LISTA III MICROECONOMIA I

2013/1º

(1)

EXERCÍCIO 1:

PROBLEMA DE MAXIMIZAÇÃO DA UTILIDADE - PMU

$$\max \left[\beta X^{\rho} + (1-\beta) Y^{\rho}\right]^{1/\rho} \qquad \text{S. A} \quad P_X X + P_Y Y = W$$

MONTANDO O LAGRANGIANO:

$$\mathcal{L}(X, Y, \lambda) = \left[\beta X^{\rho} + (1-\beta) Y^{\rho}\right]^{1/\rho} + \lambda (W - P_X X - P_Y Y)$$

$$\frac{\partial \mathcal{L}}{\partial X} = \frac{1}{\rho}\left[\beta X^{\rho} + (1-\beta) Y^{\rho}\right]^{\frac{1}{\rho}-1}\left[\beta X^{\rho} + (1-\beta) Y^{\rho}\right]' - \lambda P_X = 0$$

$$= \frac{1}{\cancel{\rho}}\left[\beta X^{\rho} + (1-\beta) Y^{\rho}\right]^{\frac{1-\rho}{\rho}}\left[\cancel{\rho}\beta X^{\rho-1}\right] - \lambda P_X = 0$$

$$= \beta X^{\rho-1}\left[\beta X^{\rho} + (1-\beta) Y^{\rho}\right]^{\frac{1-\rho}{\rho}} - \lambda P_X = 0$$

LOGO,

$$\lambda P_X = \beta X^{\rho-1}\left[\beta X^{\rho} + (1-\beta) Y^{\rho}\right]^{\frac{1-\rho}{\rho}}$$

$$\lambda = \frac{\beta X^{\rho-1}\left[\beta X^{\rho} + (1-\beta) Y^{\rho}\right]^{\frac{1-\rho}{\rho}}}{P_X} \qquad (1)$$

$$\frac{\partial \mathcal{L}}{\partial Y} = \frac{1}{\rho}\left[\beta X^{\rho} + (1-\beta) Y^{\rho}\right]^{\frac{1}{\rho}-1}\left[\beta X^{\rho} + (1-\beta) Y^{\rho}\right]' - \lambda P_Y = 0$$

$$= \frac{1}{\cancel{\rho}}\left[\beta X^{\rho} + (1-\beta) Y^{\rho}\right]^{\frac{1-\rho}{\rho}}\left[\cancel{\rho}(1-\beta) Y^{\rho-1}\right] - \lambda P_Y = 0$$

$$= (1-\beta) Y^{\rho-1}\left[\beta X^{\rho} + (1-\beta) Y^{\rho}\right]^{\frac{1-\rho}{\rho}} - \lambda P_Y = 0$$

LOGO,

$$\lambda P_Y = (1-\beta) Y^{\rho-1}\left[\beta X^{\rho} + (1-\beta) Y^{\rho}\right]^{\frac{1-\rho}{\rho}}$$

$$\lambda = \frac{(1-\beta) Y^{\rho-1}\left[\beta X^{\rho} + (1-\beta) Y^{\rho}\right]^{\frac{1-\rho}{\rho}}}{P_Y} \qquad (2)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = W - P_X X - P_Y Y = 0 \therefore W = P_X X + P_Y Y \qquad (3)$$ 1.A

IGUALANDO AS EQUAÇÕES (1) E (2) OBTÉM-SE:

$$\frac{\beta X^{\rho-1} \cancel{[\beta X^{\rho} + (1-\beta)Y^{\rho}]^{\frac{1-\rho}{\rho}}}}{P_X} = \frac{(1-\beta)Y^{\rho-1} \cancel{[\beta X^{\rho} + (1-\beta)Y^{\rho}]^{\frac{1-\rho}{\rho}}}}{P_Y}$$

$$\frac{\beta X^{\rho-1}}{P_X} = \frac{(1-\beta)Y^{\rho-1}}{P_Y}$$

$$X^{\rho-1} = \frac{(1-\beta)}{\beta} \cdot \frac{P_X}{P_Y} \cdot Y^{\rho-1}$$

$$X = \left[\frac{(1-\beta)}{\beta} \cdot \frac{P_X}{P_Y} \cdot Y^{\rho-1}\right]^{\frac{1}{\rho-1}}$$

$$= Y \cdot \left[\frac{(1-\beta)}{\beta} \cdot \frac{P_X}{P_Y}\right]^{\frac{1}{\rho-1}} \qquad (4)$$

POR SUA VEZ,

$$Y^{\rho-1} = \frac{\beta}{(1-\beta)} \cdot \frac{P_Y}{P_X} \cdot X^{\rho-1}$$

$$Y = \left[\frac{\beta}{(1-\beta)} \cdot \frac{P_Y}{P_X} \cdot X^{\rho-1}\right]^{\frac{1}{\rho-1}}$$

$$= X \cdot \left[\frac{\beta}{(1-\beta)} \cdot \frac{P_Y}{P_X}\right]^{\frac{1}{\rho-1}} \qquad (5)$$

SUBSTITUINDO (4) EM (3) OBTÉM-SE:

$$W = P_X \cdot \left\{ Y \cdot \left[\frac{(1-\beta)}{\beta} \cdot \frac{P_X}{P_Y}\right]^{\frac{1}{\rho-1}} \right\} + P_Y \cdot Y$$

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$$w = P_x \cdot Y \left[\frac{(1-\beta)}{\beta}\right]^{\frac{1}{\rho-1}} \cdot \frac{P_x^{\frac{1}{\rho-1}}}{P_y^{\frac{1}{\rho-1}}} + P_y \cdot Y$$

$$w = Y \left\{ P_x^{\frac{\rho}{\rho-1}} \left[\frac{(1-\beta)}{\beta}\right]^{\frac{1}{\rho-1}} \cdot \frac{1}{P_y^{\frac{1}{\rho-1}}} + P_y \right\}$$

$$w = Y \left\{ \frac{P_x^{\frac{\rho}{\rho-1}} \cdot (1-\beta)^{\frac{1}{\rho-1}} \cdot \beta^{\frac{1}{1-\rho}}}{P_y^{\frac{1}{\rho-1}}} + P_y \right\}$$

$$w = Y \left\{ \frac{P_x^{\frac{\rho}{\rho-1}} \cdot (1-\beta)^{\frac{1}{\rho-1}} \cdot \beta^{\frac{1}{1-\rho}} + P_y \cdot P_y^{\frac{1}{\rho-1}}}{P_y^{\frac{1}{\rho-1}}} \right\}$$

$$w = Y \left\{ \frac{P_x^{\frac{\rho}{\rho-1}} \cdot (1-\beta)^{\frac{1}{\rho-1}} \cdot \beta^{\frac{1}{1-\rho}} + P_y^{\frac{\rho}{\rho-1}}}{P_y^{\frac{1}{\rho-1}}} \right\}$$

$$w \cdot P_y^{\frac{1}{\rho-1}} = Y \left[P_x^{\frac{\rho}{\rho-1}} \cdot (1-\beta)^{\frac{1}{\rho-1}} \cdot \beta^{\frac{1}{1-\rho}} + P_y^{\frac{\rho}{\rho-1}} \right.$$

$$Y = \frac{w \cdot P_y^{\frac{1}{\rho-1}}}{P_x^{\frac{\rho}{\rho-1}} \cdot (1-\beta)^{\frac{1}{\rho-1}} \cdot \beta^{\frac{1}{1-\rho}} + P_y^{\frac{\rho}{\rho-1}}}$$

$$= \frac{w \cdot P_y^{\frac{1}{\rho-1}}}{P_x^{\frac{\rho}{\rho-1}} \cdot \left(\frac{1-\beta}{\beta}\right)^{\frac{1}{\rho-1}} + P_y^{\frac{\rho}{\rho-1}}} \qquad (6)$$

SUBSTITUINDO (6) EM (4) OBTÉM-SE:

$$X = \frac{w \cdot P_y^{\frac{1}{\rho-1}} \left(\frac{1-\beta}{\beta} \cdot \frac{P_x}{P_y}\right)^{\frac{1}{\rho-1}}}{P_x^{\frac{\rho}{\rho-1}} \cdot \left(\frac{1-\beta}{\beta}\right)^{\frac{1}{\rho-1}} + P_y^{\frac{\rho}{\rho-1}}}$$

2.A

$$X = \frac{W \cdot \cancel{P_Y^{\frac{1}{\rho-1}}} \left(\frac{1-\beta}{\beta} \cdot P_X\right)^{\frac{1}{\rho-1}} \cdot \frac{1}{\cancel{P_Y^{\frac{1}{\rho-1}}}}}{P_X^{\frac{\rho}{\rho-1}} \left(\frac{1-\beta}{\beta}\right)^{\frac{1}{\rho-1}} + P_Y^{\frac{\rho}{\rho-1}}}$$

$$= \frac{W \left(\frac{1-\beta}{\beta} \cdot P_X\right)^{\frac{1}{\rho-1}}}{P_X^{\frac{\rho}{\rho-1}} \left(\frac{1-\beta}{\beta}\right)^{\frac{1}{\rho-1}} + P_Y^{\frac{\rho}{\rho-1}}}$$

$$= \frac{W \left(\frac{1-\beta}{\beta}\right)^{\frac{1}{\rho-1}} \cdot P_X^{\frac{1}{\rho-1}}}{P_X^{\frac{\rho}{\rho-1}} \left(\frac{1-\beta}{\beta}\right)^{\frac{1}{\rho-1}} + P_Y^{\frac{\rho}{\rho-1}}}$$

$$= \frac{W \cdot P_X^{\frac{1}{\rho-1}}}{\left(\frac{1-\beta}{\beta}\right)^{\frac{-1}{\rho-1}} \left[P_X^{\frac{\rho}{\rho-1}} \left(\frac{1-\beta}{\beta}\right)^{\frac{1}{\rho-1}} + P_Y^{\frac{\rho}{\rho-1}}\right]}$$

$$= \frac{W \cdot P_X^{\frac{1}{\rho-1}}}{P_X^{\frac{\rho}{\rho-1}} + P_Y^{\frac{\rho}{\rho-1}} \left(\frac{1-\beta}{\beta}\right)^{\frac{-1}{\rho-1}}}$$

$$= \frac{W \cdot P_X^{\frac{1}{\rho-1}}}{P_X^{\frac{\rho}{\rho-1}} + P_Y^{\frac{\rho}{\rho-1}} \left(\frac{\beta}{1-\beta}\right)^{\frac{1}{\rho-1}}}$$

ENCONTRANDO A FUNÇÃO UTILIDADE INDIRETA:

$$v(P_X, P_Y, W) = \left\{ \beta \left[\frac{W \left(\frac{1-\beta}{\beta}\right)^{\frac{1}{\rho-1}} \cdot P_X^{\frac{1}{\rho-1}}}{P_X^{\frac{\rho}{\rho-1}} \left(\frac{1-\beta}{\beta}\right)^{\frac{1}{\rho-1}} + P_Y^{\frac{\rho}{\rho-1}}} \right]^{\rho} + (1-\beta) \left[\frac{W \cdot P_Y^{\frac{1}{\rho-1}}}{P_X^{\frac{\rho}{\rho-1}} \left(\frac{1-\beta}{\beta}\right)^{\frac{1}{\rho-1}} + P_Y^{\frac{\rho}{\rho-1}}} \right]^{\rho} \right\}^{1/\rho}$$

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$$v(P_x, P_y, w) = \left\{ \beta \left[\frac{w^{\rho} \cdot \left(\frac{1-\beta}{\beta}\right)^{\frac{\rho}{\rho-1}} \cdot P_x^{\frac{\rho}{\rho-1}}}{\left[P_x^{\frac{\rho}{\rho-1}} \left(\frac{1-\beta}{\beta}\right)^{\frac{1}{\rho-1}} + P_y^{\frac{\rho}{\rho-1}}\right]^{\rho}} \right] + \right.$$

$$\left. + (1-\beta) \left[\frac{w^{\rho} \cdot P_y^{\frac{\rho}{\rho-1}}}{\left[P_x^{\frac{\rho}{\rho-1}} \left(\frac{1-\beta}{\beta}\right)^{\frac{1}{\rho-1}} + P_y^{\frac{\rho}{\rho-1}}\right]^{\rho}} \right] \right\}^{1/\rho}$$

$$= \left\{ \frac{\beta \cdot w^{\rho} \left(\frac{1-\beta}{\beta}\right)^{\frac{\rho}{\rho-1}} \cdot P_x^{\frac{\rho}{\rho-1}} + (1-\beta) \cdot w^{\rho} \cdot P_y^{\frac{\rho}{\rho-1}}}{\left[P_x^{\frac{\rho}{\rho-1}} \left(\frac{1-\beta}{\beta}\right)^{\frac{1}{\rho-1}} + P_y^{\frac{\rho}{\rho-1}}\right]^{\rho}} \right\}^{1/\rho}$$

$$= \left\{ \frac{w^{\rho} \left[\beta \cdot \left(\frac{1-\beta}{\beta}\right)^{\frac{\rho}{\rho-1}} \cdot P_x^{\frac{\rho}{\rho-1}} + (1-\beta) \cdot P_y^{\frac{\rho}{\rho-1}}\right]}{\left[P_x^{\frac{\rho}{\rho-1}} \left(\frac{1-\beta}{\beta}\right)^{\frac{1}{\rho-1}} + P_y^{\frac{\rho}{\rho-1}}\right]^{\rho}} \right\}^{1/\rho}$$

$$= w \left\{ \frac{\beta \cdot \left(\frac{1-\beta}{\beta}\right)^{\frac{\rho}{\rho-1}} \cdot P_x^{\frac{\rho}{\rho-1}} + (1-\beta) \cdot P_y^{\frac{\rho}{\rho-1}}}{\left[P_x^{\frac{\rho}{\rho-1}} \left(\frac{1-\beta}{\beta}\right)^{\frac{1}{\rho-1}} + P_y^{\frac{\rho}{\rho-1}}\right]^{\rho}} \right\}^{1/\rho}$$

$$= w \left\{ \frac{\beta \cdot \left(\frac{1-\beta}{\beta}\right)^{\frac{\rho}{\rho-1}} \cdot P_x^{\frac{\rho}{\rho-1}} + (1-\beta) \cdot P_y^{\frac{\rho}{\rho-1}}}{\left[P_x^{\frac{\rho}{\rho-1}} \left(\frac{1-\beta}{\beta}\right)^{\frac{1}{\rho-1}} + \left(\frac{1-\beta}{1-\beta}\right) P_y^{\frac{\rho}{\rho-1}}\right]^{\rho}} \right\}^{1/\rho}$$

$$= w \left\{ \frac{\beta \cdot \left(\frac{1-\beta}{\beta}\right)^{\frac{\rho}{\rho-1}} \cdot P_x^{\frac{\rho}{\rho-1}} + (1-\beta) \cdot P_y^{\frac{\rho}{\rho-1}}}{\left[\left[P_x^{\frac{\rho}{\rho-1}} (1-\beta)^{\frac{1}{\rho-1}} \cdot \beta^{\frac{1}{1-\rho}} \cdot (1-\beta) + (1-\beta) P_y^{\frac{\rho}{\rho-1}}\right] \frac{1}{1-\beta}\right]^{\rho}} \right\}^{1/\rho}$$

