cdot(Y-\gamma_Y)+\gamma_X \qquad (4)$$

SUBSTITUINDO (4) EM (3):

$$W = P_X\cdot\left[A^{\sigma}\cdot P^{\sigma}\cdot(Y-\gamma_Y)+\gamma_X\right] + P_Y\cdot Y$$

$$W = P_X\cdot A^{\sigma}\cdot P^{\sigma}\cdot(Y-\gamma_Y) + P_X\cdot\gamma_X + P_Y\cdot Y$$

$$W = P_X\cdot A^{\sigma}\cdot \beta^{\sigma}\cdot Y - P_X\cdot A^{\sigma}\cdot P^{\sigma}\cdot\gamma_Y + P_X\cdot\gamma_X + P_Y\cdot Y$$

$$W = (P_X\cdot A^{\sigma}\cdot P^{\sigma} + P_Y)Y - P_X\cdot A^{\sigma}\cdot P^{\sigma}\cdot\gamma_Y + P_X\cdot\gamma_X$$

$$W = (P_X\cdot A^{\sigma}\cdot P^{\sigma} + P_Y)Y - P_X\cdot A^{\sigma}\cdot P^{\sigma}\cdot\gamma_Y - P_Y\cdot\gamma_Y + P_Y\cdot\gamma_Y + P_X\cdot\gamma_X$$

$$W = (P_X\cdot A^{\sigma}\cdot P^{\sigma} + P_Y)Y - (P_X\cdot A^{\sigma}\cdot P^{\sigma} + P_Y)\gamma_Y + P_Y\cdot\gamma_Y + P_X\cdot\gamma_X$$

$$W = (P_X\cdot A^{\sigma}\cdot P^{\sigma} + P_Y)\cdot(Y-\gamma_Y) + P_Y\cdot\gamma_Y + P_X\cdot\gamma_X$$

$$Y - \gamma_Y = \frac{W - P_X \cdot \gamma_X - P_Y \cdot \gamma_Y}{P_X \cdot A^{\sigma} \cdot p^{\sigma} + P_Y}$$

$$\boxed{Y = \frac{W - P_X \cdot \gamma_X - P_Y \cdot \gamma_Y}{P_X \cdot A^{\sigma} \cdot p^{\sigma} + P_Y} + \gamma_Y} \qquad (5)$$

SUBSTITUINDO (5) EM (4):

$$X = A^{\sigma} \cdot p^{\sigma} \cdot \left(\frac{W - P_X \cdot \gamma_X - P_Y \cdot \gamma_Y}{P_X \cdot A^{\sigma} \cdot p^{\sigma} + P_Y} + \cancel{\gamma_X} - \cancel{\gamma_X} \right) + \gamma_X$$

$$= \frac{W - P_X \cdot \gamma_X - P_Y \cdot \gamma_Y}{A^{-\sigma} \cdot p^{-\sigma} \cdot (P_X \cdot A^{\sigma} \cdot p^{\sigma} + P_Y)} + \gamma_X$$

$$\boxed{= \frac{W - P_X \cdot \gamma_X - P_Y \cdot \gamma_Y}{P_X + P_Y \cdot A^{-\sigma} \cdot p^{-\sigma}} + \gamma_X} \qquad (6)$$

ENCONTRANDO A FUNÇÃO DE UTILIDADE INDIRETA - $v(P,W)$:

$$v(P,W) = \left\{ \beta \left[\frac{W - P_X \cdot \gamma_X - P_Y \cdot \gamma_Y}{P_X + P_Y \cdot A^{-\sigma} \cdot p^{-\sigma}} + \cancel{\gamma_X} - \cancel{\gamma_X} \right]^{\rho} + (1-\beta) \left(\frac{W - P_X \cdot \gamma_X - P_Y \cdot \gamma_Y}{P_X \cdot A^{\sigma} \cdot p^{\sigma} + P_Y} + \cancel{\gamma_Y} - \cancel{\gamma_Y} \right)^{\rho} \right\}^{1/\rho}$$

$$= \left\{ (W - P_X \cdot \gamma_X - P_Y \cdot \gamma_Y)^{\rho} \left[\beta \left(\frac{1}{P_X + P_Y \cdot A^{-\sigma} \cdot p^{-\sigma}} \right)^{\rho} + (1-\beta) \left(\frac{1}{P_X \cdot A^{\sigma} \cdot p^{\sigma} + P_Y} \right)^{\rho} \right] \right\}^{1/\rho}$$

$$= (W - P_X \cdot \gamma_X - P_Y \cdot \gamma_Y) \left\{ \beta \left[\frac{1}{(P_X \cdot A^{\sigma} \cdot p^{\sigma} + P_Y) \cdot A^{-\sigma} \cdot p^{-\sigma}} \right]^{\rho} + (1-\beta) \left(\frac{1}{P_X \cdot A^{\sigma} \cdot p^{\sigma} \cdot P_Y} \right)^{\rho} \right\}^{1/\rho}$$