$$= w \left\{ \frac{\beta \left(\frac{1-\beta}{\beta}\right)^{\frac{\rho}{\rho-1}} \cdot P_x^{\frac{\rho}{\rho-1}} + (1-\beta) \cdot P_y^{\frac{\rho}{\rho-1}}}{\left[\left[P_x^{\frac{\rho}{\rho-1}} \cdot (1-\beta)^{\frac{\rho}{\rho-1}} \cdot \beta^{\frac{1}{1-\rho}} + (1-\beta) \cdot P_y^{\frac{\rho}{\rho-1}}\right]^{\rho} \frac{1}{1-\beta}\right]^{\rho}} \right\}^{1/\rho}$$

$$v(P_X,P_Y,w) = w\left\{\frac{\beta\cdot\beta^{-\frac{\rho}{\rho-1}}(1-\beta)^{\frac{\rho}{\rho-1}}\cdot P_X^{\frac{\rho}{\rho-1}}+(1-\beta)\cdot P_Y^{\frac{\rho}{\rho-1}}}{\left[\left[P_X^{\frac{\rho}{\rho-1}}\cdot(1-\beta)^{\frac{\rho}{\rho-1}}\cdot\beta^{\frac{1}{1-\rho}}+(1-\beta)\,P_Y^{\frac{\rho}{\rho-1}}\right]\frac{1}{(1-\beta)}\right]^{\rho}}\right\}^{\frac{1}{\rho}} \quad (3.A)$$

$$= w\left\{\frac{P_X^{\frac{\rho}{\rho-1}}(1-\beta)^{\frac{\rho}{\rho-1}}\cdot\beta^{\frac{1}{1-\rho}}+(1-\beta)P_Y^{\frac{\rho}{\rho-1}}}{\left[\left[P_X^{\frac{\rho}{\rho-1}}\cdot(1-\beta)^{\frac{\rho}{\rho-1}}\cdot\beta^{\frac{1}{1-\rho}}+(1-\beta)P_Y^{\frac{\rho}{\rho-1}}\right]\frac{1}{(1-\beta)}\right]^{\rho}}\right\}^{\frac{1}{\rho}}$$

$$= w\left\{\frac{\left[P_X^{\frac{\rho}{\rho-1}}(1-\beta)^{\frac{\rho}{\rho-1}}\cdot\beta^{\frac{1}{1-\rho}}+(1-\beta)\cdot P_Y^{\frac{\rho}{\rho-1}}\right]}{\left[P_X^{\frac{\rho}{\rho-1}}(1-\beta)^{\frac{\rho}{\rho-1}}\cdot\beta^{\frac{1}{1-\rho}}+(1-\beta)P_Y^{\frac{\rho}{\rho-1}}\right]^{\rho}\left(\frac{1}{1-\beta}\right)^{\rho}}\right\}^{\frac{1}{\rho}}$$

$$= w\left\{\left[P_X^{\frac{\rho}{\rho-1}}(1-\beta)^{\frac{\rho}{\rho-1}}\cdot\beta^{\frac{1}{1-\rho}}+(1-\beta)\cdot P_Y^{\frac{\rho}{\rho-1}}\right]^{1-\rho}\left(\frac{1}{1-\beta}\right)^{-\rho}\right\}^{\frac{1}{\rho}}$$

$$= w\left\{\left[\frac{P_X^{\frac{\rho}{\rho-1}}\cancel{(1-\beta)^{\frac{\rho}{\rho-1}}}\cdot\beta^{\frac{1}{1-\rho}}}{\cancel{(1-\beta)^{\frac{\rho}{\rho-1}}}}+\frac{(1-\beta)\cdot P_Y^{\frac{\rho}{\rho-1}}}{(1-\beta)^{\frac{\rho}{\rho-1}}}\right]^{1-\rho}\right\}^{\frac{1}{\rho}}$$

$$= w\left\{\left[P_X^{\frac{\rho}{\rho-1}}\cdot\beta^{\frac{1}{1-\rho}}+(1-\beta)^{\frac{1}{1-\rho}}\cdot P_Y^{\frac{\rho}{\rho-1}}\right]^{1-\rho}\right\}^{1/\rho}$$

$$= w\left[P_X^{\frac{\rho}{\rho-1}}\cdot\beta^{\frac{1}{1-\rho}}+(1-\beta)^{\frac{1}{1-\rho}}\cdot P_Y^{\frac{\rho}{\rho-1}}\right]^{\frac{1-\rho}{\rho}}$$

ENCONTRANDO A FUNÇÃO DESPESA:

$$\bar{u}(P_X,P_Y,w) = e\left[P_X^{\frac{\rho}{\rho-1}}\cdot\beta^{\frac{1}{1-\rho}}+(1-\beta)^{\frac{1}{1-\rho}}\cdot P_Y^{\frac{\rho}{\rho-1}}\right]^{\frac{1-\rho}{\rho}}$$

$$e(P_X,P_Y,\bar{u}) = \bar{u}\cdot\left[P_X^{\frac{\rho}{\rho-1}}\cdot\beta^{\frac{1}{1-\rho}}+(1-\beta)^{\frac{1}{1-\rho}}\cdot P_Y^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}}$$

CALCULANDO AS DEMANDAS HICKSIANAS:

$$x_H(P_X,P_Y,\bar{u}) = \frac{\partial e(P_X,P_Y,\bar{u})}{\partial P_{X_H}}$$

$$= \frac{\rho-1}{\rho}\cdot\bar{u}\cdot\left[P_X^{\frac{\rho}{\rho-1}}\cdot\beta^{\frac{1}{1-\rho}}+(1-\beta)^{\frac{1}{1-\rho}}\cdot P_Y^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}-1}\cdot$$

$$\cdot\left[P_X^{\frac{\rho}{\rho-1}}\cdot\beta^{\frac{1}{1-\rho}}+(1-\beta)^{\frac{1}{1-\rho}}\cdot P_Y^{\frac{\rho}{\rho-1}}\right]'$$

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$$x_H(P_x,P_y,\bar{u}) = \frac{\cancel{\rho-1}}{\rho}\cdot\bar{u}\cdot\left[P_x^{\frac{\rho}{\rho-1}}\cdot\beta^{\frac{1}{1-\rho}}+(1-\beta)^{\frac{1}{1-\rho}}\cdot P_y^{\frac{\rho}{\rho-1}}\right]^{-\frac{1}{\rho}}\cdot$$

$$\cdot\frac{\cancel{\rho}}{\rho-1}\cdot P_x^{\frac{\rho}{\rho-1}-1}\cdot\beta^{\frac{1}{1-\rho}}$$

$$= \bar{u}\cdot P_x^{\frac{1}{\rho-1}}\cdot\beta^{\frac{1}{1-\rho}}\left[P_x^{\frac{\rho}{\rho-1}}\cdot\beta^{\frac{1}{1-\rho}}+(1-\beta)^{\frac{1}{1-\rho}}\cdot P_y^{\frac{\rho}{\rho-1}}\right]^{-\frac{1}{\rho}}$$

$$y_H(P_x,P_y,\bar{u}) = \frac{\partial e(P_x,P_y,\bar{u})}{\partial P_{y_H}} \Rightarrow \text{LEMA DE SHEPARD}$$

$$= \frac{\rho-1}{\rho}\cdot\bar{u}\cdot\left[P_x^{\frac{\rho}{\rho-1}}\cdot\beta^{\frac{1}{1-\rho}}+(1-\beta)^{\frac{1}{1-\rho}}\cdot P_y^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}-1}\cdot$$

$$\cdot\left[P_x^{\frac{\rho}{\rho-1}}\cdot\beta^{\frac{1}{1-\rho}}+(1-\beta)^{\frac{1}{1-\rho}}\cdot P_y^{\frac{\rho}{\rho-1}}\right]'$$

$$= \frac{\cancel{\rho-1}}{\rho}\cdot\bar{u}\left[P_x^{\frac{\rho}{\rho-1}}\cdot\beta^{\frac{1}{1-\rho}}+(1-\beta)^{\frac{1}{1-\rho}}\cdot P_y^{\frac{\rho}{\rho-1}}\right]^{-\frac{1}{\rho}}\cdot$$

$$\cdot\frac{\cancel{\rho}}{\cancel{\rho-1}}\cdot(1-\beta)^{\frac{1}{1-\rho}}\cdot P_y^{\frac{\rho}{\rho-1}-1}$$

$$= \bar{u}\cdot(1-\beta)^{\frac{1}{1-\rho}}\cdot P_y^{\frac{1}{\rho-1}}\left[P_x^{\frac{\rho}{\rho-1}}\cdot\beta^{\frac{1}{1-\rho}}+(1-\beta)^{\frac{1}{1-\rho}}\cdot P_y^{\frac{\rho}{\rho-1}}\right]^{-\frac{1}{\rho}}$$

MOSTRANDO QUE A FUNÇÃO UTILIDADE INDIRETA É HOMOGÊNEA DE GRAU ZERO NOS PREÇOS E RENDA:

$$v(P_x,P_y,w) = w\left[P_x^{\frac{\rho}{\rho-1}}\cdot\beta^{\frac{1}{1-\rho}}+(1-\beta)^{\frac{1}{1-\rho}}\cdot P_y^{\frac{\rho}{\rho-1}}\right]^{\frac{1-\rho}{\rho}}$$

$$= tw\left[(tP_x)^{\frac{\rho}{\rho-1}}\cdot\beta^{\frac{1}{1-\rho}}+(1-\beta)^{\frac{1}{1-\rho}}\cdot(t\cdot P_y)^{\frac{\rho}{\rho-1}}\right]^{\frac{1-\rho}{\rho}}$$

$$= tw\left[t^{\frac{\rho}{\rho-1}}\cdot P_x^{\frac{\rho}{\rho-1}}\cdot\beta^{\frac{1}{1-\rho}}+(1-\beta)^{\frac{1}{1-\rho}}\cdot t^{\frac{\rho}{\rho-1}}\cdot P_y^{\frac{\rho}{\rho-1}}\right]^{\frac{1-\rho}{\rho}}$$

$$= tw\left\{t^{\frac{\rho}{\rho-1}}\left[P_x^{\frac{\rho}{\rho-1}}\cdot\beta^{\frac{1}{1-\rho}}+(1-\beta)^{\frac{1}{1-\rho}}\cdot P_y^{\frac{\rho}{\rho-1}}\right]^{\frac{1-\rho}{\rho}}\right\}$$

$$= tw\left\{t^{\frac{\rho}{-(1-\rho)}}\left[P_x^{\frac{\rho}{\rho-1}}\cdot\beta^{\frac{1}{1-\rho}}+(1-\beta)^{\frac{1}{1-\rho}}\cdot P_y^{\frac{\rho}{\rho-1}}\right]^{\frac{1-\rho}{\rho}}\right\}$$

$$v(P_x, P_y, w) = t \cdot t^{-1} \cdot w \left[P_x^{\frac{\rho}{\rho-1}} \cdot \beta^{\frac{1}{1-\rho}} + (1-\beta)^{\frac{1}{1-\rho}} \cdot P_y^{\frac{\rho}{\rho-1}} \right]^{\frac{1-\rho}{\rho}}$$

$$= t^{1-1} \cdot w \ldots$$

$$= t^{0} \cdot w \ldots$$

$$= 1 \cdot w \ldots$$

$$= w \ldots \quad ■$$

DEMONSTRANDO QUE AS DEMANDAS HICKSIANAS SÃO HOMOGÊNEAS DE GRAU ZERO NOS PREÇOS:

$$X_H(P_x, P_y, \bar{u}) = \bar{u} \cdot (1-\beta)^{\frac{1}{1-\rho}} \cdot P_y^{\frac{1}{\rho-1}} \left[P_x^{\frac{\rho}{\rho-1}} \cdot \beta^{\frac{1}{1-\rho}} + (1-\beta)^{\frac{1}{1-\rho}} \cdot P_y^{\frac{\rho}{\rho-1}} \right]^{-1/\rho}$$

$$= \bar{u} \cdot (1-\beta)^{\frac{1}{1-\rho}} \cdot (t \cdot P_y)^{\frac{1}{\rho-1}} \left[(t \cdot P_x)^{\frac{\rho}{\rho-1}} \cdot \beta^{\frac{1}{1-\rho}} + (1-\beta)^{\frac{1}{1-\rho}} \cdot (t \cdot P_y)^{\frac{\rho}{\rho-1}} \right]^{-1/\rho}$$

$$= \bar{u} \cdot (1-\beta)^{\frac{1}{1-\rho}} \cdot t^{\frac{1}{\rho-1}} \cdot P_y^{\frac{1}{\rho-1}} \left[t^{\frac{\rho}{\rho-1}} \cdot P_x^{\frac{\rho}{\rho-1}} \cdot \beta^{\frac{1}{1-\rho}} + (1-\beta)^{\frac{1}{1-\rho}} \cdot t^{\frac{\rho}{\rho-1}} \cdot P_y^{\frac{\rho}{\rho-1}} \right]^{-1/\rho}$$

$$= \bar{u} \cdot (1-\beta)^{\frac{1}{1-\rho}} \cdot t^{\frac{1}{\rho-1}} \cdot P_y^{\frac{1}{\rho-1}} \left\{ t^{\frac{\rho}{\rho-1}} \left[P_x^{\frac{\rho}{\rho-1}} \cdot \beta^{\frac{1}{1-\rho}} + (1-\beta)^{\frac{1}{1-\rho}} \cdot P_y^{\frac{\rho}{\rho-1}} \right] \right\}^{-1/\rho}$$

$$= \bar{u} \cdot (1-\beta)^{\frac{1}{1-\rho}} \cdot \cancel{t^{\frac{1}{\rho-1}}} \cdot \cancel{t^{-\frac{1}{\rho-1}}} \cdot P_y^{\frac{1}{\rho-1}} \left[P_x^{\frac{\rho}{\rho-1}} \ldots \right.$$

$$= \bar{u} \cdot (1-\beta)^{\frac{1}{1-\rho}} \cdot P_y^{\frac{1}{\rho-1}} \left[P_x^{\frac{\rho}{\rho-1}} \ldots \right. \quad ■$$

A MESMA OPERAÇÃO APLICADA EM X_H PODE SER APLICADA EM Y_H DEMONSTRANDO TAMBÉM QUE ESSA DEMANDA TAMBÉM É HOMOGÊNEA DE GRAU ZERO NOS PREÇOS.

DEMONSTRANDO QUE A FUNÇÃO DESPESA É HOMOGÊNEA DE GRAU UM NOS PREÇOS:

$$e(P_x, P_y, \bar{u}) = \bar{u} \cdot \left[P_x^{\frac{\rho}{\rho-1}} \cdot \beta^{\frac{1}{1-\rho}} + (1-\beta)^{\frac{1}{1-\rho}} \cdot P_y^{\frac{\rho}{\rho-1}} \right]^{\frac{\rho-1}{\rho}}$$

$$= \bar{u} \cdot \left[(t \cdot P_x)^{\frac{\rho}{\rho-1}} \cdot \beta^{\frac{1}{1-\rho}} + (1-\beta)^{\frac{1}{1-\rho}} \cdot (t \cdot P_y)^{\frac{\rho}{\rho-1}} \right]^{\frac{\rho-1}{\rho}}$$

$$= \bar{u} \cdot \left[t^{\frac{\rho}{\rho-1}} \cdot P_x^{\frac{\rho}{\rho-1}} \cdot \beta^{\frac{1}{1-\rho}} + (1-\beta)^{\frac{1}{1-\rho}} \cdot t^{\frac{\rho}{\rho-1}} \cdot P_y^{\frac{\rho}{\rho-1}} \right]^{\frac{\rho-1}{\rho}}$$

$$= \bar{u} \cdot \left\{ t^{\frac{\rho}{\rho-1}} \left[P_x^{\frac{\rho}{\rho-1}} \cdot \beta^{\frac{1}{1-\rho}} + (1-\beta)^{\frac{1}{1-\rho}} \cdot P_y^{\frac{\rho}{\rho-1}} \right] \right\}^{\frac{\rho-1}{\rho}}$$

$$= \bar{u} \cdot t \left[P_x^{\frac{\rho}{\rho-1}} \cdot \beta^{\frac{1}{1-\rho}} + (1-\beta)^{\frac{1}{1-\rho}} \cdot P_y^{\frac{\rho}{\rho-1}} \right]^{\frac{\rho-1}{\rho}} \quad ■$$

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EXERCÍCIO 3:

(3.A) $u(x_1, x_2) = x_1^{1/2} x_2^{1/2}$

PROBLEMA DE MAXIMIZAÇÃO DA UTILIDADE - PMU

$\max x_1^{1/2} x_2^{1/2}$ S. A $P_1 \cdot x_1 + P_2 \cdot x_2 = w$

x_1 E $x_2 \geq 0$

MONTANDO O LAGRANGIANO:

$$\mathcal{L}(x_1, x_2, \lambda) = x_1^{1/2} x_2^{1/2} + \lambda (w - P_1 x_1 - P_2 x_2)$$

$$\frac{\partial \mathcal{L}}{\partial x_1} = \frac{1}{2} x_1^{-1/2} \cdot x_2^{1/2} - \lambda . P_1 = 0 \therefore \lambda = \frac{1}{2} \frac{x_1^{-1/2} \cdot x_2^{1/2}}{P_1} \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = \frac{1}{2} x_1^{1/2} \cdot x_2^{-1/2} - \lambda . P_2 = 0 \therefore \lambda = \frac{1}{2} \frac{x_1^{1/2} \cdot x_2^{-1/2}}{P_2} \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = w - P_1 \cdot x_1 - P_2 \cdot x_2 = 0 \therefore w = P_1 \cdot x_1 + P_2 \cdot x_2 \quad (3)$$

IGUALANDO AS EQUAÇÕES (1) E (2) OBTÉM-SE:

$$\frac{1}{2} \frac{x_1^{-1/2} \cdot x_2^{1/2}}{P_1} = \frac{1}{2} \frac{x_1^{1/2} \cdot x_2^{-1/2}}{P_2}$$

$$\frac{x_2^{1/2}}{x_2^{-1/2}} = \frac{2}{2} \cdot \frac{P_1}{P_2} \cdot \frac{x_1^{1/2}}{x_1^{-1/2}}$$

$$x_2 = \frac{P_1}{P_2} \cdot x_1 \quad (4)$$

SUBSTITUINDO (4) EM (3) OBTÉM-SE

$$w = P_1 \cdot x_1 + \cancel{P_2} \cdot \left(\frac{P_1}{\cancel{P_2}} \cdot x_1 \right)$$

(5.A)

$$W = P_1 \cdot X_1 + P_1 \cdot X_1$$

$$W = 2 P_1 \cdot X_1 \quad \therefore \quad \boxed{X_1 = \frac{1}{2} \cdot \frac{W}{P_1}} \qquad (5)$$

SUBSTITUINDO (5) EM (4) OBTÉM-SE:

$$X_2 = \frac{\cancel{P_1}}{P_2} \cdot \frac{1}{2} \cdot \frac{W}{\cancel{P_1}}$$

$$\boxed{= \frac{1}{2} \frac{W}{P_2}}$$

CALCULANDO A FUNÇÃO DE UTILIDADE INDIRETA:

$$v(P_1, P_2, W) = \left(\frac{1}{2} \cdot \frac{W}{P_1}\right)^{1/2} \cdot \left(\frac{1}{2} \cdot \frac{W}{P_2}\right)^{1/2}$$

$$= \left(\frac{1}{2}\right)^{1/2} \cdot W^{1/2} \cdot \left(\frac{1}{P_1}\right)^{1/2} \cdot \left(\frac{1}{2}\right)^{1/2} \cdot W^{1/2} \cdot \left(\frac{1}{P_2}\right)^{1/2}$$

$$\boxed{= \frac{1}{2} \cdot W \cdot \left(\frac{1}{P_1 \cdot P_2}\right)^{1/2}}$$

CALCULANDO A FUNÇÃO DESPESA:

$$\bar{u} = \frac{1}{2} \cdot e \cdot \left(\frac{1}{P_1 \cdot P_2}\right)^{1/2}$$

$$2 \cdot (P_1 \cdot P_2)^{1/2} \cdot \bar{u} = e(P_1, P_2, u)$$

$$\boxed{e(P_1, P_2, u) = 2 \cdot \bar{u} \cdot (P_1 \cdot P_2)^{1/2}}$$

CALCULANDO AS DEMANDAS HICKSIANAS:

$$X_1^H(P_1, P_2, \bar{u}) = \frac{\partial e(P_1, P_2, \bar{u})}{\partial P_1} \quad \Rightarrow \text{LEMA DE SHEPARD}$$

$$X_1^H(P_1,P_2,\bar{u}) = \frac{1}{\cancel{2}1}\cdot\cancel{2}^1\cdot\bar{u}\cdot(P_1\cdot P_2)^{-1/2}(P_1\cdot P_2)'$$

$$= \bar{u}\cdot P_2\cdot(P_1\cdot P_2)^{-1/2}$$

$$\boxed{= \bar{u}\cdot P_1^{-1/2}\cdot P_2^{1/2}}$$

$$X_2^H(P_1,P_2,\bar{u}) = \frac{\partial e(P_1,P_2,\bar{u})}{\partial P_2} \Rightarrow \text{LEMA DE SHEPARD}$$

$$= \frac{1}{\cancel{2}^1}\cdot\cancel{2}^1\cdot\bar{u}\cdot(P_1\cdot P_2)^{-1/2}(P_1\cdot P_2)'$$

$$= \bar{u}\cdot P_1\cdot(P_1\cdot P_2)^{-1/2}$$

$$\boxed{= \bar{u}\cdot P_1^{1/2}\cdot P_2^{-1/2}}$$

3.3 $u(X_1,X_2) = (X_1^{-1}+X_2^{-1})^{-1}$

PROBLEMA DE MAXIMIZAÇÃO DA UTILIDADE - PMU

$\max\ (X_1^{-1}+X_2^{-1})^{-1}$ S. A $P_1X_1+P_2X_2 = w$

$X_1 \text{ e } X_2 \geq 0$

MONTANDO O LAGRANGEANO:

$$\mathcal{L}(x_1,x_2,\lambda) = (x_1^{-1}+x_2^{-1})^{-1} + \lambda(w - P_1x_1 - P_2x_2)$$

$$\frac{\partial\mathcal{L}}{\partial x_1} = -1(x_1^{-1}+x_2^{-1})^{-2}(x_1^{-1}+x_2^{-1})' - \lambda P_1 = 0$$

$$= x_1^{-2}(x_1^{-1}+x_2^{-1})^{-2} - \lambda P_1 = 0 \therefore \lambda = \frac{x_1^{-2}(x_1^{-1}+x_2^{-1})^{-2}}{P_1} \quad (1)$$

$$\frac{\partial\mathcal{L}}{\partial x_2} = -1(x_1^{-1}+x_2^{-1})^{-2}(x_1^{-1}+x_2^{-1})' - \lambda P_2 = 0$$

$$= x_2^{-2}(x_1^{-1}+x_2^{-1})^{-2} - \lambda P_2 = 0 \therefore \lambda = \frac{x_2^{-2}(x_1^{-1}+x_2^{-1})^{-2}}{P_2} \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = W - P_1 X_1 - P_2 X_2 = 0 \quad \therefore \quad W = P_1 X_1 + P_2 X_2 \qquad (3)$$

6.A

IGUALANDO AS EQUAÇÕES (1) E (2) OBTÉM-SE:

$$\frac{X_1^{-2}\cancel{(X_1^{-1} + X_2^{-1})^{-2}}}{P_1} = \frac{X_2^{-2}\cancel{(X_1^{-1} + X_2^{-1})^{-2}}}{P_2}$$

$$\frac{1}{X_1^2} = \frac{P_1}{P_2} \cdot \frac{1}{X_2^2}$$

$$X_2^2 = \frac{P_1}{P_2} \cdot X_1^2 \quad \therefore \quad X_2 = \left(\frac{P_1}{P_2}\right)^{1/2} \cdot X_1 \qquad (4)$$

SUBSTITUINDO (4) EM (3) OBTÉM-SE:

$$W = P_1 \cdot X_1 + P_2 \cdot \left(\frac{P_1}{P_2}\right)^{1/2} \cdot X_1$$

$$W = P_1 \cdot X_1 + P_2 \cdot \frac{P_1^{1/2}}{P_2^{1/2}} \cdot X_1$$

$$W = P_1 \cdot X_1 + P_2^{1/2} \cdot P_1^{1/2} \cdot X_1$$

$$W = X_1 \cdot (P_1 + P_2^{1/2} \cdot P_1^{1/2})$$

$$\boxed{X_1 = \frac{W}{P_1 + (P_1 \cdot P_2)^{1/2}}} \qquad (5)$$

SUBSTITUINDO (5) EM (4) OBTÉM-SE:

$$X_2 = \left(\frac{P_1}{P_2}\right)^{1/2} \cdot \frac{W}{P_1 + (P_1 \cdot P_2)^{1/2}}$$

$$= \frac{P_1^{1/2} \cdot P_2^{-1/2} \cdot W}{P_1 + (P_1 \cdot P_2)^{1/2}}$$

$$\rightarrow X_2 = \frac{P_1^{1/2} \cdot P^{-1/2} \cdot W}{P_1(1 + P_1^{-1/2} \cdot P_2^{1/2})}$$

$$\boxed{= \frac{W}{(P_1 \cdot P_2)^{1/2} + P_2}}$$

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(17)

CALCULANDO A FUNÇÃO DE UTILIDADE INDIRETA:

$$v(P_1, P_2, w) = \left[\left(\frac{w}{P_1 + (P_1 \cdot P_2)^{1/2}}\right)^{-1} + \left(\frac{w}{(P_1 \cdot P_2)^{1/2} + P_2}\right)^{-1}\right]^{-1}$$

$$= \left\{\frac{w^{-1}}{[P_1 + (P_1 \cdot P_2)^{1/2}]^{-1}} + \frac{w^{-1}}{[(P_1 \cdot P_2)^{1/2} + P_2]^{-1}}\right\}^{-1}$$

$$= \{w^{-1}[P_1 + (P_1 \cdot P_2)^{1/2}] + w^{-1}[(P_1 \cdot P_2)^{1/2} + P_2]\}^{-1}$$

$$= \{w^{-1}[P_1 + (P_1 \cdot P_2)^{1/2} + (P_1 \cdot P_2)^{1/2} + P_2]\}^{-1}$$

$$\boxed{= w[P_1 + P_2 + 2(P_1 \cdot P_2)^{1/2}]^{-1}}$$

OBTENDO A FUNÇÃO DESPESA:

$$\bar{u} = e[P_1 + P_2 + 2(P_1 \cdot P_2)^{1/2}]^{-1}$$

$$e(P_1, P_2, \bar{u}) = \frac{\bar{u}}{[P_1 + P_2 + 2(P_1 \cdot P_2)^{1/2}]^{-1}}$$

$$\boxed{= \bar{u}\,[P_1 + P_2 + 2(P_1 \cdot P_2)^{1/2}]}$$

CALCULANDO AS DEMANDAS HICKSIANAS:

$$X_1^H = \frac{\partial e(P_1, P_2, \bar{u})}{\partial P_1} \quad \Rightarrow \text{LEMA DE SHEPARD}$$

$$= \bar{u} \cdot [P_1 + P_2 + 2(P_1 \cdot P_2)^{1/2}]' + \cancel{[P_1 + P_2 + 2(P_1 \cdot P_2)^{1/2}](\bar{u})'}$$

$$= \bar{u} \cdot [1 + \cancel{2} \cdot \frac{1}{\cancel{2}} \cdot (P_1 \cdot P_2)^{-1/2} \cdot (P_1 \cdot P_2)']$$

$$= \bar{u} \cdot [1 + (P_1 \cdot P_2)^{-1/2} \cdot P_2] \quad \therefore \quad \boxed{X_1^H = \bar{u} \cdot [1 + P_1^{-1/2} \cdot P_2^{1/2}]}$$

(7.A)

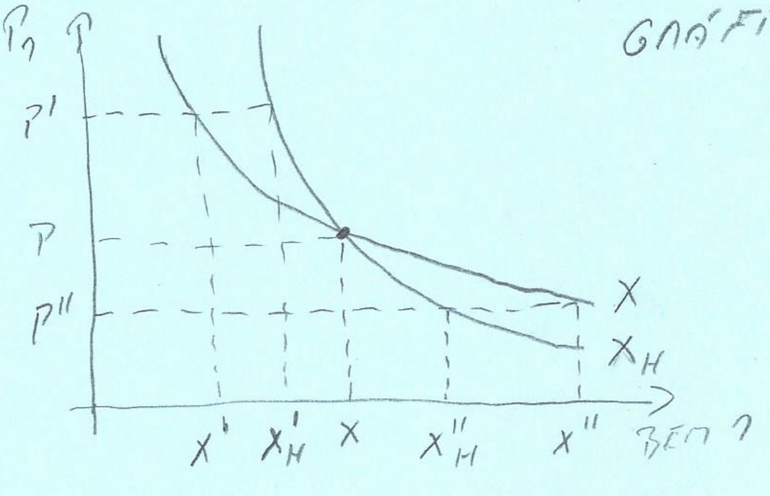
$$X_2^H(P_1,P_2,\bar{u}) = \frac{\partial e(P_1,P_2,\bar{u})}{\partial P_2} \quad \text{2, LEMA DE SHEPARD}$$

$$= \bar{u}.[P_1+P_2+2(P_1.P_2)^{1/2}]' + \cancel{[P_1+P_2+2(P_1.P_2)^{1/2}](\bar{u})'}$$

EXERCÍCIO 4:

REPRESENTAÇÃO GRÁFICA PARA AS DEMANDAS MARSHALIANAS E HICKSIANAS PARA UM BEM NORMAL:

GRÁFICO 1



QUANDO O BEM É NORMAL A CURVA DE DEMANDA MARSHALIANA É MENOS INCLINADA NEGATIVAMENTE DO QUE A CURVA DE DEMANDA HICKSIANA. SENDO ASSIM, SE O PREÇO DO BEM 1 AUMENTAR, O QUE TERÁ UM EFEITO DE UMA REDUÇÃO NA RENDA DO CONSUMIDOR, A REDUÇÃO DA DEMANDA MARSHALIANA SERÁ MAIOR DO QUE NA HICKSIANA HAJA VISTO QUE NESSA ÚLTIMA HÁ UMA COMPENSAÇÃO - COMPENSAÇÃO DE HICKS - PARA QUE O CONSUMIDOR SE MANTENHA NA MESMA UTILIDADE EM QUE ESTAVA ANTES DO AUMENTO DO PREÇO. POR SUA VEZ, SE HOUVER UMA REDUÇÃO DO PREÇO DO BEM 1, O QUE TERÁ UM EFEITO DE UM AUMENTO NA RENDA DO CONSUMIDOR, O AUMENTO DA DEMANDA MARSHALIANA SERÁ MAIOR DO QUE NA DEMANDA HICKSIANA, POIS A COMPENSAÇÃO HICKSIANA NESSE CASO IRÁ TIRAR

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RENDA DO CONSUMIDOR PARA MANTE-LO SOBRE A CURVA DE UTILIDADE EM QUE ESTAVA ANTES DA REDUÇÃO DO PREÇO DO BEM 1.