$$= (W - P_X \cdot \gamma_X - P_Y \cdot \gamma_Y) \left\{ \left(\frac{1}{P_X \cdot A^{\sigma} \cdot p^{\sigma} + P_Y} \right)^{\rho} \cdot \left[\beta \cdot (A^{\sigma} \cdot p^{\sigma})^{\rho} + (1-\beta) \right] \right\}^{1/\rho}$$

$$= \frac{W - P_X \cdot \gamma_X - P_Y \cdot \gamma_Y}{P_X \cdot A^{\sigma} \cdot p^{\sigma} + P_Y} \cdot (\beta \cdot A^{\sigma\rho} \cdot p^{\sigma\rho} + 1 - \beta)^{1/\rho}$$

$$\sigma = \frac{1}{1-\rho}$$

$$\sigma - \sigma\rho = 1$$

$$\sigma\rho = \sigma - 1$$

3

$$v(P,w) = (\cdot)\cdot(1-\beta)^{1/\rho}\cdot\left(\frac{\beta}{1-\beta}\cdot A^{\sigma-1}\cdot P^{\sigma-1}+1\right)^{1/\rho}$$

$$= (\cdot)\cdot(1-\beta)^{1/\rho}\cdot\left(A\cdot A^{\sigma-1}\cdot P^{\sigma-1}+1\right)^{1/\rho}$$

$$= (\cdot)\cdot(1-\beta)^{1/\rho}\cdot\left(A^{\sigma}\cdot P^{\sigma-1}+1\right)^{1/\rho}$$

$$= \frac{w-P_x\cdot\gamma_x-P_y\cdot\gamma_y}{P_y\left(\frac{P_x}{P_y}\cdot A^{\sigma}\cdot P^{\sigma}+1\right)}\cdot(1-\beta)^{1/\rho}\cdot\left(A^{\sigma}\cdot P^{\sigma-1}+1\right)^{1/\rho}$$

$$= \frac{w-P_x\cdot\gamma_x-P_y\cdot\gamma_y}{P_y\left(P^{-1}\cdot A^{\sigma}\cdot P^{\sigma}+1\right)}\cdot(1-\beta)^{1/\rho}\cdot\left(A^{\sigma}\cdot P^{\sigma-1}+1\right)^{1/\rho}$$

$$= (w-P_x\cdot\gamma_x-P_y\cdot\gamma_y)\cdot(1-\beta)^{1/\rho}\cdot\left(A^{\sigma}\cdot P^{\sigma-1}+1\right)^{\frac{1-\rho}{\rho}}\cdot P_y^{-1}$$

$$= (w-P_x\cdot\gamma_x-P_y\cdot\gamma_y)\cdot\left[(1-\beta)^{\frac{1}{1-\rho}}\cdot A^{\sigma}\cdot P^{\sigma-1}\cdot P_y^{\frac{\rho}{\rho-1}}+(1-\beta)^{\frac{1}{1-\rho}}\cdot P_y^{\frac{\rho}{\rho-1}}\right]^{\frac{1-\rho}{\rho}}$$

$$= (\cdot)\cdot\left[\cancel{(1-\beta)^{\sigma}}\cdot\frac{\beta^{\sigma}}{\cancel{(1-\beta)^{\sigma}}}\cdot\frac{P_y^{\sigma-1}}{P_x^{\sigma-1}}\cdot P_y^{1-\sigma}+(1-\beta)^{\sigma}\cdot P_y^{1-\sigma}\right]^{\frac{1-\rho}{\rho}}$$

$$= (\cdot)\cdot\left(\beta^{\sigma}\cdot P_x^{1-\sigma}+(1-\beta)^{\sigma}\cdot P_y^{1-\sigma}\right)^{\frac{1-\rho}{\rho}}$$

$$\boxed{= (w-P_x\cdot\gamma_x-P_y\cdot\gamma_y)\cdot\left[\beta^{\sigma}\cdot P_x^{1-\sigma}+(1-\beta)^{\sigma}\cdot P_y^{1-\sigma}\right]^{\frac{1}{\sigma-1}}}$$

ENCONTRANDO A FUNÇÃO DISPÊNDIO PELA DUALIDADE COM A FUNÇÃO UTILIDADE INDIRETA:

$$\bar{u} = (e-P_x\cdot\gamma_x-P_y\cdot\gamma_y)\cdot\left[\beta^{\sigma}\cdot P_x^{1-\sigma}+(1-\beta)^{\sigma}\cdot P_y^{1-\sigma}\right]^{\frac{1}{\sigma-1}}$$

$$e-P_x\cdot\gamma_x-P_y\cdot\gamma_y = \bar{u}\cdot\left[\beta^{\sigma}\cdot P_x^{1-\sigma}+(1-\beta)^{\sigma}\cdot P_y^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

$$\boxed{e(P,u) = \bar{u}\cdot\left[\beta^{\sigma}\cdot P_x^{1-\sigma}+(1-\beta)^{\sigma}+P_y^{1-\sigma}\right]^{\frac{1}{1-\sigma}}+P_x\cdot\gamma_x+P_y\cdot\gamma_y}$$