REPRESENTAÇÃO GRÁFICA PARA AS DEMANDAS MARSHALIANA E HICKSIANA DE UM BEM INFERIOR:

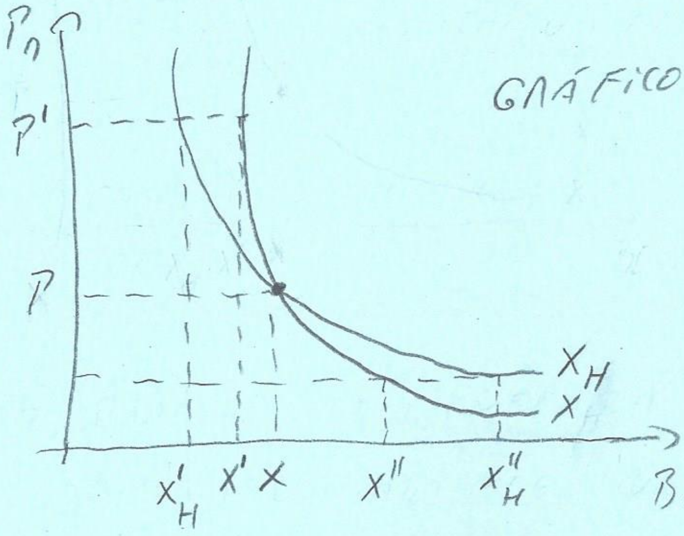

GRÁFICO 2

POR SUA VEZ, QUANDO O BEM É INFERIOR A CURVA DE DEMANDA HICKSIANA É QUE SERÁ MENOS INCLINADA NEGATIVAMENTE DO QUE A DEMANDA MARSHALIANA. NESSE CASO, SE HOUVER UM AUMENTO DO PREÇO DO BEM 1, O QUE TERÁ UM EFEITO DE UMA REDUÇÃO NA RENDA DO CONSUMIDOR, A DEMANDA HICKSIANA REDUZIRÁ MAIS DO QUE A DEMANDA MARSHALIANA. POIS NAQUELA DEMANDA HÁ A COMPENSAÇÃO DE HICKS PARA MANTER O CONSUMIDOR NO NÍVEL INICIAL DE UTILIDADE, ISTO É, NA UTILIDADE ANTES DO AUMENTO DO PREÇO. PORTANTO, A COMPENSAÇÃO HICKSIANA TERÁ UM EFEITO DE AUMENTO NA RENDA DO CONSUMIDOR E UMA VEZ QUE SE TRATA DE UM BEM INFERIOR A DEMANDA POR ESSE DIMINUIRÁ.

POR OUTRO LADO, SE O PREÇO DO BEM 1 DIMINUIR, CUJO EFEITO SERÁ DE UM AUMENTO NA RENDA DO CONSUMIDOR, O AUMENTO DA DEMANDA MARSHALIANA SERÁ INFERIOR A DA DEMANDA HICKSIANA, POIS, NESSA ÚLTIMA, O EFEITO DA COMPENSAÇÃO DE HICKS SERÁ DE UMA RE-

DUÇÃO NA RENDA DO CONSUMIDOR E, DESSA FORMA, COM MENOS (8.A)
RENDA ESSE IRÁ CONSUMIR BENS INFERIORES.

EXERCÍCIO 5:

O MULTIPLICADOR DE LAGRANGE NO PROBLEMA DE MINIMIZAÇÃO DE DESPESA É CONHECIDO COMO DESPESA MARGINAL DA UTILIDADE QUE PODE SER INTERPRETADO COMO SENDO QUAL A VARIAÇÃO MÍNIMA DA UTILIDADE QUE RESULTARÁ NUMA VARIAÇÃO MÍNIMA DA DESPESA. $\lambda = \dfrac{\partial e}{\partial u}$.
$\lambda = \dfrac{\partial u}{\partial w} \Rightarrow$ PMU ($\lambda \Rightarrow$ UTILIDADE MARGINAL DA RENDA).

EXERCÍCIO 6:

SEGUNDO A IDENTIDADE DE ROY É POSSÍVEL RECUPERAR AS DEMANDAS MARSHALIANAS POR MEIO DA FUNÇÃO UTILIDADE INDIRETA SENDO ESSA RELAÇÃO DADA POR:

$$X_\ell(P,w) = -\frac{\dfrac{\partial v(P,w)}{\partial P_\ell}}{\dfrac{\partial v(P,w)}{\partial w}}$$

POR SUA VEZ, PELO LEMA DE SHEPARD É POSSÍVEL RECUPERAR AS DEMANDAS HICKSIANAS POR MEIO DA FUNÇÃO DESPESA SENDO ESSA RELAÇÃO DADA POR:

$$X_\ell^H(P,\bar{u}) = \frac{\partial e(P,\bar{u})}{\partial P_\ell}$$

EXERCÍCIO 7:

(7.A) VERIFICANDO A PROPOSIÇÃO 3.6.1 DO MAS-COLELL

$$\min_{X_1 \text{ E } X_2 \geq 0} P_1X_1 + P_2X_2 \qquad \text{S. A} \quad u(x) = X_1^{1/2} \cdot X_2^{1/2} = \bar{u}$$

MONTANDO O LAGRANGIANO:

$$\mathcal{L}(X_1, X_2, \lambda) = P_1X_1 + P_2X_2 - \lambda(u(x) - \bar{u} \geq 0)$$

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$$\frac{\partial \mathcal{L}}{\partial x_1} = P_1 - \frac{1}{2}\cdot\lambda\cdot x_1^{-1/2}\cdot x_2^{1/2} = 0 \therefore \lambda = 2\cdot P_1\cdot x_1^{1/2}\cdot x_2^{-1/2} \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = P_2 - \frac{1}{2}\cdot\lambda\cdot x_1^{1/2}\cdot x_2^{-1/2} = 0 \therefore \lambda = 2\cdot P_2\cdot x_1^{-1/2}\cdot x_2^{1/2} \quad (2)$$

IGUALANDO (1) E (2) OBTÉM-SE:

$$2\cdot P_1\cdot x_1^{1/2}\cdot x_2^{-1/2} = 2\cdot P_2\cdot x_1^{-1/2}\cdot x_2^{1/2}$$

$$\frac{x_1^{1/2}}{x_1^{-1/2}} = \frac{2}{2}\cdot\frac{P_2}{P_1}\cdot\frac{x_2^{1/2}}{x_2^{-1/2}}$$

$$x_1 = \frac{P_2}{P_1}\cdot x_2 \quad (3)$$

SUBSTITUINDO (3) NA RESTRIÇÃO OBTÉM-SE:

$$\left(\frac{P_2}{P_1}\cdot x_2^H\right)^{1/2}\cdot x_2^{H^{1/2}} = \bar{u}$$

$$\left(\frac{P_2}{P_1}\right)^{1/2}\cdot x_2^{H^{1/2}}\cdot x_2^{H^{1/2}} = \bar{u}$$

$$\boxed{x_2^H = \bar{u}\cdot\left(\frac{P_1}{P_2}\right)^{1/2}} \quad (4)$$

SUBSTITUINDO (4) EM (3) OBTÉM-SE:

$$x_1^H = \frac{P_2}{P_1}\cdot\bar{u}\cdot\frac{P_1^{1/2}}{P_2^{1/2}}$$

$$= \bar{u}\cdot\frac{P_2^{1/2}}{P_1^{1/2}}$$

$$\boxed{x_1^H = \bar{u}\cdot\left(\frac{P_2}{P_1}\right)^{1/2}}$$

9.A

CALCULANDO A FUNÇÃO DESPESA:

$$e(P_1, P_2, \bar{u}) = P_1 \left[\bar{u} \cdot \left(\frac{P_2}{P_1} \right)^{1/2} \right] + P_2 \left[\bar{u} \cdot \left(\frac{P_1}{P_2} \right)^{1/2} \right]$$

$$= \bar{u} \cdot P_1 \cdot \frac{P_2^{1/2}}{P_1^{1/2}} + \bar{u} \cdot P_2 \cdot \frac{P_1^{1/2}}{P_2^{1/2}}$$

$$= \bar{u} \cdot P_1^{1/2} \cdot P_2^{1/2} + \bar{u} \cdot P_2^{1/2} \cdot P_1^{1/2}$$

$$\boxed{= 2 \cdot \bar{u} \cdot P_1^{1/2} \cdot P_2^{1/2}}$$

RECUPERANDO AS DEMANDAS HICKSIANAS ATRAVÉS DA FUNÇÃO DESPESA:

$$X_1^H(P_1, P_2, \bar{u}) = \frac{\partial e(P_1, P_2, \bar{u})}{\partial P_1} \Rightarrow \text{LEMA DE SHEPARD}$$

$$= \overset{1}{\cancel{2}} \cdot \frac{1}{\underset{1}{\cancel{2}}} \cdot \bar{u} \cdot P_1^{-1/2} \cdot P_2^{1/2}$$

$$= \bar{u} \cdot \frac{P_2^{1/2}}{P_1^{1/2}}$$

$$\boxed{= \bar{u} \cdot \left(\frac{P_2}{P_1} \right)^{1/2}} \quad \blacksquare$$

$$X_2^H(P_1, P_2, \bar{u}) = \frac{\partial e(P_1, P_2, \bar{u})}{\partial P_2} \Rightarrow \text{LEMA DE SHEPARD}$$

$$= \overset{1}{\cancel{2}} \cdot \frac{1}{\underset{1}{\cancel{2}}} \cdot \bar{u} \cdot P_1^{1/2} \cdot P_2^{-1/2}$$

$$= \bar{u} \cdot \frac{P_1^{1/2}}{P_2^{1/2}}$$

$$\boxed{= \bar{u} \cdot \left(\frac{P_1}{P_2} \right)^{1/2}} \quad \blacksquare$$

7.B) VERIFICANDO A PROPOSIÇÃO 3.G.2 DO MAS-COLELL

i) $D_p x_H(P_1, P_2, \bar{u}) = D_p^2 e(P, u)$

$$\frac{\partial x_1^H(P_1,P_2,\bar{u})}{\partial P_1} = -\frac{1}{2}\,\bar{u}.\,P_1^{-3/2}.\,P_2^{1/2} \qquad \frac{\partial x_1^H(P_1,P_2,\bar{u})}{\partial P_2} = \frac{1}{2}.\,\bar{u}.\,P_1^{-1/2}.\,P_2^{-1/2}$$

$$\frac{\partial x_2^H(P_1,P_2,\bar{u})}{\partial P_1} = \frac{1}{2}.\,\bar{u}.\,P_1^{-1/2}.\,P_2^{-1/2} \qquad \frac{\partial x_2^H(P_1,P_2,\bar{u})}{\partial P_2} = -\frac{1}{2}.\,\bar{u}.\,P_1^{1/2}.\,P_2^{-3/2}$$

$$\frac{\partial e(P_1,P_2,\bar{u})}{\partial P_1} = \bar{u}.\,P_1^{-1/2}.\,P_2^{1/2} \qquad \frac{\partial e(P_1,P_2,\bar{u})}{\partial P_2} = \bar{u}.\,P_1^{1/2}.\,P_2^{-1/2}$$

$$\frac{\partial^2 e(P_1,P_2,\bar{u})}{\partial P_1^2} = -\frac{1}{2}.\,\bar{u}.\,P_1^{-3/2}.\,P_2^{1/2} \qquad \frac{\partial^2 e(P_1,P_2,\bar{u})}{\partial P_1 \partial P_2} = \frac{1}{2}.\,\bar{u}.\,P_1^{-1/2}.\,P_2^{-1/2}$$

$$\frac{\partial^2 e(P_1,P_2,\bar{u})}{\partial P_2^2} = -\frac{1}{2}.\,\bar{u}.\,P_1^{1/2}.\,P_2^{-3/2} \qquad \frac{\partial^2 e(P_1,P_2,\bar{u})}{\partial P_2 \partial P_1} = \frac{1}{2}.\,\bar{u}.\,P_1^{-1/2}.\,P_2^{-1/2}$$

$$\begin{bmatrix} \partial x_1^H/\partial P_1 & \partial x_1^H/\partial P_2 \\ \partial x_2^H/\partial P_1 & \partial x_2^H/\partial P_2 \end{bmatrix} = \begin{bmatrix} \partial^2 e/\partial P_1^2 & \partial^2 e/\partial P_1 \partial P_2 \\ \partial^2 e/\partial P_2 \partial P_1 & \partial^2 e/\partial P_2^2 \end{bmatrix} \quad \blacksquare$$

ii) $D_P x_H(P_1, P_2, u)$ É UMA MATRIZ SEMIDEFINIDA NEGATIVA

$$\frac{\partial x_1^H(P_1,P_2,\bar{u})}{\partial P_1} = -\frac{1}{2}.\,\bar{u}.\,P_1^{-3/2}.\,P_2^{1/2}$$

$$\frac{\partial x_1^H(P_1,P_2,\bar{u})}{\partial P_2} = [\,\bar{u}.\,P_1^{-1/2}.\,P_2^{1/2}]'$$

$$= \frac{1}{2}.\,\bar{u}.\,P_1^{-1/2}.\,P_2^{-1/2}$$

10.A

$$\frac{\partial x_2^H(P_1,P_2,\bar{m})}{\partial P_2} = \left[\bar{m}\cdot\left(\frac{P_1}{P_2}\right)^{1/2}\right]'$$

$$= [\bar{m}\cdot P_1^{1/2}\cdot P_2^{-1/2}]'$$

$$= -\frac{1}{2}\cdot\bar{m}\cdot P_1^{1/2}\cdot P_2^{-3/2}$$

$$\frac{\partial x_2^H(P_1,P_2,\bar{m})}{\partial P_1} = [\bar{m}\cdot P_1^{1/2}\cdot P_2^{-1/2}]'$$

$$= \frac{1}{2}\cdot\bar{m}\cdot P_1^{-1/2}\cdot P_2^{-1/2}$$

MONTANDO A MATRIZ DAS DERIVADAS EM RELAÇÃO AOS PREÇOS:

$$D_P = \begin{bmatrix} \frac{\partial x_1}{\partial P_1} & \frac{\partial x_1}{\partial P_2} \\ \frac{\partial x_2}{\partial P_1} & \frac{\partial x_2}{\partial P_2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}\cdot\bar{m}\cdot P_1^{-3/2}\cdot P_2^{1/2} & 1/2\cdot\bar{m}\cdot P_1^{-1/2}\cdot P_2^{-1/2} \\ \frac{1}{2}\cdot\bar{m}\cdot P_1^{-1/2}\cdot P_2^{-1/2} & -\frac{1}{2}\cdot\bar{m}\cdot P_1^{1/2}\cdot P_2^{-3/2} \end{bmatrix}$$

CALCULANDO OS MENORES PRINCIPAIS:

$$\Delta_1 = \left| -\frac{1}{2}\cdot\bar{m}\cdot P_1^{-3/2}\cdot P_2^{1/2} \right|$$

$$= -\frac{1}{2}\cdot\bar{m}\cdot P_1^{-3/2}\cdot P_2^{1/2} < 0 \quad \text{POIS } \bar{m} > 0 \text{ E } P_1 \text{ E } P_2 \gg 0$$

$$\Delta_2 = \begin{vmatrix} -\frac{1}{2}\cdot\bar{m}\cdot P_1^{-3/2}\cdot P_2^{1/2} & \frac{1}{2}\cdot\bar{m}\cdot P_1^{-1/2}\cdot P_2^{-1/2} \\ \frac{1}{2}\cdot\bar{m}\cdot P_1^{-1/2}\cdot P_2^{-1/2} & -\frac{1}{2}\cdot\bar{m}\cdot P_1^{1/2}\cdot P_2^{-3/2} \end{vmatrix}$$

$$= \left[\left(-\frac{1}{2}\cdot\bar{m}\cdot P_1^{-3/2}\cdot P_2^{1/2}\right)\left(-\frac{1}{2}\cdot\bar{m}\cdot P_1^{1/2}\cdot P_2^{-3/2}\right) - \left(\frac{1}{2}\cdot\bar{m}\cdot P_1^{-1/2}\cdot P_2^{-1/2}\right)\left(\frac{1}{2}\cdot\bar{m}\cdot P_1^{-1/2}\cdot P_2^{-1/2}\right)\right]$$

$$= \frac{1}{4}\cdot\bar{m}^2\cdot P_1^{-1}\cdot P_2^{-1} - \frac{1}{4}\cdot\bar{m}^2\cdot P_1^{-1}\cdot P_2^{-1}$$

$$= 0$$

$$\Delta_1 = \left| -\frac{1}{2} \cdot \bar{u} \cdot P_1^{1/2} \cdot P_2^{-3/2} \right|$$

$$= -\frac{1}{2} \cdot \bar{u} \cdot P_1^{1/2} \cdot P_2^{-3/2} < 0 \quad \text{POIS } \bar{u} > 0 \text{ E } P_1 \text{ E } P_2 \gg 0$$

SENDO ASSIM, COMO TODOS OS MENORES PRINCIPAIS SÃO MENORES AO IGUAIS A ZERO CONCLUI-SE QUE A MATRIZ DAS DERIVADAS DE 1ª ORDEM EM RELAÇÃO A P_1 E P_2 É SEMIDEFINIDA NEGATIVA.

iii) $D_p X_H(P_1, P_2, \bar{u})$ É UMA MATRIZ SIMÉTRICA.

$$D_p = \begin{bmatrix} -\frac{1}{2} \cdot \bar{u} \cdot P_1^{-3/2} \cdot P_2^{1/2} & \frac{1}{2} \cdot \bar{u} \cdot P_1^{-1/2} \cdot P_2^{-1/2} \\ \frac{1}{2} \cdot \bar{u} \cdot P_1^{-1/2} \cdot P_2^{-1/2} & -\frac{1}{2} \cdot \bar{u} \cdot P_1^{1/2} \cdot P_2^{-3/2} \end{bmatrix} \quad a_{12} = a_{21}$$

iv) $D_p X_H(P_1, P_2, \bar{u}) P = 0$

$$\begin{bmatrix} -\frac{1}{2} \cdot \bar{u} \cdot P_1^{-3/2} \cdot P_2^{1/2} & \frac{1}{2} \cdot \bar{u} \cdot P_1^{-1/2} \cdot P_2^{-1/2} \\ \frac{1}{2} \cdot \bar{u} \cdot P_1^{-1/2} \cdot P_2^{-1/2} & -\frac{1}{2} \cdot \bar{u} \cdot P_1^{1/2} \cdot P_2^{-3/2} \end{bmatrix} \cdot \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} =$$

$$= \begin{bmatrix} -\frac{1}{2} \cdot \bar{u} \cdot P_1^{-1/2} \cdot P_2^{1/2} + \frac{1}{2} \cdot \bar{u} \cdot P_1^{-1/2} \cdot P_2^{1/2} \\ \frac{1}{2} \cdot \bar{u} \cdot P_1^{1/2} \cdot P_2^{-1/2} - \frac{1}{2} \cdot \bar{u} \cdot P_1^{1/2} \cdot P_2^{-1/2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \blacksquare$$

7.C VERIFICANDO A PROPOSIÇÃO 3.G.3 DO MAS-COLELL

$\max X_1^{1/2} \cdot X_2^{1/2}$ S. A $P_1 \cdot X_1 + P_2 \cdot X_2 = W$

X_1 E $X_2 \geq 0$

11.A

MONTANDO O LAGRANGEANO:

$$\mathcal{L}(x_1, x_2, \lambda) = x_1^{1/2} \cdot x_2^{1/2} + \lambda(W - P_1 \cdot x_1 - P_2 \cdot x_2)$$

$$\frac{\partial \mathcal{L}}{\partial x_1} = \frac{1}{2} \cdot x_1^{-1/2} \cdot x_2^{1/2} - \lambda P_1 = 0 \therefore \lambda = \frac{1}{2} \cdot \frac{x_1^{-1/2} \cdot x_2^{1/2}}{P_1} \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = \frac{1}{2} \cdot x_1^{1/2} \cdot x_2^{-1/2} - \lambda P_2 = 0 \therefore \lambda = \frac{1}{2} \cdot \frac{x_1^{1/2} \cdot x_2^{-1/2}}{P_2} \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = W - P_1 x_1 - P_2 x_2 = 0 \therefore W = P_1 \cdot x_1 + P_2 \cdot x_2 \quad (3)$$

IGUALANDO (1) E (2) OBTÉM-SE:

$$\frac{1}{2} \cdot \frac{x_1^{-1/2} \cdot x_2^{1/2}}{P_1} = \frac{1}{2} \cdot \frac{x_1^{1/2} \cdot x_2^{-1/2}}{P_2}$$

$$\frac{x_2^{1/2}}{x_2^{-1/2}} = \frac{\cancel{2}^1}{\cancel{2}^1} \cdot \frac{P_1}{P_2} \cdot \frac{x_1^{1/2}}{x_1^{-1/2}}$$

$$x_2 = \frac{P_1}{P_2} \cdot x_1 \quad (4)$$

SUBSTITUINDO (4) EM (3) TEM-SE QUE:

$$W = P_1 \cdot x_1 + \cancel{P_2}^1 \left(\frac{P_1}{\cancel{P_2}_1} \cdot x_1 \right)$$

$$W = P_1 \cdot x_1 + P_1 \cdot x_1$$

$$W = 2 \cdot P_1 \cdot x_1$$

$$\boxed{x_1 = \frac{1}{2} \cdot \frac{W}{P_1}} \quad (5)$$

SUBSTITUINDO (5) EM (4) TEM-SE QUE:

$$X_2 = \frac{P_1}{P_2} \cdot \frac{1}{2} \cdot \frac{W}{P_1}$$

$$\boxed{= \frac{1}{2} \cdot \frac{W}{P_2}}$$

A FUNÇÃO UTILIDADE INDIRETA É DADA POR: $v(P,w) = \frac{1}{2} \cdot w \cdot P_1^{-1/2} \cdot P_2^{-1/2}$

RECUPERANDO AS DEMANDAS MARSHALIANAS POR MEIO DA $v(P,w)$:

PELA IDENTIDADE DE ROY TEM-SE QUE: $X_\ell(P,w) = -\dfrac{\frac{\partial v(P,w)}{\partial P_\ell}}{\frac{\partial v(P,w)}{\partial w}}$

SENDO ASSIM,

$$X_1 = -\frac{\frac{\partial v(P,w)}{P_1}}{\frac{\partial v(P,w)}{\partial w}} \qquad X_2 = -\frac{\frac{\partial v(P,w)}{\partial P_2}}{\frac{\partial v(P,w)}{\partial w}}$$

$$\boxed{= \frac{1}{2} \cdot W \cdot P_1^{-1}} \ \blacksquare \qquad \boxed{= \frac{1}{2} \cdot W \cdot P_2^{-1}} \ \blacksquare$$

7.D) VERIFICANDO A PROPOSIÇÃO ~~3.6.4~~ 3.6.3 DO MAS-COLELL

AS DERIVADAS PARCIAIS DE PRIMEIRA ORDEM DAS DEMANDAS HICKSIANAS JÁ FORAM CALCULADAS NA PÁGINA 10 DESTE ARQUIVO E SÃO DADAS POR:

$$\frac{\partial X_1^H(P_1,P_2,\bar{u})}{\partial P_1} = -\frac{1}{2}\bar{u} \cdot P_1^{-3/2} \cdot P_2^{1/2} \qquad \frac{\partial X_1^H(P_1,P_2,\bar{u})}{\partial P_2} = \frac{1}{2} \cdot \bar{u} \cdot P_1^{-1/2} \cdot P_2^{-1/2}$$

$$\frac{\partial X_2^H(P_1,P_2,\bar{u})}{\partial P_1} = \frac{1}{2} \cdot \bar{u} \cdot P_1^{-1/2} \cdot P_2^{-1/2} \qquad \frac{\partial X_2^H(P_1,P_2,\bar{u})}{\partial P_2} = -\frac{1}{2} \cdot \bar{u} \cdot P_1^{1/2} \cdot P_2^{-3/2}$$

CALCULANDO AS DERIVADAS PARCIAIS DE PRIMEIRA ORDEM DAS DEMANDAS MARSHALIANAS EM RELAÇÃO AOS PREÇOS: 12.A

$$\frac{\partial x_1(p_1,p_2,w)}{\partial p_1} = -\frac{1}{4}\cdot w \cdot p_1^{-3/2} \qquad \frac{\partial x_1(p_1,p_2,w)}{\partial p_2} = 0$$

$$\frac{\partial x_2(p_1,p_2,w)}{\partial p_1} = 0 \qquad \frac{\partial x_2(p_1,p_2,w)}{\partial p_2} = -\frac{1}{4}\cdot w \cdot p_2^{-3/2}$$

CALCULANDO AS DERIVADAS PARCIAIS DE PRIMEIRA ORDEM DAS DEMANDAS MARSHALIANAS EM RELAÇÃO À RENDA:

$$\frac{\partial x_1(p_1,p_2,w)}{\partial w} = \frac{1}{2}\cdot p_1^{-1/2} \qquad \frac{\partial x_2(p_1,p_2,w)}{\partial w} = \frac{1}{2}\cdot p_2^{-1/2}$$

MONTANDO AS MATRIZES:

$$D_p x_H = \begin{bmatrix} -\frac{1}{2}\cdot \bar{u}\cdot p_1^{-3/2}\cdot p_2^{1/2} & \frac{1}{2}\cdot \bar{u}\cdot p_1^{-1/2}\cdot p_2^{-1/2} \\ \frac{1}{2}\cdot \bar{u}\cdot p_1^{-1/2}\cdot p_2^{-1/2} & -\frac{1}{2}\cdot \bar{u}\cdot p_1^{1/2}\cdot p_2^{-3/2} \end{bmatrix}$$

$$D_p x = \begin{bmatrix} -\frac{1}{2}\cdot w\cdot p_1^{-2} & 0 \\ 0 & -\frac{1}{2}\cdot w\cdot p_2^{-2} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{\cancel{2}_1}\cdot \cancel{2}^1\cdot \bar{u}\cdot p_1^{1/2}\cdot p_2^{1/2}\cdot p_1^{-2} & 0 \\ 0 & -\frac{1}{\cancel{2}_1}\cdot \cancel{2}^1\cdot \bar{u}\cdot p_1^{1/2}\cdot p_2^{1/2}\cdot p_2^{-2} \end{bmatrix}$$

$$= \begin{bmatrix} -\bar{u}\cdot p_1^{-3/2}\cdot p_2^{1/2} & 0 \\ 0 & -\bar{u}\cdot p_1^{1/2}\cdot p_2^{-3/2} \end{bmatrix}$$

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$$D_w x = \begin{bmatrix} \frac{1}{2} \cdot p_1^{-1} \\ \\ \frac{1}{2} \cdot p_2^{-1} \end{bmatrix} \qquad x = \begin{bmatrix} \frac{1}{2} \cdot \frac{w}{p_1} \\ \\ \frac{1}{2} \cdot \frac{w}{p_2} \end{bmatrix} \quad \therefore \; x^T = \begin{bmatrix} \frac{1}{2} \cdot \frac{w}{p_1} & \frac{1}{2} \cdot \frac{w}{p_2} \end{bmatrix}$$

$$D_w x \cdot x^T = \begin{bmatrix} \frac{1}{2} \cdot p_1^{-1} \\ \\ \frac{1}{2} \cdot p_2^{-1} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} \cdot w \cdot p_1^{-1} & \frac{1}{2} \cdot w \cdot p_2^{-1} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{4} \cdot w \cdot p_1^{-2} & \frac{1}{4} \cdot w \cdot p_1^{-1} \cdot p_2^{-1} \\ \\ \frac{1}{4} \cdot w \cdot p_1^{-1} \cdot p_2^{-1} & \frac{1}{4} \cdot w \cdot p_2^{-2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{4_2} \cdot 2^1 \cdot \bar{u} \cdot p_1^{1/2} \cdot p_2^{1/2} \cdot p_1^{-2} & \frac{1}{4_2} \cdot 2^1 \cdot \bar{u} \cdot p_1^{1/2} \cdot p_2^{1/2} \cdot p_1^{-1} \cdot p_2^{-1} \\ \\ \frac{1}{4_2} \cdot 2^1 \cdot \bar{u} \cdot p_1^{1/2} \cdot p_2^{1/2} \cdot p_1^{-1} \cdot p_2^{-1} & \frac{1}{4_2} \cdot 2^1 \cdot \bar{u} \cdot p_1^{1/2} \cdot p_2^{1/2} \cdot p_2^{-2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} \cdot \bar{u} \cdot p_1^{-3/2} \cdot p_2^{1/2} & \frac{1}{2} \cdot \bar{u} \cdot p_1^{-1/2} \cdot p_2^{-1/2} \\ \\ \frac{1}{2} \cdot \bar{u} \cdot p_1^{-1/2} \cdot p_2^{-1/2} & \frac{1}{2} \cdot \bar{u} \cdot p_1^{1/2} \cdot p_2^{-3/2} \end{bmatrix}$$

SENDO ASSIM,

(13.A)

$$D_p X + D_w X \cdot X^T = \begin{bmatrix} -\frac{1}{2}\cdot \bar{u}\cdot P_1^{-3/2}\cdot P_2^{1/2} & \frac{1}{2}\cdot \bar{u}\cdot P_1^{-1/2}\cdot P_2^{-1/2} \\ \frac{1}{2}\cdot \bar{u}\cdot P_1^{-1/2}\cdot P_2^{-1/2} & -\frac{1}{2}\cdot \bar{u}\cdot P_1^{1/2}\cdot P_2^{-3/2} \end{bmatrix}.$$ ∎

EXERCÍCIO 8:

(8.A) QUESTÃO 3.D.6 DO MAS-COLELL

(3.D.6.A) SUPONHA $\tilde{u}(X) = u(X)^{\frac{1}{\alpha+\beta+\gamma}} = (X_1-b_1)^{\alpha'}\cdot(X_2-b_2)^{\beta'}\cdot(X_3-b_3)^{\gamma'}$

EM QUE: $\alpha' = \frac{\alpha}{\alpha+\beta+\gamma}$; $\beta' = \frac{\beta}{\alpha+\beta+\gamma}$ E $\gamma' = \frac{\gamma}{\alpha+\beta+\gamma}$. LOGO, SOMANDO $\alpha'+\beta'+\gamma' = 1$. UMA VEZ QUE $\tilde{u}(X)$ É UMA FUNÇÃO UTILIDADE QUE APRESENTA AS MESMAS PREFERÊNCIAS DE $u(X)$, POIS AQUELA É UMA TRANSFORMAÇÃO MONOTÔNICA DESSA PODE-SE ASSUMIR SEM PERDA DE GENERALIDADE QUE $\alpha+\beta+\gamma=1$.

(3.D.6.B) PROBLEMA DE MAXIMIZAÇÃO DA UTILIDADE - PMU

$$\max\ (X_1-b_1)^{\alpha}(X_2-b_2)^{\beta}(X_3-b_3)^{\gamma}$$

X_1, X_2 E $X_3 \geq 0$

S. A $P_1X_1 + P_2X_2 + P_3X_3 = W$

PARA FACILITAR TOMA-SE O NEPERIANO DA FUNÇÃO UTILIDADE:

$$\ln(X_1-b_1)^{\alpha}(X_2-b_2)^{\beta}(X_3-b_3)^{\gamma}$$

$$\ln(X_1-b_1)^{\alpha} + \ln(X_2-b_2)^{\beta} + \ln(X_3-b_3)^{\gamma}$$

$$\alpha\ln(X_1-b_1) + \beta\ln(X_2-b_2) + \gamma\ln(X_3-b_3)$$

MONTANDO O LAGRANGEANO:

$$\mathcal{L}(X_1, X_2, X_3, \lambda) = \alpha\ln(X_1-b_1) + \beta\ln(X_2-b_2) + \gamma\ln(X_3-b_3) + \lambda(W - P_1X_1 - P_2X_2 - P_3X_3)$$

$$\frac{\partial\mathcal{L}}{\partial X_1} = \alpha\cdot\frac{1}{X_1-b_1}\cdot(X_1-b_1)' - \lambda P_1 = 0 \therefore \lambda = \frac{\alpha}{P_1\cdot(X_1-b_1)} \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = \beta \cdot \frac{1}{x_2 - b_2} \cdot (x_2 - b_2)' - \lambda P_2 = 0 \;\therefore\; \lambda = \frac{\beta}{P_2 \cdot (x_2 - b_2)} \qquad (2)$$

$$\frac{\partial \mathcal{L}}{\partial x_3} = \gamma \cdot \frac{1}{x_3 - b_3} \cdot (x_3 - b_3)' - \lambda P_3 = 0 \;\therefore\; \lambda = \frac{\gamma}{P_3 \cdot (x_3 - b_3)} \qquad (3)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = w - P_1 x_1 - P_2 x_2 - P_3 x_3 = 0 \;\therefore\; w = P_1 x_1 + P_2 x_2 + P_3 x_3 \qquad (4)$$

IGUALANDO (1) E (2) OBTÉM-SE:

$$\frac{\alpha}{P_1 \cdot (x_1 - b_1)} = \frac{\beta}{P_2 \cdot (x_2 - b_2)}$$

$$\alpha \cdot P_2 (x_2 - b_2) = \beta \cdot P_1 \cdot (x_1 - b_1)$$

$$\alpha \cdot P_2 \cdot x_2 - \alpha \cdot P_2 \cdot b_2 = \beta \cdot P_1 \cdot (x_1 - b_1)$$

$$\alpha \cdot P_2 \cdot x_2 = \beta \cdot P_1 \cdot (x_1 - b_1) + \alpha \cdot P_2 \cdot b_2$$

$$P_2 \cdot x_2 = \frac{\beta \cdot P_1 \cdot (x_1 - b_1) + \alpha \cdot P_2 \cdot b_2}{\alpha} \qquad (5)$$

IGUALANDO (1) E (3) OBTÉM-SE:

$$\frac{\alpha}{P_1 \cdot (x_1 - b_1)} = \frac{\gamma}{P_3 \cdot (x_3 - b_3)}$$

$$\alpha \cdot P_3 \cdot (x_3 - b_3) = \gamma \cdot P_1 \cdot (x_1 - b_1)$$

$$\alpha \cdot P_3 \cdot x_3 - \alpha \cdot P_3 \cdot b_3 = \gamma \cdot P_1 \cdot (x_1 - b_1)$$

$$\alpha \cdot P_3 \cdot x_3 = \gamma \cdot P_1 \cdot (x_1 - b_1) + \alpha \cdot P_3 \cdot b_3$$

$$P_3 \cdot x_3 = \frac{\gamma \cdot P_1 \cdot (x_1 - b_1) + \alpha \cdot P_3 \cdot b_3}{\alpha} \qquad (6)$$

(14.A)

SUBSTITUINDO (5) E (6) EM (4) TEM-SE QUE:

$$W = P_1 . X_1 + \frac{\beta . P_1 . (X_1 - b_1) + \alpha . P_2 . b_2}{\alpha} + \frac{\gamma . P_1 . (X_1 - b_1) + \alpha . P_3 . b_3}{\alpha}$$

$$\alpha W = \frac{\alpha . P_1 . X_1 + \beta . P_1 . X_1 - \beta . P_1 . b_1 + \alpha . P_2 . b_2 + \gamma . P_1 . X_1 - \gamma . P_1 . b_1 + \alpha . P_3 . b_3}{\cancel{\alpha}}$$

$$\alpha W = \alpha . P_1 . X_1 + \beta . P_1 . X_1 + \gamma . P_1 . X_1 - \beta . P_1 . b_1 + \alpha . P_2 . b_2 - \gamma . P_1 . b_1 + \alpha . P_3 . b_3$$

$$= (\alpha + \beta + \gamma) . P_1 . X_1 - \beta . P_1 . b_1 + \alpha . P_2 . b_2 - \gamma . P_1 . b_1 + \alpha . P_3 . b_3$$

COMO $\alpha + \beta + \gamma = 1$ TEM-SE QUE:

$$\alpha W = P_1 . X_1 - P_1 . b_1 (\beta + \gamma) + \alpha . P_2 . b_2 + \alpha . P_3 . b_3$$

UMA VEZ QUE,

$$\alpha + \beta + \gamma = 1 \therefore (\alpha + \beta + \gamma) P_1 . b_1 = P_1 . b_1 \therefore (\beta + \gamma) P_1 . b_1 = P_1 . b_1 - \alpha . P_1 . b_1$$

$$\alpha W = P_1 . X_1 - (P_1 . b_1 - \alpha . P_1 . b_1) + \alpha . P_2 . b_2 + \alpha . P_3 . b_3$$

$$\alpha W = P_1 . X_1 - P_1 . b_1 + \alpha . P_1 . b_1 + \alpha . P_2 . b_2 + \alpha . P_3 . b_3$$

$$\alpha W = P_1 . X_1 - P_1 . b_1 + \alpha . (P_1 . b_1 + P_2 . b_2 + P_3 . b_3)$$

$$P_1 . X_1 = \alpha W + P_1 . b_1 - \alpha . \Sigma P_i . b_i$$

$$P_1 . X_1 = \alpha . (W - \Sigma P_i . b_i) + P_1 . b_1$$

$$\boxed{X_1 = \frac{\alpha}{P_1} . (W - \Sigma P_i . b_i) + b_1} \qquad (7)$$

SUBSTITUINDO (7) EM (5) TEM-SE QUE:

$$P_2 . X_2 = \frac{\beta . \cancel{P_1} . \left[\frac{\alpha}{\cancel{P_1}} . (W - \Sigma P_i . b_i) + \cancel{b_1} - \cancel{b_1} \right] + \alpha . P_2 . b_2}{\alpha}$$

$$= \frac{\beta . [\alpha . (W - \Sigma P_i . b_i)] + \alpha . P_2 . b_2}{\alpha}$$

$$P_2 . X_2 = \frac{\cancel{\alpha} . [\beta . (W - \Sigma P_i . b_i) + P_2 . b_2]}{\cancel{\alpha}}$$

$$X_2 = \frac{\beta . (W - \Sigma P_i . b_i) + P_2 . b_2}{P_2}$$

$$\boxed{= \frac{\beta}{P_2} . (W - \Sigma P_i . b_i) + b_2} \quad (8)$$

SUBSTITUINDO (7) EM (6) TEM-SE QUE:

$$P_3 . X_3 = \frac{\gamma . \cancel{P_1} . \left[\frac{\alpha}{\cancel{P_1}} . (W - \Sigma P_i . b_i) + \cancel{b_1} - \cancel{b_1} \right] + \alpha . P_3 . b_3}{\alpha}$$

$$P_3 . X_3 = \frac{\gamma . [\alpha . (W - \Sigma P_i . b_i] + \alpha . P_3 . b_3}{\alpha}$$

$$P_3 . X_3 = \frac{\cancel{\alpha} . [\gamma . (W - \Sigma P_i . b_i) + P_3 . b_3]}{\cancel{\alpha}}$$

$$X_3 = \frac{\gamma . (W - \Sigma P_i . b_i) + P_3 . b_3}{P_3}$$

$$\boxed{= \frac{\gamma}{P_3} . (W - \Sigma P_i . b_i) + b_3}$$

ENCONTRANDO A FUNÇÃO UTILIDADE INDIRETA:

$$v(P_1, P_2, P_3, W) = \left[\frac{\alpha}{P_1} . (W - \Sigma P_i . b_i) + \cancel{b_1} - \cancel{b_1} \right]^{\alpha} . \left[\frac{\beta}{P_2} . (W - \Sigma P_i . b_i) + \cancel{b_2} - \cancel{b_2} \right]^{\beta} .$$

$$. \left[\frac{\gamma}{P_3} . (W - \Sigma P_i . b_i) + \cancel{b_3} - \cancel{b_3} \right]^{\gamma}$$

(13.A)

$$v(P_1,P_2,P_3,W)=\left[\frac{\alpha}{P_1}\cdot(W-\sum P_i\cdot b_i)\right]^{\alpha}\cdot\left[\frac{\beta}{P_2}\cdot(W-\sum P_i\cdot b_i)\right]^{\beta}\cdot\left[\frac{\gamma}{P_3}\cdot(W-\sum P_i\cdot b_i)\right]^{\gamma}$$

$$=(W-\sum P_i\cdot b_i)^{\alpha}\cdot(W-\sum P_i\cdot b_i)^{\beta}\cdot(W-\sum P_i\cdot b_i)^{\gamma}\cdot\left(\frac{\alpha}{P_1}\right)^{\alpha}\cdot\left(\frac{\beta}{P_2}\right)^{\beta}\cdot\left(\frac{\gamma}{P_3}\right)^{\gamma}$$

$$=(W-\sum P_i\cdot b_i)^{\alpha+\beta+\gamma}\cdot\left(\frac{\alpha}{P_1}\right)^{\alpha}\cdot\left(\frac{\beta}{P_2}\right)^{\beta}\cdot\left(\frac{\gamma}{P_3}\right)^{\gamma}$$

$$\boxed{=(W-\sum P_i\cdot b_i)\cdot\left(\frac{\alpha}{P_1}\right)^{\alpha}\cdot\left(\frac{\beta}{P_2}\right)^{\beta}\cdot\left(\frac{\gamma}{P_3}\right)^{\gamma}}$$

3.D.6.C

i) VERIFICANDO SE A FUNÇÃO DEMANDA MARSHALIANA É HOMOGÊNEA DE GRAU ZERO NOS PREÇOS E NA RENDA.

$$x_1(tP,tW)=\frac{\alpha}{t\cdot P_1}\cdot(t\cdot W-\sum t\cdot P_i\cdot b_i)+b_1$$

$$=\frac{\alpha}{t\cdot P_1}\cdot(t\cdot W-t\sum P_i\cdot b_i)+b_1$$

$$=\frac{\alpha}{\cancel{t}\cdot P_1}\cdot[\cancel{t}(W-\sum P_i\cdot b_i)]+b_1$$

$$\boxed{=\frac{\alpha}{P_1}\cdot(W-\sum P_i\cdot b_i]+b_1}$$ $\blacksquare$

UMA VEZ QUE O FORMATO PARA AS TRÊS DEMANDAS É IDÊNTICO CONCLUI-SE QUE X_2 E X_3 TAMBÉM SÃO HOMOGÊNEAS DE GRAU ZERO NOS PREÇOS E NA RENDA.

ii) VERIFICANDO SE SATISFAZ A LEI DE WALRAS:

$$W=P_1X_1+P_2X_2+P_3X_3$$

$$W=\cancel{P_1}\cdot\left[\frac{\alpha}{\cancel{P_1}}\cdot(W-\sum P_i\cdot b_i)+b_1\right]+\cancel{P_2}\cdot\left[\frac{\beta}{\cancel{P_2}}\cdot(W-\sum P_i\cdot b_i)+b_2\right]+\cancel{P_3}\cdot\left[\frac{\gamma}{\cancel{P_3}}\cdot(W-\sum P_i\cdot b_i)+b_3\right]$$

$$W = \alpha\left(W - \sum P_i \cdot b_i\right) + P_1 \cdot b_1 + \beta\left(W - \sum P_i \cdot b_i\right) + P_2 \cdot b_2 + \gamma\left(W - \sum P_i \cdot b_i\right) + P_3 \cdot b_3$$

$$= (\alpha + \beta + \gamma)\left(W - \sum P_i \cdot b_i\right) + P_1 \cdot b_1 + P_2 \cdot b_2 + P_3 \cdot b_3$$

COMO $\alpha + \beta + \gamma = 1$ TEM-SE QUE:

$$W = W - \cancel{\sum P_i} \cdot b_i + \cancel{\sum P_i \cdot b_i}$$

$$\boxed{W = W}$$

iii) VERIFICANDO A CONVEXIDADE / UNICIDADE

i) VERIFICANDO SE A FUNÇÃO DE UTILIDADE INDIRETA É HOMOGÊNEA DE GRAU ZERO NOS PREÇOS E NA RENDA: 16.A

$$v(\lambda P_1, \lambda P_2, \lambda P_3, \lambda w) = (\lambda . w - \sum \lambda . P_i . b_i) . \left(\frac{\alpha}{\lambda . P_1}\right)^{\alpha} . \left(\frac{\beta}{\lambda . P_2}\right)^{\beta} . \left(\frac{\gamma}{\lambda . P_3}\right)^{\gamma}$$

$$= (\lambda . w - \lambda \sum P_i . b_i) . \frac{1}{\lambda^{\alpha}} . \left(\frac{\alpha}{P_1}\right)^{\alpha} . \frac{1}{\lambda^{\beta}} . \left(\frac{\beta}{P_2}\right)^{\beta} . \frac{1}{\lambda^{\gamma}} . \left(\frac{\gamma}{P_3}\right)^{\gamma}$$

$$= [\lambda (w - \sum P_i . b_i)] . \frac{1}{\lambda^{(\alpha+\beta+\gamma)}} . \left(\frac{\alpha}{P_1}\right)^{\alpha} . \left(\frac{\beta}{P_2}\right)^{\beta} . \left(\frac{\gamma}{P_3}\right)^{\gamma}$$

COMO $\alpha + \beta + \gamma = 1$ TEM-SE QUE:

$$v(\lambda P_1, \lambda P_2, \lambda P_3, \lambda w) = [\cancel{\lambda}^{1} (w - \sum P_i . b_i)] . \frac{1}{\cancel{\lambda}^{1}} . \left(\frac{\alpha}{P_1}\right)^{\alpha} . \left(\frac{\beta}{P_2}\right)^{\beta} . \left(\frac{\gamma}{P_3}\right)^{\gamma}$$

$$= (w - \sum P_i . b_i) . \left(\frac{\alpha}{P_1}\right)^{\alpha} . \left(\frac{\beta}{P_2}\right)^{\beta} . \left(\frac{\gamma}{P_3}\right)^{\gamma} \quad \blacksquare$$

ii) VERIFICANDO SE A FUNÇÃO UTILIDADE INDIRETA É ESTRITAMENTE CRESCENTE NA RENDA E NÃO-CRESCENTE NOS PREÇOS

$$\frac{\partial v(P_1, P_2, P_3, w)}{\partial w} = \left(\frac{\alpha}{P_1}\right)^{\alpha} . \left(\frac{\beta}{P_2}\right)^{\beta} . \left(\frac{\gamma}{P_3}\right)^{\gamma}$$

COMO α, β E $\gamma > 0$ E P_1, P_2 E $P_3 \gg 0$ CONCLUI-SE QUE:

$$\frac{\partial v(P_1, P_2, P_3, w)}{\partial w} > 0$$

$$\frac{\partial v(P_1, P_2, P_3, w)}{\partial P_1} = -\alpha . (w - \sum P_i . b_i) . \alpha^{\alpha} . P_1^{-\alpha-1} . \left(\frac{\beta}{P_2}\right)^{\beta} . \left(\frac{\gamma}{P_3}\right)^{\gamma} +$$

$$+ \left(\frac{\alpha}{P_1}\right)^{\alpha} . \left(\frac{\beta}{P_2}\right)^{\beta} . \left(\frac{\gamma}{P_3}\right)^{\gamma} . b_1$$

$$\partial v(p_1, p_2, p_3, w) = -\alpha.(w - \Sigma p_i.b_i).\alpha^{\alpha}.p_1^{-\alpha}.p_1^{-1}.\left(\frac{\beta}{p_2}\right)^{\beta}.\left(\frac{\gamma}{p_3}\right)^{\gamma} +$$

$$+\left(\frac{\alpha}{p_1}\right)^{\alpha}.\left(\frac{\beta}{p_2}\right)^{\beta}.\left(\frac{\gamma}{p_3}\right)^{\gamma}.b_1$$

$$= -\alpha.(w - \Sigma p_i.b_i).\left(\frac{\alpha}{p_1}\right)^{\alpha}.p_1^{-1}.\left(\frac{\beta}{p_2}\right)^{\beta}.\left(\frac{\gamma}{p_3}\right)^{\gamma} +$$

$$+\left(\frac{\alpha}{p_1}\right)^{\alpha}.\left(\frac{\beta}{p_2}\right)^{\beta}.\left(\frac{\gamma}{p_3}\right)^{\gamma}.b_1$$

$$= \left(\frac{\alpha}{p_1}\right)^{\alpha}.\left(\frac{\beta}{p_2}\right)^{\beta}.\left(\frac{\gamma}{p_3}\right)^{\gamma}.b_1.\left[-\alpha.(w - \Sigma p_i.b_i).p_1^{-1} + b_1\right]$$

COMO α, β E $\gamma > 0$, p_1, p_2 E $p_3 >> 0$ E $w > 0$ CONCLUI-SE QUE:

$$\frac{\partial v(p_1, p_2, p_3, w)}{\partial p_1} < 0, \text{ POIS } -\frac{\alpha.(w - \Sigma p_i.b_i)}{p_1} < 0 \quad \blacksquare$$

AS DERIVADAS PARCIAIS DA FUNÇÃO UTILIDADE INDIRETA EM RELAÇÃO AOS PREÇOS POSSUI O FORMATO GERAL DADO POR:

$$\partial v(p, w) = \left(\frac{\alpha}{p_1}\right)^{\alpha}.\left(\frac{\beta}{p_2}\right)^{\beta}.\left(\frac{\gamma}{p_3}\right)^{\gamma}.b_i\left[-\alpha.(w - \Sigma p_i.b_i).p_i^{-1} + b_i\right]$$

SENDO ASSIM, CONCLUI-SE QUE TODAS ESSAS DERIVADAS SERÃO MENORES QUE ZERO

iii) VERIFICANDO A QUASICONVEXIDADE DA FUNÇÃO UTILIDADE INDIRETA:

$$v(p_1, p_2, p_3, w) = (w - \Sigma p_i.b_i).\left(\frac{\alpha}{p_1}\right)^{\alpha}.\left(\frac{\beta}{p_2}\right)^{\beta}.\left(\frac{\gamma}{p_3}\right)^{\gamma}$$

LINEARIZANDO A FUNÇÃO UTILIDADE INDIRETA:

$$\ln v(p, w) = \ln (w - \Sigma p_i.b_i).\left(\frac{\alpha}{p_1}\right)^{\alpha}\left(\frac{\beta}{p_2}\right)^{\beta}.\left(\frac{\gamma}{p_3}\right)^{\gamma}$$

$$= \ln(w - \Sigma p_i.b_i) + \alpha \ln \alpha/p_1 + \beta \ln \beta/p_2 + \gamma \ln \gamma/p_3$$

ENTÃO A FUNÇÃO É LINEAR, OU SEJA, É CONVEXA PARA TODO $v(P,w) \in \mathbb{R}$. (17.A)

iv) VERIFICANDO A CONTINUIDADE EM P E W:

UMA VEZ QUE A FUNÇÃO UTILIDADE INDIRETA É DIFERENCIÁVEL IMPLICA QUE É CONTÍNUA EM P E W.

8.B QUESTÃO 3.D.8 DO MAS-COLELL

$$w \cdot \frac{\partial v(P,w)}{\partial w} = -P \cdot \nabla_p v(P,w)$$

PELA IDENTIDADE DE ROY SABE-SE QUE

$$x(P,w) = -\frac{\frac{\partial v(P,w)}{\partial P}}{\frac{\partial v(P,w)}{\partial w}}$$

ENTRETANTO,

$$\frac{\partial v(P,w)}{\partial P} = \nabla_p v(P,w)$$

LOGO,

$$x(P,w) = -\frac{\nabla_p v(P,w)}{\frac{\partial v(P,w)}{\partial w}}$$

PELA LEI DE WALRAS SABE-SE QUE:

$$w = P \cdot x \therefore x = \frac{w}{P}$$

ENTÃO,

$$\frac{w}{P} = -\frac{\nabla_p v(P,w)}{\frac{\partial v(P,w)}{\partial w}}$$

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$$W \cdot \frac{\partial x(P,W)}{\partial W} = -P \cdot D_p x(P,W)$$

8.C QUESTÃO 3.E.8 DO MAS-COLELL

$u(X,Y) = X^{\alpha} \cdot Y^{\beta}$

PROBLEMA DE MAXIMIZAÇÃO DA UTILIDADE - PMU

$\max X^{\alpha} \cdot Y^{\beta}$ S. A $P_X \cdot X + P_Y \cdot Y = W$

X E $Y \geq 0$

MONTANDO O LAGRANGEANO:

$\mathcal{L}(X,Y,\lambda) = X^{\alpha} \cdot Y^{\beta} + \lambda(W - P_X \cdot X - P_Y \cdot Y)$

$$\frac{\partial \mathcal{L}}{\partial X} = \alpha \cdot X^{\alpha-1} \cdot Y^{\beta} - \lambda \cdot P_X = 0 \therefore \lambda = \frac{\alpha}{P_X} \cdot X^{\alpha-1} \cdot Y^{\beta} \qquad (1)$$

$$\frac{\partial \mathcal{L}}{\partial Y} = \beta \cdot X^{\alpha} \cdot Y^{\beta-1} - \lambda \cdot P_Y = 0 \therefore \lambda = \frac{\beta}{P_Y} \cdot X^{\alpha} \cdot Y^{\beta-1} \qquad (2)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = W - P_X \cdot X - P_Y \cdot Y = 0 \therefore W = P_X \cdot X + P_Y \cdot Y \qquad (3)$$

IGUALANDO AS EQUAÇÕES (1) E (2) TEM-SE QUE:

$$\frac{\alpha}{P_X} \cdot X^{\alpha-1} \cdot Y^{\beta} = \frac{\beta}{P_Y} \cdot X^{\alpha} \cdot Y^{\beta-1}$$

$$\frac{Y^{\beta}}{Y^{\beta-1}} = \frac{\beta}{\alpha} \cdot \frac{P_X}{P_Y} \cdot \frac{X^{\alpha}}{X^{\alpha-1}}$$

$$Y = \frac{\beta}{\alpha} \cdot \frac{P_X}{P_Y} \cdot X \qquad (4)$$

1P.A

SUBSTITUINDO (4) EM (3) TEM-SE QUE:

$$W = P_x \cdot X + \cancel{P_Y} \left(\frac{\beta}{\alpha} \cdot \frac{P_x}{\cancel{P_Y}} \cdot X \right) \qquad W = P_x \cdot X \left(\frac{\alpha + \beta}{\alpha} \right)$$

$$W = P_x \cdot X + P_x \cdot X \cdot \frac{\beta}{\alpha} \qquad W = P_x \cdot X \left(\frac{1}{\alpha} \right)$$

$$W = P_x \cdot X \left(1 + \frac{\beta}{\alpha} \right) \qquad \alpha \cdot W = P_x \cdot X$$

$$\boxed{X = \frac{\alpha \cdot W}{P_x}} \qquad (5)$$

SUBSTITUINDO A EXPRESSÃO (5) EM (4) OBTÉM-SE:

$$Y = \frac{\beta}{\cancel{\alpha}} \cdot \frac{\cancel{P_x}}{P_Y} \cdot \frac{\cancel{\alpha} \cdot W}{\cancel{P_x}}$$

$$\boxed{= \frac{\beta \cdot W}{P_Y}} \qquad (6)$$

ENCONTRANDO A FUNÇÃO UTILIDADE INDIRETA:

$$v(P_x, P_Y, W) = \left(\frac{\alpha \cdot W}{P_x} \right)^{\alpha} \cdot \left(\frac{\beta \cdot W}{P_Y} \right)^{\beta}$$

$$= W^{\alpha} \cdot W^{\beta} \cdot \left(\frac{\alpha}{P_x} \right)^{\alpha} \cdot \left(\frac{\beta}{P_Y} \right)^{\beta}$$

$$= W^{\alpha + \beta} \cdot \left(\frac{\alpha}{P_x} \right)^{\alpha} \cdot \left(\frac{\beta}{P_Y} \right)^{\beta}$$

$$\boxed{= W \cdot \left(\frac{\alpha}{P_x} \right)^{\alpha} \cdot \left(\frac{\beta}{P_Y} \right)^{\beta}}$$

PROBLEMA DE MINIMIZAÇÃO DA DESPESA - PMD:

$\min P_x \cdot X + P_Y \cdot Y \quad$ S. A $\quad u(X,Y) = X^{\alpha} \cdot Y^{\beta} = \bar{u}$

$X \text{ E } Y \geq 0$

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MONTANDO O LAGRANGEANO:

$$\mathcal{L}(X, Y, \lambda) = P_X \cdot X + P_Y \cdot Y - \lambda(u(X,Y) - \bar{u} \geq 0)$$

$$\frac{\partial \mathcal{L}}{\partial X} = P_X - \lambda \alpha \cdot X^{\alpha-1} \cdot Y^{\beta} = 0 \therefore \lambda = \frac{P_X}{\alpha \cdot X^{\alpha-1} \cdot Y^{\beta}} \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial Y} = P_Y - \lambda \cdot \beta \cdot X^{\alpha} \cdot Y^{\beta-1} = 0 \therefore \lambda = \frac{P_Y}{\beta \cdot X^{\alpha} \cdot Y^{\beta-1}} \quad (2)$$

IGUALANDO AS EQUAÇÕES (1) E (2) OBTÉM-SE:

$$\frac{P_X}{\alpha \cdot X^{\alpha-1} \cdot Y^{\beta}} = \frac{P_Y}{\beta \cdot X^{\alpha} \cdot Y^{\beta-1}}$$

$$\frac{X^{\alpha}}{X^{\alpha-1}} = \frac{\alpha}{\beta} \cdot \frac{P_Y}{P_X} \cdot \frac{Y^{\beta}}{Y^{\beta-1}}$$

$$X = \frac{\alpha}{\beta} \cdot \frac{P_Y}{P_X} \cdot Y \quad (3)$$

SUBSTITUINDO (3) NA RESTRIÇÃO OBTÉM-SE:

$$\left(\frac{\alpha}{\beta} \cdot \frac{P_Y}{P_X} \cdot Y_H\right)^{\alpha} \cdot Y_H^{\beta} = \bar{u}$$

$$\left(\frac{\alpha}{\beta} \cdot \frac{P_Y}{P_X}\right)^{\alpha} \cdot Y_H^{\alpha} \cdot Y_H^{\beta} = \bar{u}$$

$$\left(\frac{\alpha}{\beta} \cdot \frac{P_Y}{P_X}\right)^{\alpha} \cdot Y_H^{\alpha+\beta} = \bar{u}$$

$$\left(\frac{\alpha}{\beta} \cdot \frac{P_Y}{P_X}\right)^{\alpha} \cdot Y_H = \bar{u}$$

$$Y_H = \frac{\bar{u}}{\left(\frac{\alpha}{\beta} \cdot \frac{P_Y}{P_X}\right)^{\alpha}} = \boxed{\bar{u} \cdot \left(\frac{\beta}{\alpha} \cdot \frac{P_X}{P_Y}\right)^{\alpha}} \quad (4)$$

SUBSTITUINDO (4) EM (3) TEM-SE QUE: 19.A

$$x_H = \frac{\alpha}{\beta} \cdot \frac{P_Y}{P_X} \cdot \bar{u} \cdot \left(\frac{\beta}{\alpha} \cdot \frac{P_X}{P_Y}\right)^{\alpha}$$

$$= \alpha \cdot \beta^{-1} \cdot P_Y \cdot P_X^{-1} \cdot \bar{u} \cdot \beta^{\alpha} \cdot \alpha^{-\alpha} \cdot P_X^{\alpha} \cdot P_Y^{-\alpha}$$

$$= \alpha^{1-\alpha} \cdot \beta^{-1+\alpha} \cdot P_Y^{1-\alpha} \cdot P_X^{-1+\alpha} \cdot \bar{u}$$

$$= \alpha^{\beta} \cdot \beta^{-(1-\alpha)} \cdot P_Y^{\beta} \cdot P_X^{-(1-\alpha)} \cdot \bar{u}$$

$$= \alpha^{\beta} \cdot \beta^{-\beta} \cdot P_Y^{\beta} \cdot P_X^{-\beta} \cdot \bar{u}$$

$$= \left(\frac{\alpha}{\beta}\right)^{\beta} \cdot \left(\frac{P_Y}{P_X}\right)^{\beta} \cdot \bar{u}$$

$$\boxed{= \bar{u} \cdot \left(\frac{\alpha}{\beta}\right)^{\beta} \cdot \left(\frac{P_Y}{P_X}\right)^{\beta}} \qquad (5)$$

ENCONTRANDO A FUNÇÃO DESPESA:

$$e(P_X, P_Y, \bar{u}) = P_X \cdot \left[\bar{u} \cdot \left(\frac{\alpha}{\beta}\right)^{\beta} \cdot \left(\frac{P_Y}{P_X}\right)^{\beta}\right] + P_Y \cdot \left[\bar{u} \cdot \left(\frac{\beta}{\alpha}\right)^{\alpha} \cdot \left(\frac{P_X}{P_Y}\right)^{\alpha}\right]$$

$$= P_X \cdot [\bar{u} \cdot \alpha^{\beta} \cdot \beta^{-\beta} \cdot P_Y^{\beta} \cdot P_X^{-\beta}] + P_Y \cdot [\bar{u} \cdot \beta^{\alpha} \cdot \alpha^{-\alpha} \cdot P_X^{\alpha} \cdot P_Y^{-\alpha}]$$

$$= \bar{u} \cdot \alpha^{\beta} \cdot \beta^{-\beta} \cdot P_Y^{\beta} \cdot P_X^{1-\beta} + \bar{u} \cdot \beta^{\alpha} \cdot \alpha^{-\alpha} \cdot P_X^{\alpha} \cdot P_Y^{1-\alpha}$$

$$= \bar{u} \cdot \alpha^{\beta} \cdot \beta^{-\beta} \cdot P_Y^{\beta} \cdot P_X^{\alpha} + \bar{u} \cdot \beta^{\alpha} \cdot \alpha^{-\alpha} \cdot P_X^{\alpha} \cdot P_Y^{\beta}$$

$$= \bar{u} \cdot P_X^{\alpha} \cdot P_Y^{\beta} \cdot (\alpha^{\beta} \cdot \beta^{-\beta} + \beta^{\alpha} \cdot \alpha^{-\alpha})$$

$$= \bar{u} \cdot P_X^{\alpha} \cdot P_Y^{\beta} \cdot \left[\left(\frac{\alpha}{\beta}\right)^{\beta} + \left(\frac{\beta}{\alpha}\right)^{\alpha}\right]$$

$$= \bar{u} \cdot P_X^{\alpha} \cdot P_Y^{\beta} \cdot \left(\frac{\alpha^{\beta}}{\beta^{\beta}} + \frac{\beta^{\alpha}}{\alpha^{\alpha}}\right)$$

$$= \bar{u} \cdot P_X^{\alpha} \cdot P_Y^{\beta} \cdot \left(\frac{\alpha^{\beta} \cdot \alpha^{\alpha} + \beta^{\alpha} \cdot \beta^{\beta}}{\beta^{\beta} \alpha^{\alpha}}\right) \therefore \boxed{e(P_X, P_Y, \bar{u}) = \bar{u} \cdot \left(\frac{P_X}{\alpha}\right)^{\alpha} \cdot \left(\frac{P_Y}{\beta}\right)^{\beta}}$$

$v(P_x, P_y, e(P,\bar{u})) = \bar{u}$

$e(P,\bar{u}) \cdot \left(\frac{\alpha}{P_x}\right)^{\alpha} \cdot \left(\frac{\beta}{P_y}\right)^{\beta} = \bar{u}$

$\bar{u} \cdot \cancel{\left(\frac{P_x}{\alpha}\right)^{\alpha}} \cdot \cancel{\left(\frac{P_y}{\beta}\right)^{\beta}} \cdot \cancel{\left(\frac{\alpha}{P_x}\right)^{\alpha}} \cdot \cancel{\left(\frac{\beta}{P_y}\right)^{\beta}} = \bar{u}$

$\boxed{\bar{u} = \bar{u}}$ ■

$e(P, v(P,w)) = w$

$v(P,w) \cdot \left(\frac{P_x}{\alpha}\right)^{\alpha} \cdot \left(\frac{P_y}{\beta}\right)^{\beta} = w$

$w \cdot \cancel{\left(\frac{\alpha}{P_x}\right)^{\alpha}} \cdot \cancel{\left(\frac{\beta}{P_y}\right)^{\beta}} \cdot \cancel{\left(\frac{P_x}{\alpha}\right)^{\alpha}} \cdot \cancel{\left(\frac{P_y}{\beta}\right)^{\beta}} = w$

$\boxed{w = w}$ ■

$x_H(P,\bar{u}) = x(P, e(P,u))$

$$x_H(P,\bar{u}) = \frac{\alpha \cdot \bar{u} \cdot \left(\frac{P_x}{\alpha}\right)^{\alpha} \cdot \left(\frac{P_y}{\beta}\right)^{\beta}}{P_x}$$

$$= \alpha \cdot \bar{u} \cdot \left(\frac{P_x}{\alpha}\right)^{\alpha} \cdot \left(\frac{P_y}{\beta}\right)^{\beta} \cdot \frac{1}{P_x}$$

$$= \alpha \cdot \bar{u} \cdot P_x^{\alpha} \cdot \alpha^{-\alpha} \cdot \left(\frac{P_y}{\beta}\right)^{\beta} \cdot P_x^{-1}$$

$$= \bar{u} \cdot P_x^{\alpha-1} \cdot \alpha^{1-\alpha} \cdot \left(\frac{P_y}{\beta}\right)^{\beta}$$

$$= \bar{u} \cdot P_x^{-(1-\alpha)} \cdot \alpha^{\beta} \cdot \left(\frac{P_y}{\beta}\right)^{\beta}$$

$$= \bar{u} \cdot P_x^{-\beta} \cdot \alpha^{\beta} \cdot \left(\frac{P_y}{\beta}\right)^{\beta}$$

$$\boxed{x_H(P,\bar{u}) = \bar{u} \cdot \left(\frac{\alpha}{\beta} \cdot \frac{P_y}{P_x}\right)^{\beta}}$$ ■

↳ IGUAL A EXPRESSÃO (5) DA PÁGINA 19.A.

$$Y_H(P,\bar{u}) = Y(P, e(P,\bar{u}))$$

$$= \frac{\beta \cdot \bar{u} \cdot \left(\frac{P_X}{\alpha}\right)^{\alpha} \cdot \left(\frac{P_Y}{\beta}\right)^{\beta}}{P_Y}$$

$$= \beta \cdot \bar{u} \cdot \left(\frac{P_X}{\alpha}\right)^{\alpha} \cdot P_Y^{\beta} \cdot \beta^{-\beta} \cdot \frac{1}{P_Y}$$

$$= \bar{u} \cdot \left(\frac{P_X}{\alpha}\right)^{\alpha} \cdot P_Y^{\beta} \cdot \beta^{1-\beta} \cdot P_Y^{-1}$$

$$= \bar{u} \cdot \left(\frac{P_X}{\alpha}\right)^{\alpha} \cdot P_Y^{\beta-1} \cdot \beta^{\alpha}$$

$$= \bar{u} \cdot \left(\frac{P_X}{\alpha}\right)^{\alpha} \cdot P_Y^{-(1-\beta)} \cdot \beta^{\alpha}$$

$$\rightarrow Y_H(P,\bar{u}) = \bar{u} \cdot \left(\frac{P_X}{\alpha}\right)^{\alpha} \cdot P_Y^{-\alpha} \cdot \beta^{\alpha}$$

$$\boxed{= \bar{u} \cdot \left(\frac{\beta}{\alpha} \cdot \frac{P_X}{P_Y}\right)^{\alpha}} \quad ■$$

↳ IGUAL A EXPRESSÃO (4) DA PÁGINA 19.

$$X(P,w) = X_H(P, v(P,w))$$

$$= w \cdot \left(\frac{\alpha}{P_X}\right)^{\alpha} \cdot \left(\frac{\beta}{P_Y}\right)^{\beta} \cdot \left(\frac{\alpha}{\beta}\right)^{\beta} \cdot \left(\frac{P_Y}{P_X}\right)^{\beta}$$

$$= w \cdot \alpha^{\alpha} \cdot P_X^{-\alpha} \cdot \beta^{\beta} \cdot P_Y^{-\beta} \cdot \alpha^{\beta} \cdot \beta^{-\beta} \cdot P_Y^{\beta} \cdot P_X^{-\beta}$$

$$= w \cdot \alpha^{\alpha+\beta} \cdot P_X^{-\alpha-\beta} \cdot \beta^{\beta-\beta} \cdot P_Y^{-\beta+\beta}$$

$$= w \cdot \alpha^{1} \cdot P_X^{-(\alpha+\beta)} \cdot \beta^{0} \cdot P_Y^{0}$$

$$= w \cdot \alpha \cdot P_X^{-1} \cdot 1 \cdot 1$$

$$\boxed{= \frac{\alpha \cdot w}{P_X}} \quad ■$$

IGUAL A EXPRESSÃO (5) DA PÁGINA 18.A

$$Y(P,w) = Y_H(P, v(P,w))$$

$$= w \cdot \left(\frac{\alpha}{P_X}\right)^{\alpha} \cdot \left(\frac{\beta}{P_Y}\right)^{\beta} \cdot \left(\frac{\beta}{\alpha} \cdot \frac{P_X}{P_Y}\right)^{\alpha}$$

$$= w \cdot \alpha^{\alpha} \cdot P_X^{-\alpha} \cdot \beta^{\beta} \cdot P_Y^{-\beta} \cdot \beta^{\alpha} \cdot \alpha^{-\alpha} \cdot P_X^{\alpha} \cdot P_Y^{-\alpha}$$

$$= w \cdot \alpha^{\alpha-\alpha} \cdot P_X^{-\alpha+\alpha} \cdot \beta^{\beta+\alpha} \cdot P_Y^{-\beta-\alpha}$$

$$= w \cdot \alpha^{0} \cdot P_X^{0} \cdot \beta^{1} \cdot P_Y^{-(\beta+\alpha)}$$

$$= w \cdot 1 \cdot 1 \cdot \beta \cdot P_Y^{-1}$$

$$\rightarrow \boxed{Y(P,w) = \frac{\beta \cdot w}{P_Y}} \quad ■$$

↳ IGUAL A EXPRESSÃO (6) DA PÁGINA 18.6

EXERCÍCIO 9:

$$v(p, w) = \mu + (\alpha + \beta)\log w - \alpha \log p_1 - \beta \log p_2 \quad \mu = \text{CONSTANTE}$$

9-A PELA IDENTIDADE DE ROY TEM-SE QUE:

$$X_\ell(p, w) = -\frac{\dfrac{\partial v(p,w)}{\partial p_\ell}}{\dfrac{\partial v(p,w)}{\partial w}}$$

ENTÃO,

$$\frac{\partial v(p,w)}{\partial p_1} = \frac{-\alpha}{p_1 . \ln 10} \quad E \quad \frac{\partial v(p,w)}{\partial w} = \frac{\alpha + \beta}{w . \ln 10}$$

LOGO,

$$X_1(p, w) = -\frac{\dfrac{-\alpha}{p_1 . \ln 10}}{\dfrac{\alpha+\beta}{w . \ln 10}}$$

$$= -\frac{-\alpha}{p_1 . \cancel{\ln 10}} . \frac{w . \cancel{\ln 10}}{\alpha + \beta}$$

$$\boxed{= \frac{\alpha . w}{p_1 . (\alpha + \beta)}}$$

POR SUA VEZ,

$$\frac{\partial v(p,w)}{\partial p_2} = \frac{-\beta}{p_2 . \ln 10}$$

(9.7.A)

ENTÃO,

$$x_2(P,W) = -\frac{\frac{\partial v(P,W)}{\partial P_2}}{\frac{\partial v(P,W)}{\partial W}}$$

$$= -\frac{\frac{-\beta}{P_2 \cdot \ln 10}}{\frac{\alpha+\beta}{W \cdot \ln 10}}$$

$$= -\frac{-\beta}{P_2 \cdot \cancel{\ln 10}} \cdot \frac{W \cdot \cancel{\ln 10}}{\alpha+\beta}$$

$$\boxed{= \frac{\beta \cdot W}{P_2 \cdot (\alpha+\beta)}}$$

(9.B)

$$\bar{u} = u + (\alpha+\beta)\log e - \alpha \log P_1 - \beta \log P_2$$

$$\bar{u} - u = \log e^{\alpha+\beta} - (\log P_1^{\alpha} + \log P_2^{\beta})$$

$$\bar{u} - u = \log e^{\alpha+\beta} - \log P_1^{\alpha} \cdot P_2^{\beta}$$

$$\bar{u} - u = \log \frac{e^{\alpha+\beta}}{P_1^{\alpha} \cdot P_2^{\beta}}$$

$$\frac{e^{\alpha+\beta}}{P_1^{\alpha} \cdot P_2^{\beta}} = 10^{\bar{u}-u}$$

$$e^{\alpha+\beta} = 10^{\bar{u}-u} \cdot P_1^{\alpha} \cdot P_2^{\beta}$$

$$\boxed{e(P,u) = 10^{\frac{\bar{u}-u}{\alpha+\beta}} \cdot P_1^{\frac{\alpha}{\alpha+\beta}} \cdot P_2^{\frac{\beta}{\alpha+\beta}}}$$

PELO LEMA DE SHEPARD:

$$X_1^H(P,\bar{u}) = \frac{\partial e(P,\bar{u})}{\partial P_1}$$

$$= \frac{\alpha}{\alpha+\beta} \cdot 10^{\frac{\bar{u}-u}{\alpha+\beta}} \cdot P_1^{\frac{-\beta}{\alpha+\beta}} \cdot P_2^{\frac{\beta}{\alpha+\beta}}$$

$$\boxed{= \frac{\alpha}{\alpha+\beta} \cdot \left(10^{\bar{u}-u} \cdot P_1^{-\beta} \cdot P_2^{\beta}\right)^{\frac{1}{\alpha+\beta}}}$$

$$X_2^H = \frac{\partial e(P,u)}{\partial P_2}$$

$$= \frac{\beta}{\alpha+\beta} \cdot 10^{\frac{\bar{u}-u}{\alpha+\beta}} \cdot P_1^{\frac{\alpha}{\alpha+\beta}} \cdot P_2^{\frac{-\alpha}{\alpha+\beta}}$$

$$\boxed{= \frac{\beta}{\alpha+\beta} \cdot \left(10^{\bar{u}-u} \cdot P_1^{\alpha} \cdot P_2^{-\alpha}\right)^{\frac{1}{\alpha+\beta}}}$$

9.C $\lambda = \frac{\partial u}{\partial w}$

$$= (\alpha+\beta) \cdot \frac{1}{w \cdot \ln 10}$$

$$\boxed{= \frac{\alpha+\beta}{w \cdot \ln 10}}$$

UMA VEZ QUE O MULTIPLICADOR DE LAGRANGE NÃO É FUNÇÃO DOS PREÇOS IMPLICA QUE MUDANÇA NOS NÍVEL DE PREÇOS NÃO AFETARÃO A UTILIDADE MARGINAL DA RENDA.

22-A